PhD Dissertation

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Stochastic Optimization approaches for trading on financial and energy markets

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Abstract

The goal of this thesis is twofold. First, for a rather broad class of financial options a stochastic model predictive control (SMPC) approach is proposed for dynamically hedging a portfolio of underlying assets. After formulating the dynamic hedging problem as a stochastic control problem with a least-squares criterion, for plain vanilla and exotic options we test its ability to replicate the payoff at expiration date. We show not only that relatively small hedging errors are obtained in spite of price realizations, but also that the approach is robust with respect to market modeling errors. The SMPC approach is then extended to hedging derivative contracts (such as plain vanilla and exotic options) in the presence of transaction costs. After proving that the least-squares approach is no longer suitable to handle this kind of market, the hedging performance obtained by three different measures is tested and compared in simulation on a European call and a barrier option. The aim in the second part of this thesis is to present a novel market design for trading energy and regulating reserves and to introduce a strategy for the optimal bidding problem in such a scenario. In the deregulated market, the presence of several market participants or Balance Responsible Parties (BRPs) entitled for trading energy, together with the increasing integration of renewable sources and price-elastic loads, shift the focus on decentralized control and reliable forecast techniques. The main feature of the considered market design is its double-sided nature. In addition to portfolio-based supply bids and based on prediction of their stochas-

tic production and load, BRPs are allowed to submit risk-limiting requests. Requesting capacity from the AS market corresponds to giving to the market an estimate of the possible deviation from the daily production schedule resulting from the day-ahead auction and from bilateral contracts, named E-Program. In this way each BRP is responsible for the balanced and safe operation of the electric grid. On the other hand, at each Program Time Unit (PTU) BRPs must also offer their available capacity under the form of bids. In this paper, a bidding strategy to the double-sided market is described, where the risk is minimized and all the constraints are fulfilled. The algorithms devised are tested in a simulation environment and compared to the current practice, where the double-sided auction is not contemplated. Results in terms of expected imbalances and reliability are presented.

Keywords

[Stochastic, Optimization, Hedging, Bidding, Electricity Markets]

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Chapter 1

Introduction

1.1 Motivation

The origin of stock trading dates back as far as the 16th century in Belgium, where brokers and moneylenders used to meet to deal in business, government and individual debt issues, even though the only products actually traded were promissory notes and bonds. A more evolved prototype of stock exchange burst out during colonialism, when investors were eager to have part of the huge profits coming from the East Indies. They often lent money to ship owners undertaking long and dangerous sea trips to obtain dividends in return. The exchange of stocks, issued on paper without any regulation, grew so quickly and disorderly that it inevitably led to a crash, when the South Sea Company (SSC) was unable to pay off the dividends to the investors.

Since then, stock exchanges regulated by governments were formed all over the world, the most important of them being the New York Stock Exchange (NYSE). The number and variety of the stocks traded on stock markets has developed tremendously without interruptions, and more and more sophisticated financial products have been created to redistribute the risk borne by investors.

Among the uncountable types of contracts arisen in the last decades, the group of financial *derivatives*, products whose value derives from the value of other stocks traded on the market, is one of the most known and controversial. Financial derivatives have spread all over the world due to the market globalization and to the introduction of automatic pricing. They are often associated with the subprime crisis in 2008, that stemming from the mortgage industry also affected the global credit market. However, if utilized consciously and with carefulness, derivatives can be a good tool for investors who want to limit their risk.

A wide class of derivatives, the *options*, can be seen as a bet on the future evolution of some quantities observable on financial markets. The option assigns a value, called *payoff* to each realization of the observed variables, which in general are called *underlying* assets. Financial institutions selling those contracts try to replicate the evolution of the option by managing a dynamic portfolio, consisting of simpler stocks or cash, in order to ensure they will be able to pay off the real value of the option to the investor at the expiration date. The most widespread and studied options are the *plain vanilla* options: a *call* (*put*) gives the owner the right to buy (sell) a given asset (the *underlying*) on a predetermined *expiration date* at an agreed price, called *strike price*. A financial institution treating options faces two main problems:

1. determining the initial price at which to sell the contracts;
2. designing a dynamic strategy for the portfolio management, whose initial value is the price at which the option was sold, indicating how to change its composition in order to hedge against the risk.

Such strategy has to make the replicating portfolio as close as possible to the option payoff, independently of the price of the underlying.

The hedging problem can be cast as an optimal control problem that given an initial condition, namely the value of the portfolio or the option price, aims at minimizing the difference between the final portfolio value and the option payoff, eliminating the effects of price uncertainty as much as possible. In the last years a new research line has been developing, which applies well known results of the linear feedback control theory in a portfolio management context [12, 14, 36, 55–57, 64, 65]. One of the possible tools that can be borrowed from control theory for the stock trading problem is Model Predictive Control (MPC). This suboptimal method solves at each sampling time an open loop optimization problem over a finite prediction horizon, based on a model of the system to be controlled. Note that if we allow the prediction horizon to be infinite, we would obtain the real optimal control scheme for the system. After the optimization problem is solved and a sequence of control moves is obtained, only the first move is actually implemented, while the remaining moves are discarded. The optimization is repeated at the next sample time, when the actual state of the system is updated based on the latest information available on the system. One limitation of MPC is that it does not provide a specific strategy to deal with uncertainty. It assumes that the prediction model is exact and neglects possible disturbances. For this reason the stochastic version of MPC is often used: in Stochastic MPC (or SMPC), the function describing the time evolution of the system is not deterministic but stochastic, the state variables are therefore associated with a probability distribution.

Since the liberalization of energy markets, electricity can be deemed as another type of tradable asset. Traditionally, it was general belief that all the phases of the electricity supply chain, from generation to distribution, had to be controlled in a centralized way to ensure reliable and efficient operations. However, since the late nineties the liberalization of electricity markets started to be implemented in many European countries, especially after the Directive 96/92/EC, which came into force in 1997 establishing common rules for the internal market, in particular concerning the generation, transmission and distribution of electricity and separating the monopoly elements of the business from the potentially competitive segments. England and Wales were the first to implement the liberalized market [33] followed by Norway in 1991, Sweden in 1995, Finland in 1997, Spain and Germany in 1999. Of course, the introduction of liberalized markets led to higher risks for energy supply companies. An overview of the stochastic models dealing with prices risks in liberalized electricity markets is given in [59].

Although electricity can be traded in a similar way as stocks at power exchanges, there are major differences between traditional and financial markets like lacking liquidity, high volatility, non-normal distributions, market incompleteness. According to [77], wholesale markets for electricity are inherently incomplete and imperfectly competitive due to two main characteristics of electrical power; power is a flow of energy that cannot be monitored perfectly, and storing electrical energy in large quantities involves high costs. Market flaws are mostly related to inadequate design of operational rules, structural and architectural problems. The way market operators behave on the electricity market differ among the various countries, and are strongly related to the market design and regulatory framework they operate in. In general, electricity markets are divided into four categories: *forward markets*, where long term bilateral contracts are issued, *day-ahead markets*, that can be deemed as the wholesale market for energy, *intraday markets* where market operators can adjust their position by correcting/withdrawing offers and requests and finally *ancillary services* markets, where additional services like emergency power and regulating capacity are offered and bought to the grid for the real-time balance of the system. A well-designed market architecture must ensure the correct and reliable operation of the power system, as well as the fairness in prices and tariffs. Efficient architectures must discourage market participants to deviate from their scheduled power generation and consumption. To this aim, market entities with regulating functions are in place. In particular, the Transmission System Operator is responsible for the system security and must guarantee that the correct amount of reserve power is allocated for the safe operation of the system.

The energy market is going through a period of transition. On the one hand the liberalization gave everyone the opportunity for an equal and fair access to the grid; small-size producers and end-users of electricity gained importance, also thanks to new technologies like smart meters and sensors, by which they can regulate consumption and production based on the state of the network. Renewable sources also play an important role in the achievement of the ambitious targets set by the European Commission on greenhouse gases emissions and energy consumption reduction. On the other hand however, these elements form a complex scenario consisting of numerous interrelated (and often stochastic) variables and the need for new control structures and market architectures emerges. Market operators entitled to trade energy are often called Balance Responsible Parties (BRPs). While participating to the market BRPs need supporting tools in their complex decision processes, from the market level to real-time operations, in order to be able to exploit the information coming from the outside world at their best, and to hedge against risks.

1.2 Goal and purpose

The goal of this thesis is to present optimization-based tools for trading on financial and energy markets. More precisely, the first part focuses on Stochastic MPC applied to dynamic hedging of derivatives with and without transaction costs, while the second part is related to bidding strategies on double-sided energy markets, i.e. markets where transactions are bidirectional (energy can be bought or sold by everyone).

Analogies and differences between the energy markets and the standard tradable products like stocks and options are investigated, and appropriate techniques are proposed and discussed to tackle these two different families of assets with particular focus on risk management.

1.3 Structure of the thesis

The thesis is divided into two main parts. The topic of the first part is hedging of financial derivatives on stock markets. Chapters 2 and 3 are introductive chapters on the main mathematical tools and risk measures used in this thesis. More specifically, in Chapter 2 the concept of risk is introduced and the main financial risk measures are defined, while Chapter 3 contains a short overview of Model Predictive Control (MPC) techniques and its stochastic variant SMPC. Chapter 4 is the central section of this first part, and focuses on dynamic option hedging. The problem is tackled with SMPC techniques based on the minimum variance criterion (in case of frictionless markets) and on three different objective functions when transaction costs are applied.

The second part of the thesis deals with trading strategies in the liberalized electricity market, focusing on the market behavior of BRPs (balance responsible parties). In particular, Chapter 5 describes the main tasks tackled by energy market operators in current market designs, with a particular focus on the Dutch situation, and presents a new architecture for double-sided ancillary services markets devised in the European project E-Price[1]. In Chapter 6 a strategy to bid on this kind of market is proposed, based on the minimization of a risk function. Mathematical modeling and formulations of the bidding problems are given, and a case study related

[1]E-Price: Price- based Control of Electrical Power Systems.

to the project simulation environment is analyzed. Finally, in Chapter 7 some conclusions are presented.

1.4 Publications

Chapter 4 is based on the following publications:

- A. Bemporad, T. Gabbriellini, L. Puglia, L. Bellucci, *Scenario-based Stochastic Model Predictive Control for Dynamic Option Hedging*, in Proc. 49-th Conference on Decision and Control, Atlanta, GA, 2010
- A. Bemporad, L. Puglia, *A Stochastic Model Predictive Control Approach to Dynamic Option Hedging with Transaction Costs*, in Proc. American Control Conference, San Francisco, CA, 2011

A journal version of the last conference paper has been submitted for review:

- A. Bemporad, L. Puglia, T. Gabbriellini, *Stochastic Model Predictive Control for Dynamic Option Hedging with Transaction Costs*, Applied Mathematical Sciences, submitted for review on October 25, 2012

Chapter 5 is largely based on the following publication:

- A. Jokic, P.P.J. van den Bosch, A. Virag, W.H.A. Hendrix, L. Puglia, W. de Boer, R. Vujanic, F. Nobel, *Reliability and Efficiency at Global Level in Power Systems*, in Proc. 10th International Conference on the European Energy Markets, Stockholm, Sweden, 2013

Chapter 6 is based on the conference paper:

- L.Puglia, A. Jokic, A. Virag. A. Bemporad, *Double-sided ancillary services markets: design and optimal bidding strategies*, in Proc. 10-th International Conference on the European Energy Markets, Stockholm, Sweden, 2013

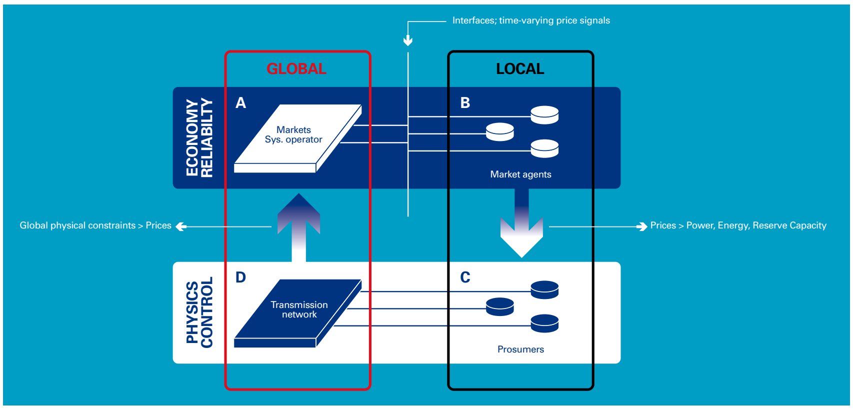


Figure 1.1: The E-Price concept

1.5 The E-Price project

The research presented in the second part of this thesis has been carried out within the framework of the European project "E-Price - Price-based Control of Electrical Power systems", FP7-IST contract no. 249096.

The main motivation of E-Price stems from the technological and societal developments arisen over recent decades encouraging the use of an increasing quantity of renewable energy sources (wind, solar) for the production of electric energy. People are starting to generate their own energy, becoming producers themselves. At the same time the predictability of both production and consumption of electric energy is decreasing, introducing larger imbalances in the electricity network. The current energy production system inadequately copes with this unpredictability and will soon reach the limit for secure and reliable operation.

E-Price is a three-year European research project aiming to develop a reliable, an efficient and a societally-acceptable control concept for the EU energy market. E-Price sets a new standard by introducing a feasible price-based control strategy. Four academic and five industrial partners are involved in the project.

The aim of E-Price is to offer an integral solution as the standard framework for trade in electrical energy, satisfying European Union policy goals. This will bring about a market and control concept that gives incentives to all participants to follow their own interests while still satisfying the societal requirements on reliability, efficiency and transparency.

The project proposes the price-based control approach as a coherent methodological framework to ensure the feasibility, the reliability and the efficiency of the future European power system, anticipate and support market based operation and decentralized decision making. It is enabled by state-of-the-art ICT technologies and by utilizing (beyond-) state-of-the-art decentralized and distributed control systems theory and modern optimization techniques.

In the E-Price project we propose a systematic scientific approach to formulate ICT and control requirements and solutions for price-based control of future power systems. In the heart of our approach are modeling, analysis and synthesis of the interplays between:

- the interconnected physical power system (C and D in figure 1.1), with time varying power requirements as prominent signals; and the economical layer (A and B in figure 1.1) with time varying price signals as the prominent information carriers.

- Local objectives of producers/consumers (prosumers) (B and C in figure 1.1) and global balance, transmission network limits and reliability constraints (A and D in figure 1.1).

The research reported in this thesis deals with the interconnection between A and B in figure 1.1, that is, with the economical optimization of market agents behavior.

1.6 Basic notation

Let $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{Z}$ and $\mathbb{Z}_+$ respectively denote the field of real numbers, nonnegative real numbers, integers and nonnegative integers. For a matrix $A \in \mathbb{R}^{m\times n}$, $[A_{ij}]$ denotes the element in the i-th row and j-th column. The transpose of a matrix A is denoted by A'. Positive and semi-positive definiteness of a matrix A are denoted respectively by $A \succ 0$ and $A \succeq 0$. We use I_n and $\mathbb{1}_n$ to indicate the identity matrix of dimension $n \times n$ and a column vector of n ones. The floor function $\lfloor\cdot\rfloor$ of $a \in \mathbb{R}$ is used to define the largest $b \in \mathbb{Z}$ such that $a \geq b$. The operators $[f(x)]^+ = \max\{0, f(x)\}$ and $[f(x)]^- = \min\{0, f(x)\}$ for a function $f : \mathbb{R}^n \to \mathbb{R}$ define respectively the positive and negative part of a function. Finally, we denote by $E[X]$ the expected value of a random variable X and by $\mathrm{Var}(X)$ its variance. We use $E[X|Y]$ and $\mathrm{Var}(X|Y)$ to define the conditional expectation and variance of X given $Y = y$.

Chapter 2

Risk management in financial and energy markets

Risk management is the identification, assessment and prioritization of risk, followed by appropriate control measures aimed to eliminate, or more often reduce, the probability or impact of unfortunate events. The risks that a firm can encounter can originate from a multitude of causes, for example the uncertainty of financial markets, credit risks, project failures, natural disasters or unknown and unforeseeable events. The strategies used to counteract risk typically include transferring the risk to another party, or avoiding the threat, reducing the possible impact of a negative event or even accepting the risk to some reasonable extent.

The most important types of risk that companies face on standard financial markets are:

- **Credit risk**: the risk that a borrower goes into default and is therefore not able to pay an obligation;
- **Liquidity risk**: the risk that a good or stock cannot be traded quickly enough to prevent a loss;
- **Market risk**: the risk that some quantity on the market, such as stock prices, interest rates or commodity prices will change;

- **Operational risk**: is the risk associated with human errors, system failures and inadequate procedures and resulting in unexpected losses;

Moreover, there are other types of risk more specific to the trading of electricity on the market, such as:

- **Volume risk**: a type of risk frequently faced by firms operating on electricity markets, related to the fact that they often do not know the exact amount of energy to be delivered or produced at a given time instant;

- **Basis risk**: the risk that the ratio between the prices of two traded commodities, for example fuel and electricity, will change;

- **Physical risk**: the risk that electricity is not delivered at the contracted time and location due to various problems affecting the grid connections, like transmission lines overflows or outages;

- **Regulatory and political risk**: changes in the regulatory and political framework can affect seriously the trading activity of a firm.

2.1 Risk measures

When performing risk management, the first step is to quantify the risk. Therefore, the need for an appropriate and coherent risk measure emerges. A first empirical approach to simply assess the effect of uncertainty is the *stress test*. One or more possible realizations of the uncertain variables (for example, a 30% shift in the fuel price) are hypothesized and the effect of such events are measured. The advantage of this approach is its simplicity, but one cannot exclude a more reliable and strict mathematical assessment when performing a thorough risk management activity. In the following sections the most common measures for risk are examined.

2.1.1 Variance

The most traditional risk measure is *variance*, quantifying the dispersion of the uncertain variables. Minimizing variance is equivalent to minimize the portfolio dispersion around the expected value. The standard mean-variance Markovitz problem [54] is the following:

$$\min_x \quad x^T \Sigma x \tag{2.1}$$
$$\text{s.t.} \quad Ax = r \tag{2.2}$$

where $\Sigma \in \mathbb{R}^{n \times n}$ is the covariance matrix in a portfolio of n assets, $r \in \mathbb{R}^n$ is a vector of expected returns and $x \in \mathbb{R}^n$ is the position in each asset composing the portfolio. The aim is to minimize the variance of a portfolio, with the requirement that a minimum expected return is obtained. By construction, Σ is a positive definite matrix and the linear constraint (2.2) defines a convex set. Therefore, a solution to the Markowitz problem exists and it is unique.

In the mean-variance approach if the investor has a quadratic objective function regardless of the probability distribution of the underlying assets only the first two moments are relevant. This strategy is therefore not well suited for the case of very skewed and asymmetric distributions like the ones characterizing energy markets.

2.1.2 Value at Risk

In the late eighties, partly triggered by the stock market crisis of 1987, a new downside-risk measure was introduced, namely *Value at Risk* (VaR). This measure was thought as an attempt to quantify the risk of extreme events from measurements of everyday price movements.

Let $f(u, s) : \mathbb{R}^{n+k} \to \mathbb{R}$ be a loss function associated with the decision vector $u \in \mathbb{R}^n$ and with the random vector $s \in \mathbb{R}^k$. Let $p(s)$ be the probability density function of s. With respect to a given probability β, $0 \leq \beta \leq 1$, the β-VaR (Value at Risk) is defined as follows.

Definition 2.1.1 *VaR with confidence level β of the loss associated with a decision variable u is the value*

$$\ell_\beta = \inf\{\ell \in \mathbb{R} : f(u, \ell) \geq \beta\} \tag{2.3}$$

In other words, VaR is the lowest value ℓ_β, such that, with probability β, the loss will not exceed ℓ_β. The number β is a fixed value, typically $\beta = 90\%$, 95%, or 99%. The main criticism moved against VaR is that the amount of loss occurring with probability $(1-\beta)$ is not taken into account directly. VaR is not capable to differentiate between large and very large losses, and moreover it lacks sub-additivity, that is, VaR of the combination of two portfolios can be higher than the sum of the risks of the individual portfolios, thus contradicting the diversification principle. To avoid these inconveniences, another asymmetric risk measure strictly related to VaR, β-CVaR, was introduced.

2.1.3 Conditional Value at Risk

Conditional Value at Risk β-CVaR is defined as follows.

Definition 2.1.2 *CVaR with confidence level β of the loss associated with a decision variable u is the value*

$$\phi_\beta(u) = (1-\beta)^{-1} \int_{f(u,s) \geq \ell_\beta} f(u, s) p(s) ds \tag{2.4}$$

Conditional Value at Risk is the conditional expectation of the loss function above ℓ_β, quantifying what the *average loss* is when one loses *more than* ℓ_β, with probability $1-\beta$ [71]. In [71] the authors show that the β-CVaR of the loss associated with any u can be determined by the formula

$$\phi_\beta(u) = \min_{\ell \in \mathbb{R}} F_\beta(u, \ell) \tag{2.5}$$

where

$$F_\beta(u, \ell) = \ell + (1 - \beta)^{-1} \int_{s \in \mathbb{R}^m} [f(u, s) - \ell]^+ p(s) ds \tag{2.6}$$

where $[\cdot]^+$ denotes the positive part of its argument, $[f]^+ = \max\{f, 0\}$. The integral in (2.6) can be approximated by sampling the probability distribution of s according to its probability density function $p(s)$. If $[s_1, \ldots, s_q]$ is a sample vector of the random variable s, then the corresponding approximation of $F_\beta(\ell)$ is

$$\tilde{F}_\beta(\ell) = \ell + \frac{1}{q(1 - \beta)} \sum_{k=1}^{q} [f(u, s_k) - \ell]^+ \tag{2.7}$$

The expression $\tilde{F}_\beta(\ell)$ is convex and piecewise linear with respect to ℓ and it can readily be minimized.

Chapter 3

Model Predictive Control

Predictive control techniques were born around 1970 and they have the scope of determining the control signal to feed into a system, minimizing a given objective function. The acronym MPC encloses the three main elements of predictive control:

- **Model**: a mathematical model of the system is needed to predict future evolutions of the system state and output. The output measured at the current instant depends on past values of the output and state, besides the control action (causality).
- **Predictive**: optimization is based on the prediction of the future evolution of the system.
- **Control**: the goal is to find a control law of a complex system.

The common strategy used to build a predictive controller is the following:

1. a time horizon N is fixed, then future values of the system output over that horizon are predicted using the model at disposal: $y(t+k|t), k = 1, \ldots, N$. The value of y depends on past values of input and output and on future input values: $u(t+k|t), k = 0, \ldots, N-1$.

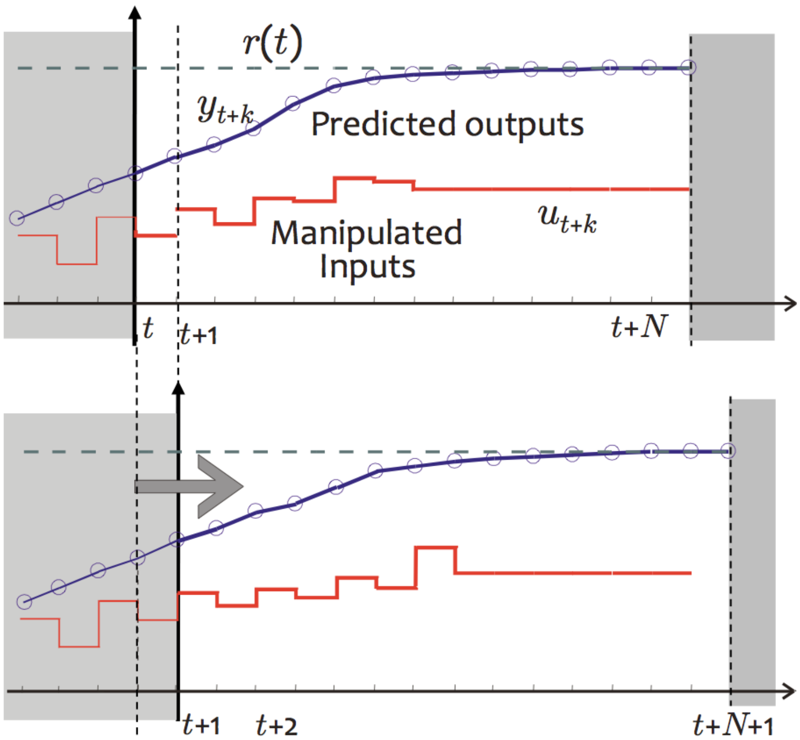


Figure 3.1: The receding horizon philosophy

2. The set of future control signals is evaluated optimizing a performance index in order to keep the process as close as possible to a given reference signal $r(t+k)$.

3. The control signal $u(t|t)$ is fed into the process, the remaining control signals $u(t+k|t), k = 1, \ldots, N-1$ are discarded. This concept is called *receding horizon philosophy*: the signal $u(t+1|t)$ is discarded because at the next sampling instant the output value $y(t+1)$ is known, the new information coming from updated measurements can be used to start again from step 1 and repeat the optimization.

The described control strategy can be schematized by Figure 3.1

If the model is linear, the optimization problem is quadratic (QP) if the objective function is in l_2 form, it is otherwise linear if the objective function is in l_1 or l_∞. For example, if we chose a quadratic objective function, the MPC controller should solve at each time instant a problem like this over an N time horizon:

$$\min_{u_t \dots u_{t+N-1}} \sum_{k=0}^{N-1} |y_{t+k} - r(t)|^2 + \rho\, |u_{t+k} - u_r(t)|^2 \tag{3.1a}$$

$$\text{s.t. } x_{t+k+1} = f(x_{t+k}, u_{t+k}) \tag{3.1b}$$

$$y_{t+k} = g(x_{t+k}, u_{t+k}) \tag{3.1c}$$

$$u_{min} \leq u_{t+k} \leq u_{max} \tag{3.1d}$$

$$y_{min} \leq y_{t+k} \leq y_{max} \tag{3.1e}$$

$$x_t = x(t), k = 0, \dots, N-1 \tag{3.1f}$$

where (3.1a) describes the objective function to be minimized, (3.1b) represents the dynamic evolution of the system, (3.1c) indicate the system output and (3.1d)-(3.1e) are input and output constraints.

3.1 Stochastic MPC

The classical MPC approach does not provide a real strategy to deal with uncertainties; it assumes that the model is exact and perfectly representing the reality. Robustness with respect to modeling errors or external disturbances can be handled with a min-max approach, where the performance index is calculated over the worst case scenario. However, controllers operating at nominal conditions often obtain poor performance, while robust approaches result in too conservative control laws [70].

The stochastic version of MPC, Stochastic MPC (SMPC), developed recently, exploiting the statistical information about system disturbances, aimed to minimize the expected value of the performance index. SMPC formulation is based on a maximum likelihood approach, where a scenario tree is built at each time step, using all the available information on the state of the system. Each node of the tree represents a scenario, appropriately weighted in the optimization problem. Starting from the root node, a series of candidate nodes are generated, each node corresponding to a possible realization of the prediction model. A given stochastic optimization index is minimized, for example based on the expected value or variance, then a series of optimal moves are obtained, starting from the current instant up to the prediction horizon. Exploiting the receding horizon philosophy, only the first move is applied, the state is updated at the following step and the optimization is repeated.

Chapter 4

Hedging of financial derivatives

This chapter is structured as follows. In section 4.1 the main derivatives traded on financial markets are presented and described, with particular focus on options. Section 4.2 contains an overview of the literature on option hedging and trading. In section 4.3 the mathematical models describing the evolution of the assets over time and the option payoffs are formulated. In section 4.4 the hedging problem will be formulated as a stochastic optimization problem and solved by means of MPC techniques. Simulation results are given with respect to simple plain vanillas and Napoleon/barrier options. In section 4.5 the hedging problem is extended to the case of transaction costs. Three alternative stochastic formulations of the problem are given and simulation results are reported as well. Finally, some conclusions are drawn in section 4.6.

4.1 Derivatives

Derivatives are financial instruments that are linked to other financial instruments or indicators or commodities, and through which financial risks can be traded ([41]). The value of a financial derivative derives from the price of an underlying item, such as an asset or index. Unlike debt instruments, little or none capital investment is required, and they are regulated at a future date. Generally speaking, derivatives can be used for three main reasons:

- *speculation*: consists of buying or selling a product with the aim to obtain short-term profits, exploiting one's vision of the market trend;
- *arbitrage*: consists of settling a risk-free profit by simultaneously entering opposite positions in two different markets exploiting their imperfect nature;
- *hedging*: a strategy by which firms try to decrease the risk linked to other financial activities, like price fluctuations of other assets.

Various types of derivatives exist and new arise every day and with any kind of underlying, such as weather conditions or raw materials prices. In relation to the type of contract, derivatives might be:

- *forwards and futures*: fixed term contracts by which firms exchange an asset (goods, financial instruments, indices or foreign currencies) on a future date at a fixed price;
- *swap*: contracts by which firms commit themselves to exchange cash flows according to a specified scheme;
- *options*: contracts giving the owner (holder) the right to buy (*call* option) or sell (*put* option) the underlying asset at a determined price (*strike* price) on a given date (*European option*) or within a certain date (*American option*). The fixed date is called *expiration date* or expiry.

Derivatives can be traded either on regulated markets, with standard rules concerning prices and conditions, or by specifying ad hoc terms between the two parties, in that case we talk about *Over The Counter* transactions (OTC).

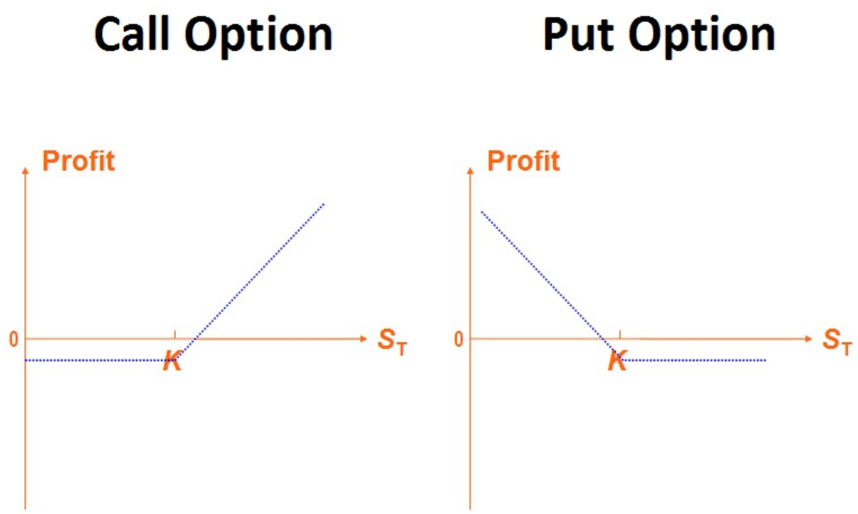


Figure 4.1: Payoff as a function of the underlying price at expiration for *plain vanilla* options

The derivatives treated in this thesis are the options. The main difference distinguishing options from the other derivatives is that they do not state any obligation for the owner (or holder) of the contract, who can decide whether to exercise her right or not based on the market conditions. The person selling the option (writer) has instead the obligation to deliver the contracted asset on time, if required by the holder.

4.1.1 *Plain Vanilla* options

The simplest and most widespread options in circulation are called *plain vanilla*. A *call* option gives the right to buy a given asset at an agreed price. Generally one buys a call option if she expects the price of the underlying to go higher than the strike price, while who writes the option usually expects the price to decrease below the strike price. A call option is exercised if the underlying price exceeds the strike price. Indicating by s the stock price and by K the strike price, the value of the option at the expiration date T is $\max\{s(T) - K, 0\}$. This value is called *payoff*.

A put option gives the holder the right to sell a given asset at a predetermined date. She is hoping that the price of the underlying will decrease below the strike price. The payoff of a put option is given by $\max\{K - s(T), 0\}$. Payoff function in relation to the final stock price for *plain vanilla* options are shown in Figure 4.1. The value of the option at a generic instant of its life t is called *intrinsic value*. The right to exercise the option has a cost, called *premium*. The holder of the option is said to be in a *long* positions, while the writer is said to be in a *short* position. Options whose strike price is close to the underlying price are called *At-the-money*, the ones whose intrinsic value is positive are called *In-the-money*, the ones with negative intrinsic value are defined *Out-of-the-money*.

A variant of the plain vanilla options are the *american options*. The holder of such an option can exercise her right at any time during the option life. Finally, *bermudan options* provide the holder with a restricted set of possible dates when the option can be exercised. Determining the price of this kind of options is more difficult than for the *plain vanilla* ones.

4.1.2 Exotic options

The contracts described in the previous section are the simplest that can be found on the market. Besides these, other types of options with much more complicated payoffs are present, often showing a dependency on past values of the underlying and not only at the exercise date; these options are called *exotic*. If the option value depends on past values of the underlying it is said to be *path-dependent*. Path dependency can be *strong* or *weak*. Contracts with a strong path dependency have a payoff based on some feature of the option underlying in the past, thus implying the impossibility to derive the option payoff only based on the present value of the underlying and time instant. For example, *asian options* depend on the mean value of the underlying prior to expiry. This requires the introduction of an additional variable to describe the state of the system. On the contrary, weak path dependency does not require an additional variable to describe the option payoff. The simplest options showing this kind of dependency are the *barrier options*, that are activated by the underlying achieving a predefined boundary. The actual time instant and the underlying value are still sufficient to describe the state of the system. Let us now briefly examine the most common exotic options.

Barrier options

The payoff of these option is the same as *plain vanilla* options, conditioned to the fact that the underlying has reached or not a given limit, called *barrier*. More specifically, barrier options are defined as follows:

- *knock-in*: a payoff is obtained only if the barrier is reached by the underlying. They are divided into:
  - *Down-and-In*: the barrier is below the current value of the asset, payoff is triggered at the achievement of this lower limit;
  - *Up-and-In*: the barrier is above the current value of the asset, payoff is triggered at the achievement of this upper limit.
- *knock-out*: pay a given payoff only if the barrier is not reached. They can be split into:
  - *Down-and-Out*: the barrier is below the underlying value, the option looses its value if this lower limit is reached;
  - *Up-and-Out*: the barrier is above the underlying value, the option looses its value if this upper limit is reached.

For example, an *Up-and-Out* call with strike 100 € and barrier 120 € gives the holder the same payoff of a call option, only if during the option life the underlying does not reach the barrier.

Asian options

Asian options have a payoff depending on the mean value of an underlying asset over a given time period. As already mentioned, they are strongly path dependent, since their value depends on the whole time path and not only on the current state. Arithmetic or geometric mean can be used to determine the current option payoff, and the mean can be calculated either continuously, so considering all the values taken by the underlying, or more simply by sampling the underlying realization at discrete time instants.

Lookback options

The payoff of this kind of options depends on the maximum and minimum observed value of the underlying over a fixed period. For example a look-back option can pay the holder with the difference between the maximum and minimum value of the underlying. Also in this case maximum and minimum value can be referred to the whole path or to a sample path.

Napoleon cliquet

The payoff of the Napoleon clique option is given by:

$$N + \max\left\{0, C + \min_{i\in\{1,\dots,N_{fix}\}} \frac{x(t_i) - x(t_i - 1)}{x(t_i - 1)}\right\} \tag{4.1}$$

where N stands for *nominal*, C is a given percentage value called *base coupon* and t_i, $i = 1, \dots, N_{fix}$ are *fixing dates*, that is, the set of dates when the underlying is checked. In other words, the napoleon clique option pays a percentage of the nominal value of the option (like an obligation), plus a quantity given by the sum of the *base coupon* and the minimum return o the title, if this quantity is positive.

4.2 State of the art

For a financial institution, hedging a derivative contract implies maintaining a self-financing portfolio of underlying assets, whose quantities need to be readjusted periodically so that at the expiration date of the contract the value of the portfolio is as close as possible to the payoff value to be paid to the customer.

The most common approach used in practice to dynamically rebalancing the portfolio replicating the option is *delta hedging*, which directly derives from the fundamental theory of Black and Scholes [9], according to which the portfolio includes a quantity of stocks equal to the derivative of the option price with respect to the price of the underlying stock. Delta hedging makes the portfolio insensitive to the indeterministic evolution of the stock price, under a series of (often unrealistic) hypotheses including continuous hedging, static volatility, and the *absence of transaction costs*. When applied in a real market context, such assumptions may lead to intolerable hedging errors.

The seminal works [9, 58] and their extensions to models with stochastic volatility [38] aim at perfect hedging by eliminating the risk at each time instant through a proper rebalancing of assets in the portfolio, usually continuously in time. *Simulation* is another method often used by investment firms to price options [11, 39]. A (large) set of scenarios for the future prices of the underlying assets is generated by Monte Carlo simulation; the final value of the asset price of each scenario is used to compute the payoff value; the average of such payoff values, discounted by the interest rate, provides the option price. In view of such a current practice for option pricing, in this thesis we focus our attention only on the hedging problem.

Approaches that instead look at the entire life of the option aim at minimizing risk at expiration date. The problem can be cast as a stochastic optimal control problem and rely on the Hamilton-Jacobi-Bellman partial differential equation. This category includes *multi-stage stochastic programming* approaches, in which the pricing and hedging problem is solved as a stochastic linear programming problem [31, 47, 79]. The approach is often limited by numerical reasons. In fact, the number of nodes in the tree is exponential in the number of trading periods, which typically limits the number of branches at each node to two or three. *Stochastic dynamic programming* approaches [8, 24] also discretize the probability space and solve the pricing and hedging problem backwards in time. While the method is appealing, its main limitation is due to numerical explosion when the number of trading periods is large and several assets are traded.

This thesis attacks the hedging problem from a feedback control viewpoint and proposes stochastic model predictive control (SMPC) ideas [18, 60, 66] to design a dynamic hedging strategy. SMPC can be seen as a suboptimal way of solving a stochastic multi-stage dynamic programming problem: rather than solving the problem for the whole option-life horizon, a smaller problem is solved repeatedly from the current time-step t up to a certain number N of time steps in the future by suitably remapping the condition at the expiration date into a value at time $t + N$. SMPC has been proposed recently in financial applications, such as in [37] for portfolio optimization, and in [3, 57] for option pricing and hedging. Other approaches that look at automatic trading as a feedback control problem were proposed in [2, 13].

In [64] dynamic hedging under transaction costs is performed from a SMPC point of-view for a plain vanilla option. A finite horizon constrained stochastic control problem is formulated and iteratively solved at each trading date by employing a semi-definite programming algorithm.

The contributions [25] and [31] proposed analytic methods based on stochastic optimization to handle transaction costs. In [25] the option price and the optimal trading strategy are jointly determined that reduce the total risk of writing the option. In [31] a trinomial process is used for generating the scenarios required to setup a stochastic control problem, in which the objective function is the expected value of a given performance index.

In [65] the hedging problem is formulated as a linear quadratic regulation (LQR) problem with constraints and two methods are proposed to cope with transaction costs. One involves penalizing transaction costs in the objective function, so that the problem can be solved as an unconstrained linear quadratic problem; the second method uses a model predictive control approach to solve a quadratic program over a specified horizon, exploiting the LQR solution from the first approach in the cost function.

In this thesis we propose a novel SMPC approach to dynamic option hedging based on a minimum variance criterion that requires a simple *least-squares optimization* to evaluate the optimal trading moves, by extending results proposed in [5]. To be able to handle very general stock price models and exotic payoffs, for which no analytic hedging policy exist, a pricing engine is used on-line to generate a finite number of future scenarios of option prices, rather than analytically deriving expected values from pricing models as in [37, 64]. To evaluate each option price, the pricing engine employs either Monte Carlo simulation (*on-line* computations), or *off-line* function approximation to approximate the option value as a function of the state of the market (such as the price of the underlying stock), so that on-line evaluation is very fast. We will then extend the SMPC approach to option hedging to handle proportional transaction costs. After showing that the minimum variance criterion is inadequate to handle transaction costs, we propose three new different approaches to solve the dynamic hedging problem via SMPC. The first is based on the scalarization of the multi-objective problem of minimizing both the variance and the expected value of the hedging error; the second minimizes the Conditional Value at Risk (CVaR) introduced in Chapter 2; the third is based on the minimization of the maximum hedging error over the set of scenarios considered in the stochastic optimization problem solved by the SMPC algorithm. The three approaches lead to, respectively, a quadratic programming (QP), a linear programming (LP), and a (smaller) LP problem to be solved at each trading date. The three SMPC formulations are tested and compared, among them and to delta hedging, on both plain vanilla and barrier exotic options.

4.3 Model formulation

Consider the problem of hedging an option $\mathcal{O}$ defined over n underlying assets, whose spot prices at time τ are $s_i(\tau)$, $i = 1, \dots, n$, satisfying the stochastic differential equations in the real-world probability measure

$$ds_i(\tau) = \mu_i^s(s_i(\tau), y_i(\tau))d\tau + \sigma_i^s(s(\tau), y(\tau))dz_i^s \tag{4.2a}$$

$$dy_i(\tau) = \mu_i^y(y_i(\tau))d\tau + \sigma_i^y(y(\tau))dz_i^y \tag{4.2b}$$

where $z_i^s(\tau), z_i^y(\tau)$ are Wiener processes, namely dz_i^s, dz_i^y are correlated Gaussian variables with zero mean and variance $d\tau$. In (4.2) we assume $s_i \geq 0$, $\forall i = 1, \dots, n$, $\forall \tau \geq 0$. Model (4.2) is a rather general form that covers several popular models, including the log-normal stock price model

$$ds_i(\tau) = (\mu d\tau + \sigma dz_i)s_i(\tau) \tag{4.3}$$

where $z_i(\tau)$ is a Wiener process, with zero mean and variance $d\tau$. More general models can be used to describe price dynamics, such as Heston's model [38]:

$$
\begin{aligned}
ds_i(\tau) &= (\mu_i^s d\tau + \sqrt{y_i(\tau)} dz_i^s) s_i(\tau) && (4.4a)\\
dy_i(\tau) &= \theta_i(k_i - y_i(\tau)) d\tau + \omega_i \sqrt{y_i(\tau)} dz_i^y && (4.4b)
\end{aligned}
$$

where (4.4b) is the Cox-Ingersoll-Ross process [19] for the variance $y_i(\tau)$, and dz_i^s has correlation ρ_i with dz_i^y.

In this thesis, we focus on the log-normal model (4.3), whose discrete-time equivalent form is

$$s_i(t+1) = e^{(\mu - \frac{1}{2}\sigma^2)\Delta_T + \sigma\sqrt{\Delta_T} z_i(t)} s_i(t) \tag{4.5}$$

where t denotes the trading instant, $t = 0, 1, \ldots$, and Δ_T is the time interval between two consecutive trading dates. We denote by $s(t) = [s_1(t) \ \ldots \ s_n(t)]' \in \mathbb{R}^n$ the overall vector of asset prices.

4.3.1 Option price and payoff function

We assume that the portfolio associated with option $\mathcal{O}$ is updated every Δ_T units of time, and denote by T the maturity of $\mathcal{O}$ expressed in terms of number of sampling steps. The payoff $p(T)$ of $\mathcal{O}$ is described by a function $\mathcal{P}$:

$$p(T) = \mathcal{P}(m(T)) \tag{4.6}$$

of the state $m(T)$ of the considered asset market at expiration date, for example $m(T) = x(T)$. We denote by $p(t)$ the price of the hedged option at a generic intermediate time $t\Delta_T$,

$$p(t) = (1+r)^{t-N} \tilde{E}\left[\mathcal{P}(m(T)) | m(t)\right] \tag{4.7}$$

where $m(T)$ is the state of the market at time t and $\mathcal{P}(m(T))$ is the expected value of the payoff in the risk-neutral measure, given the market at time t. In (4.7) $r = e^{r_a \Delta T} - 1$ is the return of the risk free investment over Δ_T, and r_a is the annualized continuously compounded interest rate, which we assume to be constant (Equation (4.7) can be also restated recursively as $p(t) = (1+r)^{-1}\tilde{E}[p(t+1)|m(t)]$). For instance, for a European call option on a single stock s with strike price K, we have

$$p(T) = \max\{s(T) - K, 0\} \tag{4.8}$$

$m(t) = \{s(t), y(t)\}$, and $p(t) = e^{-r(N-t)}\tilde{E}\left[\max\{s(T) - K, 0\}|s(t), y(t)\right]$. In particular, for log-normal price models, $m(t) = s(t)$. For "Napoleon cliquet" path-dependent exotic options

$$p(T) = \max\left\{0, C + \min_{i \in \{1,\ldots,N_{\text{fix}}\}} \frac{s(t_i) - s(t_{i-1})}{s(t_{i-1})}\right\} \tag{4.9}$$

where t_i, $i = 1, \ldots, N_{\text{fix}}$ are the fixing dates, and C is a fixed value. In this case $m(t) = \{s(t_0), \ldots, s(t_k), s(t), y(t)\}$, where k is the fixing index such that $t_k \leq t < t_{k+1}$. For *weak* path-dependent "Barrier" exotic options

$$\begin{aligned} p(T) &= \begin{cases} \max(s(T) - K, 0) & \text{if } s(t) < s_u, \ \forall t \leq T \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} \max(s(T) - K, 0) & \text{if } s_\ell(t) = 0 \\ 0 & \text{if } s_\ell(t) = 1 \end{cases} \end{aligned} \tag{4.10}$$

where s_u define the upper barrier level, and $s_\ell(t) \in \{0, 1\}$ is a logic state with dynamics $s_\ell(t+1) = s_\ell(t)$ OR $[s(t) \geq s_u]$, $s_\ell(0) = 0$. In this case $m(t) = \{s(t), s_\ell(t), y(t)\}$.

4.3.2 Portfolio dynamics

Assume that there are no transaction costs, and that the standard *self-financing* condition holds, i.e., that the wealth $w(t)$ of the portfolio replicating option $\mathcal{O}$ is always totally reinvested. Then, the dynamics of the wealth $w(t)$ of the portfolio is

$$w(t+1) = (1+r)w(t) + \sum_{i=1}^{n} b_i(t)u_i(t) \tag{4.11}$$

where $u_i(t)$ is the quantity of asset i held at time t and $b_i(t) \triangleq s_i(t+1) - (1+r)s_i(t)$ is the excess return, i.e., how much the asset gains (or loses) with respect to the risk-free rate. The initial condition $w(0)$ is set equal to the price paid by the customer to purchase option $\mathcal{O}$, $w(0) = (1+r)^{-N}\tilde{E}[p(T)|m(0)]$.

4.4 Stochastic MPC formulation

Dynamic hedging aims at making the final wealth $w(T)$ as close as possible to $p(T)$ for all possible market realizations. The hedging problem can be restated as a stochastic control problem, where the wealth $w(t) \in \mathbb{R}$ represents the state and output of the regulated process, the traded asset quantities $u(t) \in \mathbb{R}^n$ are the inputs, the option price $p(t)$ the target reference for $w(t)$. By defining the tracking error $e(t) \triangleq w(t) - p(t)$, the objective can be restated as the one of minimizing $e(t)$ for all possible asset price realizations. This can be achieved by minimizing the variance of the hedging error.

$$J(e(T)) = E\left[(e(T) - E[e(T)])^2\right] \tag{4.12}$$

by solving the one-step ahead minimum-variance problem

$$\begin{aligned}
\min_{\{u(t)\}} \quad & \mathrm{Var}_{m_{t+1}}\left[w(t+1, m_{t+1}) - p(t+1, m_{t+1})\right] && \text{(4.13a)} \\
\text{s.t.} \quad & w(t+1, m_{t+1}) = (1+r)w(t) \\
& + \sum_{i=0}^{n} b_i(t, m_{t+1})u_i(t) && \text{(4.13b)}
\end{aligned}$$

with respect to the portfolio composition $u(t)$ at each trading date $t\Delta_T$. Note that expectations and variances are conditioned to the particular market realization m_t at time t; we omit here the conditional notation for simplicity. Since now on we will use the notation $w(t+1)$ as a shortcut for the future wealth $w(t+1, m_{t+1})$. The formulation in (4.13) is equivalent to a stochastic model predictive control approach with prediction horizon $N = 1$, under the terminal condition of perfect hedging between prediction time $t+N$ and expiration date T. Problem (4.13) can be solved by enumerating a number M of scenarios, each one corresponding to a different realization of a certain sequence of prices, and optimize the resulting sample variance. Each scenario j has probability π_j of occurring, $j = 1, \dots, M$, $\pi_j > 0$, $\pi_j \leq 1$, $\sum_{k=1}^{M} \pi_j = 1$. Scenarios can be generated via Monte Carlo simulation [5], where $\pi^j = \frac{1}{M}$, or by discretizing a given probability density function that describes the disturbance process $z_i(t)$ [6]. Assuming that $z_i(t)$ follows a Gaussian normal distribution $\pi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$, for a fixed set of limit values $\bar{z}_j$, $j = 0, \dots, M$, $\bar{z}_0 = -\infty$, $\bar{z}_M = +\infty$, we obtain:

$$\pi^j = \int_{\bar{z}_j}^{\bar{z}_{j+1}} \pi(z)dz = \frac{1}{2}\left(\text{erf}\left(\frac{\bar{z}_{j+1}}{\sqrt{2}}\right) - \text{erf}\left(\frac{\bar{z}_j}{\sqrt{2}}\right)\right) \tag{4.14}$$

$$\begin{aligned} s^j(t+1) &= \frac{1}{\pi^j}\int_{\bar{z}_j}^{\bar{z}_{j+1}} s(z)\pi(z)dz \\ &= s(t)\frac{k_1}{\pi^j} e^{\frac{k_2}{2}} \int_{\bar{z}_j - k_2}^{\bar{z}_{j+1} - k_2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}v^2} dv \\ &= s(t)\frac{k_1}{2\pi^j} e^{\frac{k_2}{2}} \left(\text{erf}\left(\frac{\bar{z}_{j+1} - k_2}{\sqrt{2}}\right) - \text{erf}\left(\frac{\bar{z}_j - k_2}{\sqrt{2}}\right)\right) \end{aligned} \tag{4.15}$$

$$\begin{aligned} p^j(t+1) &= \frac{1}{\pi^j}\int_{\bar{z}_j}^{\bar{z}_{j+1}} p(t+1, m(z))\pi(z)dz \\ &\approx \frac{p(t+1, m(\bar{z}_{j+1})) + p(t+1, m(\bar{z}_j))}{2\pi^j(\bar{z}_{j+1} - \bar{z}_j)} \end{aligned} \tag{4.16}$$

Where $k_1 = e^{\mu - \frac{1}{2}\sigma^2 \Delta t}$, $k_2 = \sigma\sqrt{\Delta t}$, $s(z) = s(t)k_1 e^{k_2} z$, and $m(z)$ is the market state of the asset.

Note that, contrarily to multi-stage stochastic programming approaches that typically limit the number M of considered scenarios to only 2 or 3 to avoid the combinatorial explosion over the optimization horizon N, here M can be quite large without incurring into prohibitive computation efforts, as the prediction horizon is simply $N = 1$.

By optimizing the sample variance of $w(t+1) - p(t+1)$, in the absence of transaction costs problem (4.13) can be rewritten as the following *least squares* problem

$$\min_{u(t)} \sum_{j=1}^{M} \pi^j \left(w^j(t+1) - p^j(t+1) - \left(\frac{1}{M} \sum_{i=1}^{M} w^i(t+1) - p^i(t+1) \right) \right)^2 \tag{4.17}$$

where $w^j(t+1) = (1+r)w(t) + \sum_{i=0}^{n} b_i^j(t)u_i(t)$ are the future values of portfolio wealth for each scenario $j = 1, \dots, M$, and π^j is the corresponding probability, $\pi^j \geq 0$, $\sum_{i=1}^{M} \pi^j = 1$. The resulting SMPC algorithm is described by Algorithm 1.

Algorithm 1 SMPC algorithm for dynamic option hedging

1. Let t=current hedging date, $w(t)$= current wealth of portfolio, $m(t)$=current market state;
2. Generate M scenarios of future market states $m^1(t+1)$, ..., $m^M(t+1)$, with corresponding probabilities π^1, ..., π^M;
3. Use a pricing engine to generate the corresponding future option prices $p^1(t+1)$, ..., $p^M(t+1)$;
4. Solve the least square problem (4.17) to minimize the sample variance of $w(t+1) - p(t+1)$;
5. Rebalance the portfolio according of the optimal solution $u^*(t)$ of problem (4.17);
6. End.

4.4.1 Pricing future option values

An option pricing engine is needed at step 2. of Algorithm 1 to compute the future option prices $p^1(t+1), \ldots, p^M(t+1)$ over the generated scenarios, which may be a bottleneck of the proposed SMPC approach for exotic options. Several approaches exist to option pricing, such as those based on Monte Carlo simulation. If each option evaluation requires the simulation of L scenarios, then one has to simulate ML paths on-line at each trading period t to build the optimization problem (4.17), which may be a time consuming task.

Although advanced techniques exist for parallel computation of Monte Carlo simulations, alternative *off-line* function approximation techniques can be used to obtain option prices for each future scenario. The idea is to construct a function that returns the option price as a function of $m(t)$ (that is, of the current asset parameters and of other option-related quantities). In this thesis we use a function approximation inspired by the Monte Carlo method of Longstaff and Schwartz [51] for pricing American derivates, in which the continuation value (the option value at a future date) is estimated by a regression of the discounted payoff on a base of functions of some state variables. This methodology proved to have superior performance with respect to other classical general purpose function approximation methods.

4.4.2 Simulation results

In this section we test the SMPC algorithm 1 on different options and asset price models.

All simulation were performed on a Asus with 1.70 GHz Intel Pentium R processor and 2 Gb RAM running MATLAB™ R2007b under Windows XP, using the following parameters: $M = 100$ scenarios (unless specified differently), prediction horizon $N = 1$, $\Delta_T = 1$ week is the time interval between consecutive reallocations of the portfolio, $T = 24$ weeks is the maturity of the option, $r_a = 4\%$ is the annualized continuously compounded interest rate so that $r = e^{0.04\frac{1}{54}} - 1 = 0.00074102$ is the return of the risk free investment over Δ_T. In every example shown below the hedging strategy is tested over $N_s = 1000$ simulations of randomly generated market evolutions.

We will consider a single stock $s_1(t)$ with initial spot price $s_1(0) = 100$ €. For European call options (4.8), we will consider the strike price $K = 100$ €. The number of traded assets is $n = 1$ when only the underlying stock is traded, or $n = 2$ when also the European call option with expiration at time $t\Delta T$ and strike price $s_1(t)(1+r)^{T-t}$ is also traded in the portfolio. For "Napoleon cliquet" options (4.9), we consider $N_{\text{fix}} = 3$ fixing dates, with $t_0 = 0$, $t_1 = 8$, $t_2 = 16$, $t_3 = 24$ weeks, and coupon $C = 0.1$. For barrier options, we have considered an UP-AND-OUT option with barrier $x_u = 120$ €, where the barrier level is checked only at trading instants. When Monte Carlo simulation is used to price "Napoleon cliquet" and Barrier options, $L = 1000$ scenarios are evaluated to compute the expected payoff.

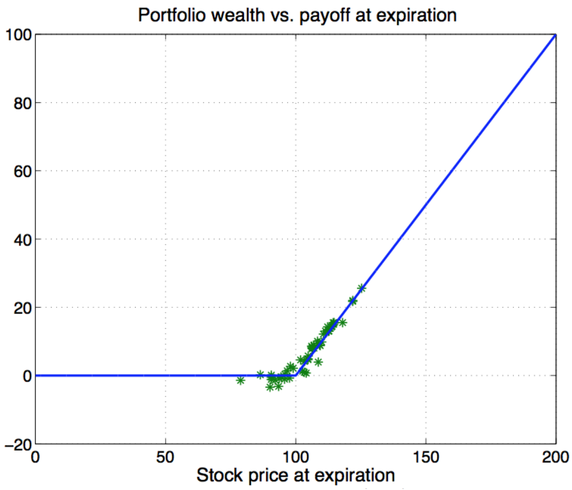
We will consider the log-normal stock price model (4.3) with[1] $\mu = r_a$, $dz_1^x \sim \mathcal{N}(0, 1)$ and volatility $\sigma = 0.5$, which will be also referred to as Black-Scholes (BS) model, and Heston (H) model (4.4), with initial variance $y_1(0) = 0.25$, and parameters $\theta_1 = 0.25$, $\kappa_1 = 1$, $\omega_1 = 0.3$, $\rho_1 = -0.5$. In all simulations we assume that the value of market volatility is estimated exactly.

[1]In this particular case, the probability measure used for asset price and portfolio dynamics coincides with the risk-neutral one. However, the reader should notice that this approach relies on the real-world probability measure for asset price and portfolio dynamics.

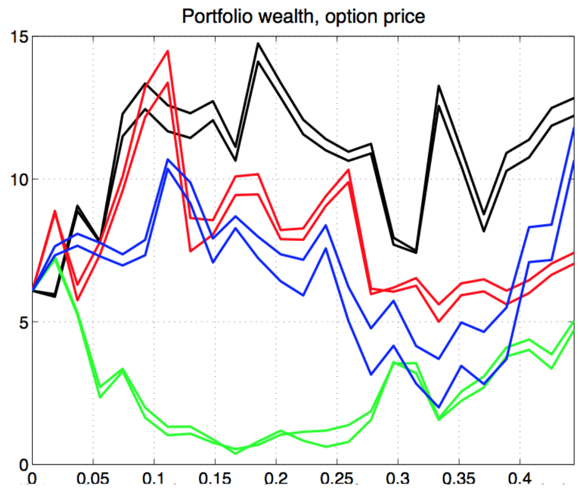
European call option

We first test the SMPC strategy (4.17) to replicate a European call option, only trading the risk-free asset and the underlying stock ($n = 1$). Heston's model [38] is used both in the MPC formulation and to generate actual market prices in simulation.

The analytical pricing formula [38] is used to compute future asset values $p^j(t+1)$, $j = 1, \dots, M$. The results are depicted in Figure 4.2, where only the first 50 simulations are reported in Figure 4.2a and 4 simulations in Figure 4.2b. The empirical distribution of the hedging error[2] computed on all N_s simulations is depicted in Figure 4.3 (purple line). The average CPU time to execute Algorithm 1 is 81.2 ms. The average hedging error



(a) payoff function $p(T)$ and final wealth $w(T)$ (€) as a function of the stock price $s1(T)$ at expiration (€)



(b) Sample trajectories of wealth $w(t)$ and option price $p(t)$

Figure 4.2: Hedging a European call using SMPC based on Heston's model (values in €)

$E[e(T)] = -0.0511$ €, $E[|e(T)|] = 1.9907$, $\max |e(T)| = 14.5699$. For comparison, Figure 4.3 also shows the error distribution when delta hedging[3] is applied (green line), which takes an average CPU time of 2.5 ms per

[2] Hedging errors are sampled with the Freedman-Diaconis rule [28].

[3] By letting $\Delta = \frac{\partial p}{\partial x}$, in *Delta hedging* at each time step the portfolio contains a quantity $-\Delta$ of asset x. In our simulations Δ is computed by differentiating the pricing formula [38] numerically.

time step. In each simulation, the difference between the hedging error $e(T)$ achieved by SMPC and the one obtained by delta hedging is within ± 3.75 €.

Exotic options

The advantage of using the SMPC strategy becomes more evident when replicating exotic options.

We use again Heston's model [38] both as a market model and a prediction model for stock prices. For the "Napoleon Cliquet" option, we only consider the case $n = 2$ and we use Longstaff-Schwartz's off-line approximation (calibrated in 251.5310 s) to estimate the option price $p(t)$ as a function of the spot price $s_1(t)$, its variance $y_1(t)$, and of the spot prices at past fixing dates $s_1(t_0)$, ..., $s_1(t_k)$, with $t_k \le t < t_{k+1}$. On-line CPU time is 0.4391 s (for comparison, when using on-line Monte Carlo simulation to compute future options CPU time is 2.49 s).

Hedging results are reported in the third and fourth rows of Table 4.1, where for comparison in the fifth row we also show the results obtained through delta hedging.

For the barrier option, off-line pricing approximation takes 114.016 s to estimate $p(t)$ as a function of $s_1(t)$ and its variance $y_1(t)$. On-line CPU time is 428.8 ms ($n = 2$). Hedging results are reported in the last two rows of Table 4.2.

SMPC model	$E[e(T)]$	$E[\lvert e(T)\rvert]$	$\max \lvert e(T)\rvert$	CPU (ms)
Fixed Black	0.0031	0.0080	0.0561	1256.28
Implied Black	0.0031	0.0079	0.0560	1293.7
Heston (MC)	0.0032	0.0075	0.0516	6717.48
Heston (LS)	0.0025	0.0110	0.4159	439.1
Δ hedging	-0.0032	0.0176	0.1344	33.7

Table 4.1: Napoleon Cliquet option (final hedging error $e(T)$ in €, MC=Monte Carlo online pricing, LS=Longstaff&Schwartz offline option price approximation)

SMPC model	$E[e(T)]$	$E[\|e(T)\|]$	$\max \|e(T)\|$	CPU (ms)
Fixed Black	-0.0324	0.6965	14.5866	113.12
Implied Black	-0.0320	0.6956	14.5829	155.64
Heston	0.1936	0.7961	16.0870	363.76
Δ hedging	-1.3060	2.4335	18.9145	103.0

Table 4.2: Barrier option (final hedging error $e(T)$ in €)

4.4.3 Robustness with respect to market modeling errors

Generating future scenarios of asset prices requires a model of their stochastic and dynamic evolution. Getting such a model is often a complex task and unavoidably affected by inaccuracy. This is due to the fact that we are trying to enclose a huge net of complicated relationships, in addition to a large source of randomness, in a small box. As complicated as the model can be, one will never be able to catch the exact dynamics of the assets, and in any case a very complicated model would lose the advantages of modelization. Therefore, in general, the asset price model will always be different from the way the real world behaves, and one must find a compromise, by using a simple enough model which allows one to keep computational complexity as low as possible.

In the previous sections we have assumed that the actual prices behave according to the same model we use to predict their evolution (nominal conditions). The hedging error was exclusively due to randomness. In this section we test numerically the robustness of the SMPC algorithm not only with respect to price stochasticity, but also when real and prediction model mismatch. In particular, we assume that real assets evolve following Heston's model [38], while the simpler Black and Scholes model (4.3) is used to generate future scenarios in SMPC.

The tool that will be used to concile the two models is the calibration of the lognormal model (4.3) using the so-called *implied volatility*, that is the market's view of future actual volatility and is updated at each trading period from observed market prices of plain vanilla options (generated by the Heston's model [38] in our setting) by inverting (numerically) the Black-Scholes pricing formula. Such a value of implied volatility will be used in our simple prediction model (4.5). This approach could be seen as a way of "projecting" the real market (which, in our case, follows Heston's model) onto the log-normal model.

4.4.4 Simulation results with respect to robustness

In the following tests we have considered that the real market evolves according to Heston's model with initial volatility $\sigma = 0.5$ ($y_1(0) = 0.25$), and that, to avoid bias in hedging errors due to wrong initial pricing, the initial wealth of the portfolio is computed correctly using Heston's model and exact $y_1(0)$. For SMPC we consider instead three different models:

1. *Fixed Black-Scholes*: The log-normal model (4.3) is used to generate future scenarios in SMPC, setting the volatility to a fixed arbitrary value, different from the actual;

2. *Implied Black-Scholes*: at each prediction step the estimated implied volatility is used in (4.3);

3. *Heston*: nominal case, both the SMPC model and the real market model coincide, and the actual volatility is observed exactly.

European Call

We first test the robustness of the SMPC algorithm on a European call option, only trading the risk-free asset and the underlying stock ($n = 1$). The analytical pricing formula [38] is used to compute future asset values $p^j(t+1)$, $j = 1, \ldots, M$.

Figure 4.3 shows the empirical discrete density function of the hedging error $e(T) = w(T) - p(T)$ in the presence of modeling errors. Note that

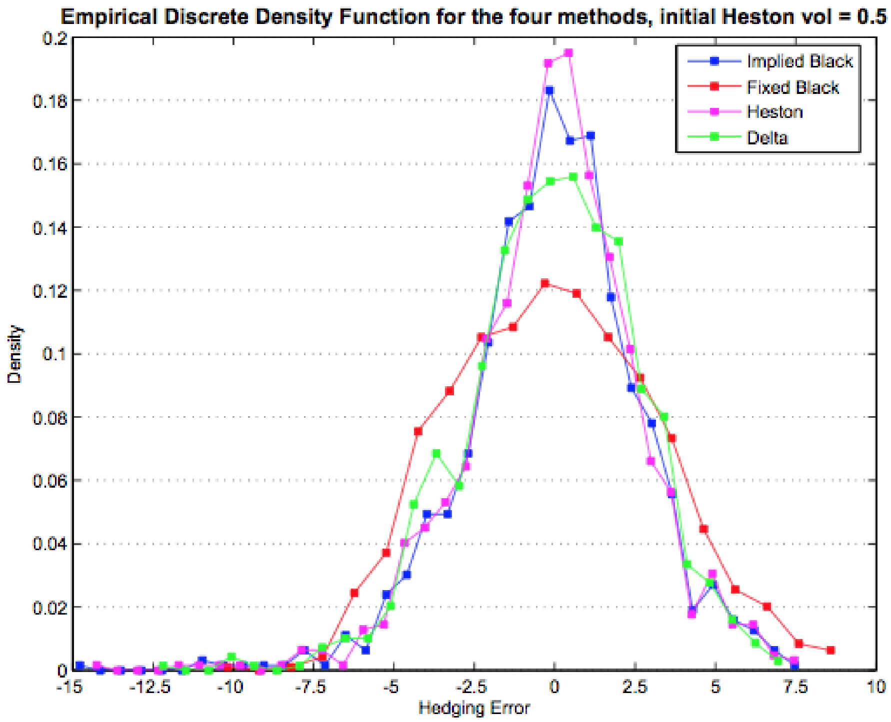


Figure 4.3: Comparing the empirical distribution of hedging errors among the four methods: *Heston*, *Implied Black*, *Fixed Black* and *Delta Hedging*.

all four distributions are bell-shaped. We can easily see that the density of *Fixed Black* (red line) has fatter tails than the others and that *Implied Black* (blue line) better follows the distribution of *Heston* (=the exact model, purple line). While *Fixed Black* and *Implied Black* take approximately the same CPU time (9.6 ms and 10.2 ms per time step, respectively), *Heston* (nominal conditions) takes 81.2 ms per time step. *Delta Hedging* is faster: only takes 2.5 ms per time step, because it simply uses finite differences.

The benefits of resorting to the discretization of the normal distribution as in (4.14) with respect to Monte Carlo simulation $\pi^j = \frac{1}{M}$ are highlighted in Table 4.3, where the *Implied Black* method is used to hedge in the SMPC algorithm.

Note that the average final hedging error and the average absolute hedging error obtained when only 3 scenarios, weighted with the corresponding

M	π^j	$E[e(T)]$	$E[\|e(T)\|]$	$\max \|e(T)\|$	CPU (ms)
100	$\frac{1}{M}$	-0.0587	2.0296	15.0929	10.2
8	$\frac{1}{M}$	-0.1177	3.7149	18.2113	3.4
8	Eq. (4.14)	-0.0914	2.1517	13.7776	4.4
5	$\frac{1}{M}$	-0.1763	5.3472	20.4058	3.2
5	Eq. (4.14)	-0.0962	2.1697	13.5410	3.8
3	$\frac{1}{M}$	-0.1717	5.2603	20.5207	3.1
3	Eq. (4.14)	-0.0501	2.0153	15.2368	3.4

Table 4.3: Montecarlo vs. discretization of probability density function in generating scenarios (European call, errors expressed in €)

probabilities as in (4.14), are used in SMPC are very similar to the case with $M = 100$ scenarios generated by Monte Carlo, but with evident savings of CPU time.

Exotic options

The robustness with respect to modeling errors in the case of path-dependent "Napoleon cliquet" options with payoff (4.9) is highlighted in (the first and second rows of) Table 4.1, where we use $M = 100$ equally probably scenarios generated by using Longstaff-Schwartz's off-line approximation. For exotic options we only consider the case $n = 2$, that is, trading both the asset and its associated call option. While all methods perform similarly, it is apparent the computational benefits of hedging using the log-normal model, in spite of the modeling error. Note that, although delta hedging is the fastest algorithm, its performance in terms of $E[|e(T)|]$ deteriorates by almost 50% with respect to Implied Black and almost 60% with respect to SMPC based on Heston's model; partly this is because delta hedging does not include options in the portfolio ($n = 1$). Similar results are obtained on the UP-AND-OUT Barrier option, as shown in Table 4.2.

4.5 Transaction costs

When trading assets on the market, one often suffers the friction of transaction costs [22]. In mathematical terms, the investor pays a quantity $h_i(t)$ of wealth to change the number of assets in the portfolio from $u_i(t-1)$ at time $t-1$ to $u(t)$ at time t, for each asset i. Such wealth $h_i(t)$ is taken away from the overall wealth $w(t)$ of the portfolio, so that (4.11) becomes (cf. [67])

$$w(t+1) = (1+r)\left(w(t) - \sum_{i=1}^{n} h_i(t)\right) + \sum_{i=1}^{n} b_i(t)u_i(t) \tag{4.18}$$

Proposition 1 *The variance of the hedging error $e(t) = w(t) - p(t)$ is not affected by transaction costs.*

Proof: Let $\omega(t) = \sum_{i=1}^{n} h_i(t)$ be the total transaction cost paid at time t. As $\omega(t)$ is a deterministic function that only depends on $u(t)$ (it does not depend on $s(t)$), we get that the expected value of the hedging error $e(t+1) = w(t+1) - p(t+1)$ is

$$\begin{aligned} E[w(t+1) - p(t+1)] &= E[(1+r)w(t) + \sum_{i=1}^{n} b_i(t)u_i(t) \\ &\quad -p(t+1) - (1+r)\omega(t)] \\ &= E[w_0(t+1) - p(t+1)] - (1+r)\omega(t) \end{aligned}$$

where $w_0(t+1)$ is the wealth at time $t+1$ in the absence of transaction costs. Therefore, while the expectation $E[e(t+1)]$ of the hedging error

$e(t+1)$ is affected by $\omega(t)$, its variance $\mathrm{Var}[e(t+1)]$ is clearly not, as

$$\begin{aligned}\mathrm{Var}[e(t+1)] &= E[(e(t+1)-E[e(t+1)])^2] \\ &= E[(w_0(t+1)-p(t+1)-(1+r)\omega(t) \\ &\quad -E[w_0(t+1)-p(t+1)]+(1+r)\omega(t))^2] \\ &= \mathrm{Var}[w_0(t+1)-p(t+1)]\end{aligned}$$

□

Proposition 1 has clearly shown that the minimum variance criterion (4.12) is inadequate to handle transaction costs.

In the simplest case, transaction costs $h_i(t)$ are proportional to the traded quantity of stock $|u_i(t)-u_i(t-1)|$

$$h_i(u_i) = \epsilon_i|u_i(t)-u_i(t-1)|s_i(t) \tag{4.19}$$

where the fixed quantity ϵ_i depends on commissions on trading asset i, $i=1,\dots,n$ (we assume that no costs are applied on transacting the risk-free asset).

4.5.1 Minimization of variance and expectation (QP-Var)

Let $x(t), y(t) \in \mathbb{R}^n$ be two vectors whose i-th components are nonnegative and defined as

$$\begin{gathered} x_i(t)-y_i(t) = u_i(t)-u_i(t-1) \\ x_i(t) \geq 0, y_i(t) \geq 0,\ \forall t=0,\dots,T \end{gathered} \tag{4.20}$$

Accordingly, the cost $h_i(t)$ for trading a quantity $u_i(t)-u_i(t-1)$ of the i-th asset is $h^i(t) = \epsilon^i|u_i(t)-u_i(t-1)|s_i(t) = \gamma_i(t)(x_i(t)+y_i(t))$, where $\gamma_i(t) \triangleq \epsilon^i s^i(t)$, $i=1,\dots,n$. The quantities $x_i(t)$ and $y_i(t)$ can be interpreted, respectively, as the amount of asset i *bought* at time t and the amount of asset i *sold* at time t. We can therefore introduce the new decision vector $v(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \in \mathbb{R}^{2n}$ of decision variables and replace $u(t) \in \mathbb{R}^n$ with

$$u(t) = u(t-1) + x(t) - y(t) \tag{4.21}$$

By letting

$$\mathbb{1} \triangleq \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \in \mathbb{R}^M, \;\; \gamma(t) \triangleq \begin{bmatrix} \gamma_1(t) \\ \vdots \\ \gamma_n(t) \end{bmatrix}$$

from (4.18) we can express the vector of future hedging errors $e(t+1) = w(t+1) - p(t+1)$ on the M different scenarios as

$$\begin{aligned}
\begin{bmatrix} e^1(t+1) \\ \vdots \\ e^M(t+1) \end{bmatrix} &= B(t)u(t) + (1+r)\left(w(t) - \gamma'(t)(x(t)+y(t))\right)\mathbb{1} - \begin{bmatrix} p^1(t+1) \\ \vdots \\ p^M(t+1) \end{bmatrix} \\
&= B(t)(u(t-1) + x(t) - y(t)) \\
&\quad -(1+r)\,\mathbb{1}\,\gamma'(t)(x(t)+y(t)) + D(t) \\
&= A_v(t)v(t) + B_v(t) - \mathbb{1}\,G_v(t)v(t)
\end{aligned}$$

where

$$\begin{aligned}
B(t) &\triangleq \begin{bmatrix} b_1^1(t) & \dots & b_n^1(t) \\ \vdots & & \vdots \\ b_1^M(t) & \dots & b_n^M(t) \end{bmatrix}, \; D(t) \triangleq (1+r)\,\mathbb{1}\,w(t) - \begin{bmatrix} p^1(t+1) \\ \vdots \\ p^M(t+1) \end{bmatrix} \\
B_v(t) &\triangleq B(t)u(t-1) + D(t), \;\; A_v(t) \triangleq [B(t)| - B(t)], \\
G_v(t) &\triangleq (1+r)[\gamma'(t)| + \gamma'(t)]
\end{aligned}$$

The hedging error $e(t+1) = w(t+1) - p(t+1)$ has therefore the following empirical expectation

$$\begin{aligned} E[e(t+1)] &= \pi'(A_v(t)v(t) + B_v(t) - \mathbb{I}\, G_v(t)v(t)) \\ &= -G_v(t)v(t) + \pi'(A_v(t)v(t) + B_v(t)) \end{aligned} \tag{4.22}$$

where $\pi' = [\pi_1 \ \ldots \pi_M]' \in \mathbb{R}^M$, $\pi' \,\mathbb{I} = 1$. Note that by (4.22) we can rewrite $E[e(t+1)] = K(t) - H(t)$, where $K(t) = \pi'[B(t)(x(t) - y(t)) + B_v(t)]$ and $H(t) = (1+r)\gamma'(x(t) + y(t))$. Therefore, $K(t)$ depends on the quantity $x(t) - y(t)$ (i.e., on the net increment $u(t) - u(t-1)$ of the underlying assets hold in portfolio from time $t-1$ to time t) and is independent of $\Lambda(t) = \min\{x(t), y(t)\}$ and of the transaction costs, while $H(t)$ depends on the actual number of transactions executed (simultaneously) to rebalance the portfolio at time t, depends on $\Lambda(t)$ and, via $\gamma(t)$, on the transaction costs (indeed, $E[e(t+1)]$ can be decreased by increasing $\Lambda(t)$).

By letting i_j be the j-th vector of the canonical basis of R^M, that is $I = [i_1| \ldots |i_M]$, and omitting the dependence of t for ease of notation we get

$$\begin{aligned} E[e^2(t+1)] &= \sum_{j=1}^{M} \pi_j \left(i_j'(A_v v + B_v - \mathbb{I}\, G_v v)\right)^2 \\ &= v'G_v'G_v v + (A_v v + B_v)' \operatorname{diag}(\pi)(A_v v + B_v) \\ &\quad -2\pi'(A_v v + B_v)G_v v \end{aligned} \tag{4.23}$$

$$\begin{aligned} E^2[e(t+1)] &= \left(\sum_{j=1}^{M} \pi_j i_j'(A_v v + B_v - \mathbb{I}\, G_v v)\right)^2 \\ &= (\pi'(A_v v + B_v) - G_v v)^2 \\ &= v'G_v'G_v v + (A_v v + B_v)'\pi\pi'(A_v v + B_v) \\ &\quad -2\pi'(A_v v + B_v)G_v v \end{aligned} \tag{4.24}$$

Hence, the variance of $e(t+1)$ is

$$\begin{aligned}
\mathrm{Var}[e(t+1)] &= E[(e(t+1) - E[e(t+1)])^2] \\
&= E[e^2(t+1)] - E^2[e(t+1)] \qquad (4.25a)\\
= (A_v(t)v(t) &+ B_v(t))'(\mathrm{diag}(\pi) - \pi\pi')(A_v(t)v(t) + B_v(t)) \qquad (4.25b)
\end{aligned}$$

Note that (4.25b) does not depend on $\gamma(t)$, in accordance with Proposition 1, and that $\mathrm{diag}(\pi) - \pi\pi'$ is a positive semidefinite matrix by definition:

$$\begin{aligned}
v'(\mathrm{diag}(\pi) - \pi\pi')v &= \sum_{i=1}^{M} \pi_i v_i^2 - \left(\sum_{i=1}^{M} \pi_i v_i\right)\left(\sum_{j=1}^{M} \pi_j v_j\right) \\
&= \sum_{i=1}^{M} \pi_i \left(v_i^2 - 2v_i \sum_{j=1}^{M} \pi_j v_j + v_i \sum_{j=1}^{M} \pi_j v_j\right) \\
&= \left(\sum_{i=1}^{M} \pi_i (v_i^2 - 2v_i \sum_{j=1}^{M} \pi_j v_j)\right) \\
&\quad + \left(\sum_{i=1}^{M} \pi_i v_i\right)\left(\sum_{j=1}^{M} \pi_j v_j\right)\left(\sum_{i=1}^{M} \pi_i\right) \\
&= \sum_{i=1}^{M} \pi_i \left(v_i - \sum_{j=1}^{M} \pi_j v_j\right)^2 \geq 0, \quad \forall v \in \mathbb{R}^M
\end{aligned}$$

Note also that $\mathrm{Var}[e(t+1)]$ does not depend on $x(t) - y(t)$, and therefore on $\Lambda(t)$, that confirms what observed earlier about $\Lambda(t)$ only affecting transaction costs, that are deterministic.

In order to minimize both the variance and the expected value of the one-step ahead hedging error $e(t+1)$ we solve the following optimization problem

$$\begin{aligned}
\min_{v(t)} \quad & \mathrm{Var}[e(t+1)] + \alpha E^2[e(t+1)] \qquad (4.26)\\
\text{s.t.} \quad & v(t) \geq 0
\end{aligned}$$

where α is a fixed scalar, $\alpha \in [0, 0.5]$. Problem (4.26) is a QP problem with $2n$ variables subject to nonnegativity constraints.

Note that the hedging strategy defined by (4.26) might lead to choosing optimal quantities $x_i(t)$ and $y_i(t)$ that are both positive, that is $\Lambda_i(t) \triangleq \min\{x_i(t), y_i(t)\} > 0$. This amounts to allow the trader to simultaneously buy and sell the same quantity $\Lambda_i(t)$ of asset i at the same trading instant t (cf. [17, p. 290]) or, in alternative, to violate the self-financing condition (4.11), by subtracting the wealth $\Lambda_i(t)\gamma_i(t)$ from the total portfolio wealth and rebalancing $u_i(t) = u_i(t-1)+\bar{x}_i(t)-\bar{y}_i(t)$, where $\bar{x}_i(t) = x_i(t)-\Lambda_i(t)$, $\bar{y}_i(t) = y_i(t)-\Lambda_i(t)$. Clearly, $\bar{x}_i(t)-\bar{y}_i(t) = x_i(t)-y_i(t)$ and either $\bar{x}_i(t) = 0$ or $\bar{y}_i(t) = 0$. Constraining $\Lambda_i(t) = 0$ would make (4.26) a nonconvex problem, therefore more complicated to solve numerically; however, leaving $\Lambda_i(t)$ unconstrained does not lead to undesired effects from a hedging viewpoint. In fact, having $x_i(t)$ and $y_i(t)$ both positive ($\Lambda_i(t) > 0$) might be a good choice to avoid super-replication without altering variance. On the other hand, if at optimality $E[e(t+1)] \leq 0$, that is one is under-replicating the option price at time t, then necessarily $\Lambda_i(t) = 0$, otherwise $\bar{x}_i(t)$, $\bar{y}_i(t)$ would be a solution with the same variance and a lower $E^2[e(t+1)]$, thus providing a lower value of the objective function in (4.26) than $x(t)$, $y(t)$.

Note also that one could minimize $\mathrm{Var}[e(t+1)] + \alpha E[e(t+1)]$ instead of (4.26), therefore not penalizing super-replication. In this setting, either $x_i(t) = 0$ or $y_i(t) = 0$ spontaneously at optimality (that is, $\Lambda_i(t) = 0$), as, as observed earlier, a positive quantity $\Lambda_i(t)$ would only increase the term $H(t)$ due to transaction costs without altering $K(t)$ and $\mathrm{Var}[e(t+1)]$.

An alternative formulation based on mixed-integer quadratic programming, related to the approach of [30] but based on the theory of hybrid dynamical systems [7], that can handle more general transaction costs is reported in Appendix A.

4.5.2 Minimization of conditional value at risk (LP-CVaR)

A drawback of the QP formulation (4.26) is that it requires the calibration of the scalar α that achieves the best tradeoff between variance (=risk) and expectation (=lack of hedging accuracy due to transaction costs). *Conditional Value at Risk* (CVaR) can be used as an alternative performance measure to penalize the hedging error $e(t+1)$, and is defined as follows. Recalling the concept of CVaR in Section 2.1.3 let $f(u,s) : \mathbb{R}^{n+k} \to \mathbb{R}$ be a loss function associated with the decision vector $u \in \mathbb{R}^n$ and with the random vector $s \in \mathbb{R}^k$. In our case $u = u(t)$, $s = m(t+1)$, $f(u,s) = |e(t+1)|$ (in case super-replication of the option price is not penalized, $f(u,s) = -e(t+1)$). Let $p(s)$ be the probability density function of s. We use CVaR to formulate the SMPC problem for dynamic hedging:

$$
\begin{aligned}
\min_{v(t),\ell(t),\{z_j(t)\}_{j=1}^M} \quad & \ell(t) + \frac{1}{1-\beta}\sum_{j=1}^M \pi_j z_j(t) && \text{(4.27a)}\\
\text{s.t.} \quad & z_j(t) \geq w^j(t+1) - p^j(t+1) - \ell && \text{(4.27b)}\\
& z_j(t) \geq -w^j(t+1) + p^j(t+1) - \ell && \text{(4.27c)}\\
& z_j(t) \geq 0,\ j = 1,\dots,M && \text{(4.27d)}\\
& v(t) \geq 0,\ j = 1,\dots,M && \text{(4.27e)}
\end{aligned}
$$

for the given fixed value of β, where $w^j(t+1) - p^j(t+1)$ is given by (4.22).Problem (4.27) is an LP problem with $M+n+1$ variables and $3M$ constraints. Note that by removing constraint (4.27b) one does not penalize super-replication of the option price, as the loss function becomes $\max\{-e(t+1), 0\}$.

4.5.3 Minimization of worst-case error (LP-MinMax)

A simpler approach than CVaR is to penalize the worst-case loss over the set of M generated scenarios, that is the largest absolute value $|e(t+1)|$

of the hedging error. The resulting formulation is the linear program

$$\min_{v(t),\ell(t)} \quad \ell(t) \tag{4.28a}$$
$$\text{s.t.} \quad \ell(t) \geq w^j(t+1) - p^j(t+1) \tag{4.28b}$$
$$\ell(t) \geq -w^j(t+1) + p^j(t+1) \tag{4.28c}$$
$$\ell(t) \geq 0 \tag{4.28d}$$
$$v(t) \geq 0, \ j = 1, \dots, M \tag{4.28e}$$

where $w^j(t+1) - p^j(t+1)$ is given by (4.22). Note that the LP (4.28) is simpler than (4.27) as it only involves $n+1$ variables and $2M+1$ constraints. In contrast, it is clear that the LP-MinMax formulation (4.28) does not exploit the available information about the probability distribution of the stochastic variables that affect the portfolio evolution.

4.5.4 Simulation results with transaction costs

We test the SMPC formulations for dynamic hedging of Section 4.5 on a European plain vanilla call option and on a barrier option. All simulations were run on a MacBook Pro 2.66 GHz Intel Core 2 Duo processor and 4 Gb RAM running MATLAB R2009b. The QP solver `QUADPROG` of the Optimization Toolbox was used to solve QP problems, while the solver GLPK [53] was used to solve LP problems.

We test the proposed three SMPC algorithms defined, respectively, by (4.26), (4.27), and (4.28) under different scenario generation settings: $M = 100$ and $M = 1000$ scenarios generated by Monte Carlo simulation ($\pi_i = \frac{1}{M}$, $\forall i = 1, \dots, M$), and $M = 5$ with π_i obtained by discretizing a Gaussian distribution of $s(t+1)$ as described (4.14) - (4.16). Let $\Delta_T = 1$ week be the time interval between two consecutive trading dates. The option expires after $T = 24$ intervals, and $r_a = 4\%$ is the annualized continuously compounded interest rate so that $r = e^{0.04\frac{1}{54}} - 1 = 0.00074102$ is the return of the risk free investment over Δ_T.

We consider a single stock $s_1(t)$ with initial spot price $s_1(0) = 100$ €. For European call options (4.8), we consider the strike price $K = 100$ €, while for barrier options, we consider an UP-AND-OUT option with barrier $x_u = 115$ €, where the barrier level is checked only at trading instants. In the following tests we will consider two different cases:

- ($n = 1$) The replicating portfolio is composed by the underlying stock and a cash position in the money market account (a set-up similar to common "delta" hedging);

- ($n = 2$) The replicating portfolio is composed, besides the previous two assets, by a position in an at-the-money (ATM) European call option whose expiry coincides with the expiration date T of the product to be hedged (a set-up similar to common "delta" and "vega" hedging).

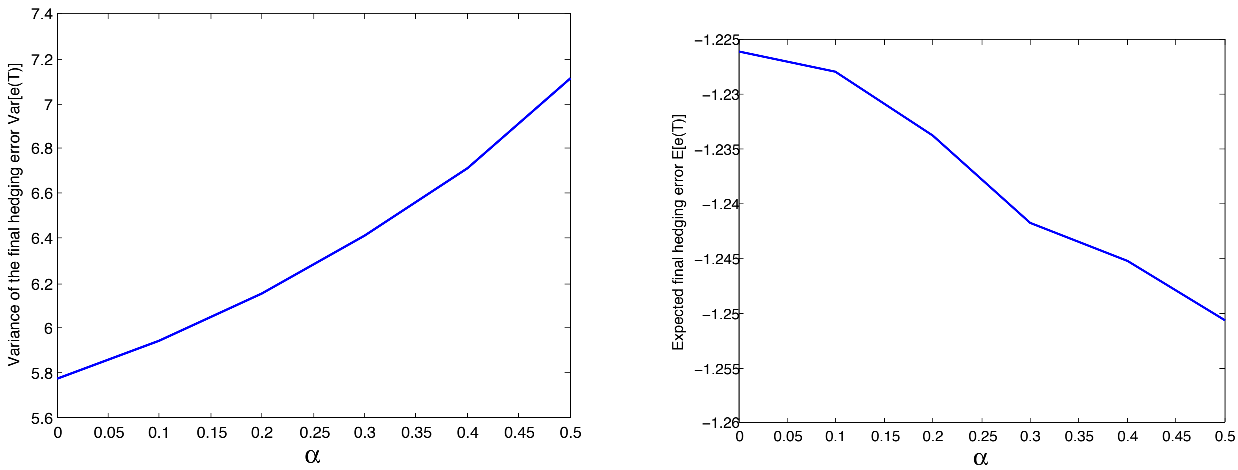
Note that, in the second case, at each trading date t the call option to be traded is ATM (i.e., strike price $= s_1(t)(1 + r)^{T-t}$), meaning that all options previously held at time $t - 1$ have been cleared in order to buy/sell the newer ATM options.

We consider the log-normal stock price model (4.3) with $\mu = r_a$, $dz_1 \sim \mathcal{N}(0, 1)$ and volatility $\sigma = 0.5$ when hedging the call option, while $\sigma = 0.3$ when hedging the barrier option and we assume the idealized case of the real market generating prices according to the same model.

We first test the SMPC algorithm on a European call option, only trading the underlying stock and the risk free asset ($n = 1$). The transaction cost to trade the underlying stock is $\epsilon_1 = 2.5\%$. The strategy is tested over $N_s = 100$ simulations.

QP-Var formulation

Consider the method based on QP described in Section 4.5.1, where problem (4.26) is solved instead of (4.17). We first need to calibrate the relative weight α in (4.26). To this end, for a set of different values of α we compute the variance and expectation of the final hedging error $e(T)$ from a set of $N_s = 100$ simulations by running the SMPC algorithm based on (4.26) with $M = 100$ scenarios. The test has been made for $0 \leq \alpha \leq 0.5$. Higher values of α would lead to an excessive risk exposure, since the expected error would be predominant.



(a) Variance of the final hedging error for $0 \leq \alpha \leq 0.5$ (b) Expected final hedging error for $0 \leq \alpha \leq 0.5$

Figure 4.4: Final variance $\text{Var}[e(T)]$ and expectation $E[e(T)]$ of the final hedging error $e(T)$, used for the calibration of parameter α

In Figure 4.4 the results of the calibration phase are highlighted. As expected, the variance increases with high values of α, while the expected hedging error $E[e(T)]$ decreases. However, the decrement in the expected error is much less dramatic than the increment of variance. For a given risk attitude of the trader, the plot of Figure 4.4 helps choosing the tradeoff parameter α. Here the value $\alpha = 0.25$ is selected to run the SMPC algorithm based on (4.26) for three different values of M (predicted scenarios): $M = 100$ ($\pi_j = \frac{1}{100}$), $M = 1000$ ($\pi_j = \frac{1}{1000}$), and $M = 5$ (π_i is obtained by

Model	**Monte Carlo** $M = 100$					**Monte Carlo** $M = 1000$					**discretized Gaussian** $M = 5$				
	$E[e(T)]$	$E[\|e(T)\|]$	$\min(e(T))$	$\mathrm{Var}[e(T)]$	CPU(s)	$E[e(T)]$	$E[\|e(T)\|]$	$\min(e(T))$	$\mathrm{Var}[e(T)]$	CPU(s)	$E[e(T)]$	$E[\|e(T)\|]$	$\min(e(T))$	$\mathrm{Var}[e(T)]$	CPU(s)
QP-Var	-1.23	2.16	-7.48	6.27	0.026	-1.17	2.22	-8.63	6.99	0.26	-1.28	2.31	-6.78	6.96	0.01
LP-CVaR	-1.27	2.27	-6.74	6.42	0.017	-1.10	2.20	-7.42	6.80	1.11	-1.28	2.39	-6.88	7.46	0.001
LP-MinMax	-1.23	2.28	-6.85	7.15	0.006	-1.28	2.35	-6.69	7.21	0.18	-1.29	2.41	-6.88	7.60	0.001
Delta Hedging	-0.1312	1.77	-5.4	4.84	0.00012	-0.1312	1.77	-5.4	4.84	0.00012	-0.1312	1.77	-5.4	4.84	0.00012

Table 4.4: SMPC results for the European call option

Model	**LS** $M = 100$					**LS** $M = 5$					**LS** $M = 1000$				
	$E[e(T)]$	$E[\|e(T)\|]$	$\min(e(T))$	$\mathrm{Var}[e(T)]$	CPU(s)	$E[e(T)]$	$E[\|e(T)\|]$	$\min(e(T))$	$\mathrm{Var}[e(T)]$	CPU(s)	$E[e(T)]$	$E[\|e(T)\|]$	$\min(e(T))$	$\mathrm{Var}[e(T)]$	CPU(s)
QP-Var	-0.55	0.97	-7.29	3.27	0.09	-0.44	1.12	-9.40	4.85	0.01	-0.55	0.98	-7.04	3.24	0.37
LP-CVaR	-0.39	1.09	-8.03	3.89	0.08	-0.34	1.25	-9.83	5.33	0.001	-0.44	1.04	-7.68	3.66	1.04
LP-MinMax	-0.56	1.22	-8.82	4.81	0.07	-0.28	1.58	-10.73	6.84	0.001	-0.52	1.32	-8.84	5.23	0.33
Delta Hedging	-0.70	1.79	-16.14	13.61	0.0041	-0.70	1.79	-16.14	13.61	0.0041	-0.70	1.79	-16.14	13.61	0.0041

Table 4.5: SMPC results for the barrier option

sampling the Gaussian function). The obtained results are shown in the first row of Table 4.4.

It is apparent that the global performance obtained with the three different values of M are comparable, with a slightly higher maximum error in the case $M = 1000$. This suggests that increasing M over a certain number of predicted scenarios does not necessarily lead to improvements. The discretization leads to some minor savings of CPU time, but the hedging performance gets worse.

LP-CVaR and LP-MinMax formulations

The last two rows of Table 4.4 highlight the performance of the two proposed LP formulations, where either the LP (4.27) with $\beta = 0.95$ or the LP (4.28) is solved instead of (4.17). The results obtained with the two approaches are similar (especially in the case $M = 5$), the LP (4.28) providing a slightly higher variance with respect to the other two formulations.

In the last row of Table 4.4 the results obtained with delta hedging on the same option are shown. We can see that for plain vanilla options this last method outperforms the SMPC approach. A plot of the wealth of the portfolio against the option value at expiration date is shown in Figure 4.5 for the case of LP-MinMax hedging.

Note that, differently from the case of absence of transaction costs, the

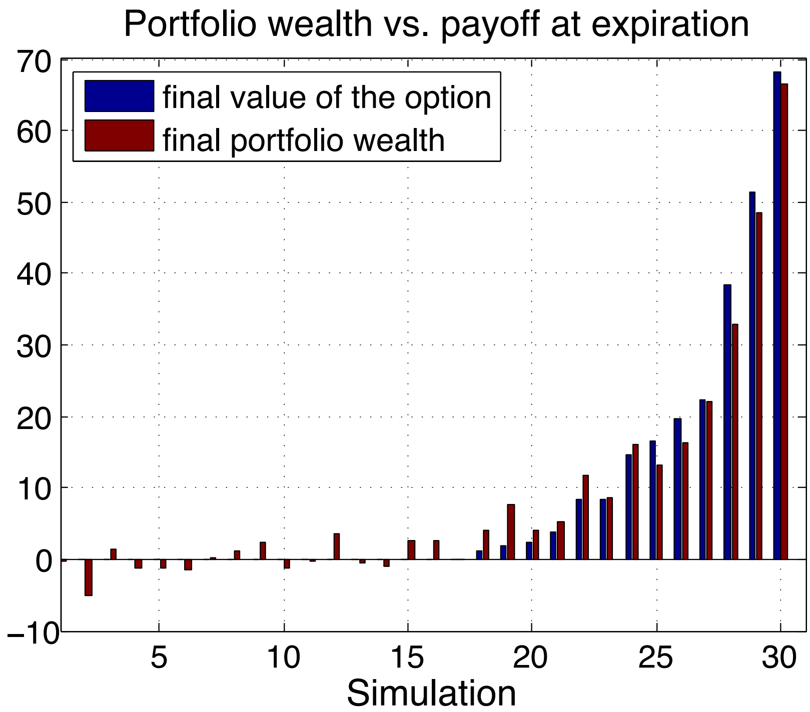


Figure 4.5: Payoff function $p(T)$ and final wealth $w(T)$ (e) for the LP-MinMax approach with discretization of the density function

final wealth of the portfolio, represented by the green asterisks, does not track exactly the payoff function, but is shifted below of an almost fixed distance (the fixed transaction cost).

Barrier option

Since the value of a barrier option is much lower than the corresponding call option, we have decreased the transaction costs at 1.5% of the underlying price to better test the SMPC algorithms. Pricing of future option values is made by using the approximation method of [51] (LS). A number $M = 100$ of future scenarios is considered, and compared to the cases of $M = 1000$ and $M = 5$ scenarios obtained by sampling the Gaussian distribution.

We have run $N_s = 50$ simulations for each setting. It can be seen that the best global performance is given by the QP-Var method. LP-CVaR

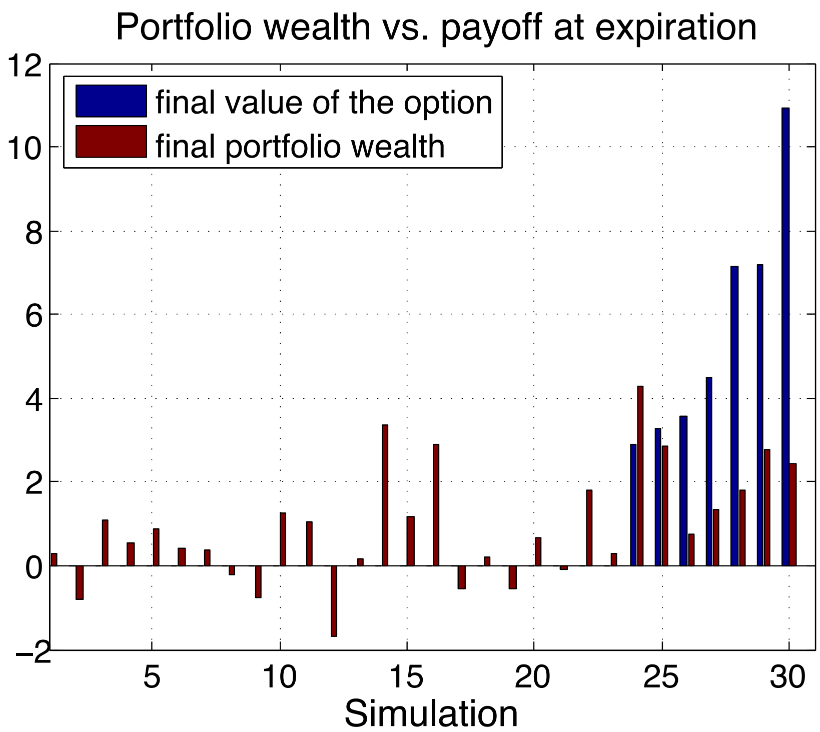


Figure 4.6: Payoff function $p(T)$ and final wealth $w(T)$ (e) for the MinMax approach with discretization of the density function

provides similar results, while LP-MinMax goes slightly worse. Longstaff-Schwartz's option pricing method with $M = 1000$ scenarios does not yield considerable improvements with respect to $M = 100$, it only worsen the required CPU time. The discretization of the Gaussian curve provides worse performance in terms of minimum error. Nevertheless, this method represents a good tradeoff between hedging performance and CPU time.

In conclusion, Longstaff-Schwartz's option pricing method with $M = 100$ scenarios is the best approach in terms of expected absolute hedging error and variance, and in particular the LP-CVaR approach, showing a comparable performances but a lower computational effort. The largest hedging errors appear when the stock price gets close to the barrier without overpassing it, as hedging becomes particularly difficult because of the discontinuity of the payoff function.

4.6 Conclusions

After recasting the dynamic hedging problem of financial options as a stochastic control problem, we have proposed a stochastic model predictive control approach based on a minimum variance criterion to rebalance periodically the portfolio underlying the option. In a first instance we assumed that transaction costs are negligible. We showed that the tool is very versatile for dynamic option hedging, as it can handle multiple assets, very general exotic options and payoff functions, and rather general stock price models, and is also robust with respect to market modeling assumptions. The computational demand of the SMPC approach is mostly due to pricing future option values, a task that can be alleviated in three ways: (*i*) by approximating the pricing function off-line, (*ii*) by using a simplified log-normal model (with implied volatility), and (*iii*) by sampling the uniform distribution instead of generating random and equally probably samples using Monte Carlo simulation.

At a later stage the proposed SMPC techniques have been extended in the case of transaction costs, although the minimum variance criterion is proven to be no longer suitable to handle this type of market. Three alternative SMPC approaches (QP-Var, LP-CVaR, and LP-MinMax) have been proposed, showing good hedging performance, but only outperforming the traditional delta hedging technique and static hedging when applied on exotic options. When CPU time is a concern, LP-CVaR is probably the best candidate formulation for SMPC, as it provides acceptable performance while involving only a small number of variables, and without requiring the calibration of the tradeoff parameter α as in the QP-Var method.

The potential use of SMPC by financial institutions is twofold. It can be used on-line to suggest trading moves to traders, or off-line to run extensive simulations and quantify the average hedging error for a given market model and option type.

Chapter 5

Optimal bidding on energy markets

This chapter is organized as follows. Section 5.1 contains a short overview on the deregulated energy markets and the new challenges addressed by market operators. In Section 5.2 the main aspects of the current market design are described with reference to the Dutch electricity market. In Section 5.3 a novel market architecture devised in E-Price for double-sided ancillary services is presented.

5.1 Introduction

The recent changes in the deregulated energy market are leading the European Community to seek a unified and common network code for the production, transmission and control of power systems [63] in European countries. The network codes developed by ENTSO-E[1] will help reach the three objectives of the Third Package, a set of directives and regulations that came into force in March 2011 for establishing binding Europe-wide network codes. These objectives are:

- the secure operation of European power systems;
- the integration of large volumes of low carbon generation;

[1] European Network of Transmission System Operators for Electricity

- and the creation of a Single Europe-wide Electricity Market.

The motivation behind is the need for a common network protocol allowing for cross-border exchanges of electricity, facilitating the competition between companies, guaranteeing the security of energy supply, and helping reach the ambitious climate change objectives imposed by the European Union [15].

The E-Price scope goes towards this concept and envisions Europe-oriented market architectures, ICT interfaces and decentralized control systems. In particular, this thesis focuses on optimal control of BRPs, legal entities allowed to trade energy on the market and bearing responsibility for the correct and safe operation of the grid. Typically, a BRP is a large-scale production plant, a set of small-size consumers or a combination of the two. Independent of their size, BRPs communicate with the market and the TSO using the same protocol, and they are therefore required to use the same standard interfaces. The behavior of each BRP is influenced by internal and structural characteristics like risk attitude, generation assets, cost structure, and by exogenous inputs like price signals, renewable sources stochasticity, uncontrollable and price-elastic loads. The real-time optimal control of BRP power injections is highly connected to capacity allocation and provision of ancillary services. In fact, the capability of the BRP to respond to real-time signals is strictly correlated to the available reserve capacity at disposal, that is the result of a thorough operation planning.

BRPs can choose several ways to deliver their output. Over-The-Counter (OTC) [68] contracts are nowadays highly exploited in the European markets. The main motivation is that they bind sure incomes for BRPs and prevent power consumers with a high and steady request of power from being negatively exposed to the volatility of energy prices. However, the constantly growing focus for real-time optimization in power production brought the attention of researchers on the day ahead and on the ancillary services markets. The more sophisticated forecast and control techniques should be able to compensate for the volatility of renewable sources and prices, bringing a consistent advantage of the real time markets over the static long term contracts. The day ahead market for electricity is also called Power Exchange (PX), where BRPs express the desired power profile and expected incomes for each hour or PTU (program time unit) of the following day. Those bids are then *cleared* by an independent system operator obtaining an E-Program for the next day. After the disclosure of the E-Program the ancillary services (AS) market auction takes place, where BRPs can buy/sell residual capacity for secondary control in real time. It is evident that DA and AS markets are strongly coupled: the energy sold on the spot market cannot be re-assigned to AS capacity reserves. In order to get the maximal benefit from the BRP operation one must take into account possible price fluctuations, stochasticity deriving from renewable sources and intermittent load. In this context, forecasting techniques and decentralized control are the key elements to provide a reliable activity.

In real-time operations, the existence of an imbalance market (or real-time market) is required to deal with unavoidable deviations from the E-Program, due to uncertainties in power demand and generation. Unlike day-ahead prices, imbalance prices are extremely more volatile, and are affected by counterintuitive phenomena, like negative values. In real time, the BRP must fulfill its E-Program, trying, insofar as possible, to avoid imbalance costs and to fulfill its own internal balance.

In this thesis we describe a novel market architecture designed for the AS bidding auction and we introduce a bidding strategy that can be used by BRPs to submit offers. The main characteristic of this market design is its *double-sided* nature, that gives to BRPs the possibility of placing, in addition to bids, also *requests* for capacity reserves, providing a confidence interval on the possible deviation from the contracted program. The main responsibility for the smooth operation of power systems is therefore shifted from the centralized TSO (or Transmission System Operator) to decentralized BRPs. This market structure has been first introduced in [74].

5.2 Current market design

In the deregulated energy market, BRPs must submit profit-maximizing energy *bids* and *offers* for the spot market (PX) and for the regulating capacity or ancillary services market (AS). The market design highly influences the bidding strategies. The current Dutch market, that has been taken as the standard benchmark for our work, besides a set of bilateral contracts, consists of a Power Exchange, an Intraday market and an ancillary services market, as shown in Figure 5.1.

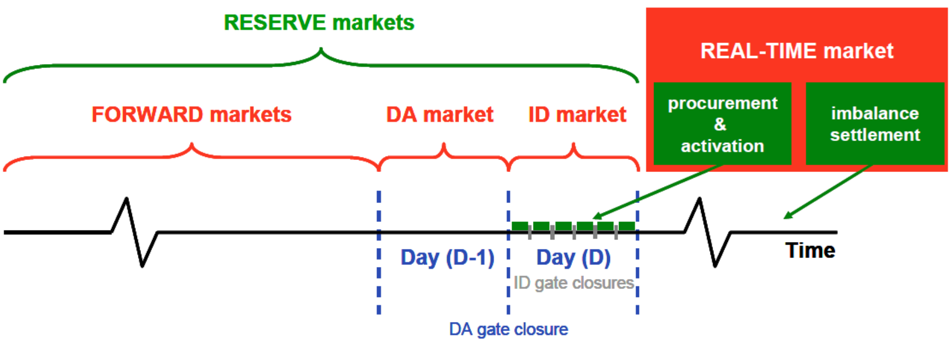


Figure 5.1: The energy markets (source: [48])

One day before the delivery generators compose and submit bids to the Power Exchange (PX). Thereafter the *market clearing price* (MCP) and the power volumes are assigned to each plant, determining the E-Program. Intraday markets are then available in multiple sessions to trade un-cleared bids or foreseen deviations closer to delivery. Up to one hour prior to delivery, BRPs submit offers for regulating capacity on the ancillary services market. Ancillary services include control reserves (which are divided into primary, secondary and tertiary resources) and emergency reserves. The supply of primary resources is compulsory for eligible generators, and is aimed to stabilize the system frequency after a disturbance. The activation time must be lower than 30 seconds. Secondary reserves are released subsequently to restore the nominal system frequency within 15 minutes (1 program time unit, PTU). Finally, tertiary reserves are those reserves whose activation time is greater than 4 PTUs, and are used to economically optimize the deployment of reserve capacity. The ancillary services dealt with in this thesis are secondary reserves. Generators with installed power higher than 60 MW are obliged to offer all the power they can increase or decrease by activating controllable generators. Those bids are sorted in ascending order as shown in Figure 5.2 and activated, the most convenient first, by the TSO to satisfy the real-time need for regulating capacity. BRPs supplying regulating capacity are rewarded at the *marginal price*, meaning that the imbalance price is the price of the last activated bid. BRPs incurring an imbalance pay their deviation from the E-Program at this price. An essential description of the background framework is contained in [72].

Generally speaking, power plants that are deemed *price-takers* tend to bid at their marginal cost. Indeed, since the behavior of these plants is not supposed to influence the final market outcome, they accept the cleared price as a result, under the condition that this price is higher than the marginal production cost, and therefore some profit is guaranteed. The imbalance settlement can be seen as a modified version of Bertrand competition [46]: the TSO is willing to buy as much as possible from the firm with the lower price (even though it will pay at the biggest cleared price). Therefore no BRP has incentive to deviate from its marginal cost.

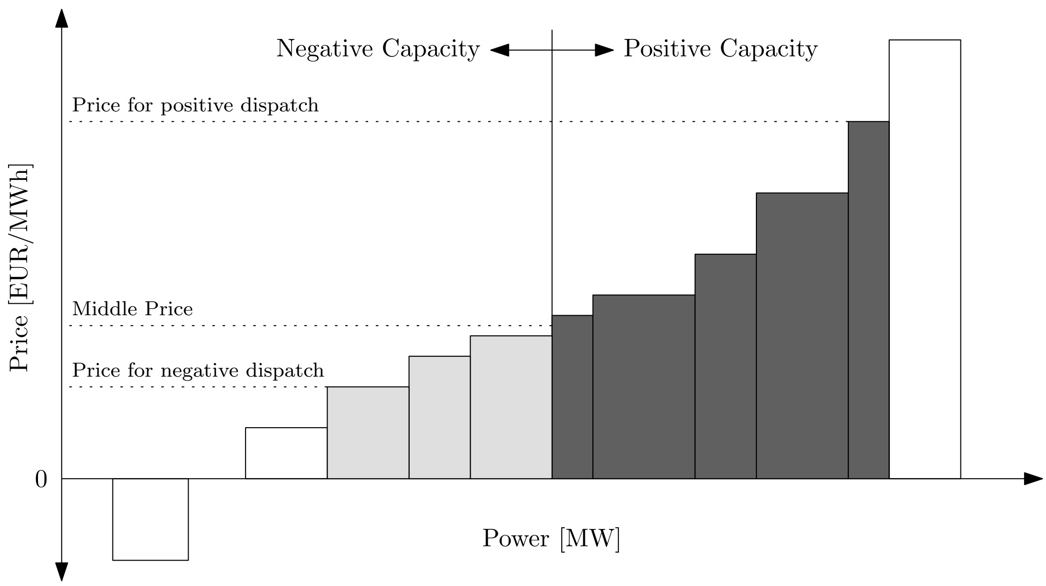


Figure 5.2: The bid ladder for ancillary services

It is an established result that when competing on prices and in transparent markets, firms tend to settle at their marginal cost (Bertrand competition, [20, 78]), while when competing on quantities (Cournot competition), the equilibrium is found above marginal costs. On the other hand, big power producers with market power have their private bidding strategies that cannot be revealed.

The bidding activity is crucial for the BRP's economic equilibrium, and involves the analysis of several sources of uncertainty. First, energy prices are highly volatile and can range from a few euros per MWh up to 1000 €. Price forecasts occupy a central role, since a bad bidding strategy can lead to severe losses. Stochastic models of electricity prices, that are affected by high volatility and jumps, are presented in [59] and [73]. The latter considers the dynamical evolution of volatility and introduces parameter-varying models such as GARCH models. Second, the generator has its own load to satisfy, which is usually stochastic (cf. [23] for a possible approach to short-term load forecasts) as well as the available amount of renewable sources, over which only (more or less reliable) predictions can be given. Third, as the Day-ahead and the ancillary services markets mutually affect each other and the production is finite, a BRP has to decide where to allocate its capacity in order to optimize an economic objective that can be the pure expected profit or a risk-based signal.

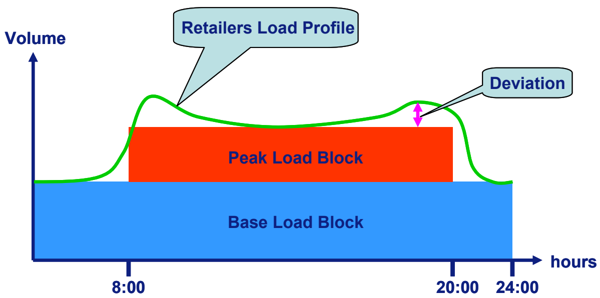
An important distinction has to be made between *integrated markets* and *sequential markets*. In integrated markets the day ahead and ancillary services auctions are cleared at the same time, while the couplings and market linkages between them are explicitly accounted for. This means that BRPs must submit two independent bids without knowing either of the two market outcomes. Although integrated markets are more mathematically insightful and can be proven to reach the optimum social welfare under some conditions (cf. [42]), they are more complex and difficult to implement, especially when renewable sources are in place. In this case BRPs must account for the stochasticity affecting prices and also renewable production and load, that can be very high if the markets are cleared far ahead in time (usually at 12.00 of the day before delivery). For this reason the market structure more often implemented in practice is sequential: the AS market is executed only after the day ahead outcome has been revealed.

To overcome the limitations related to integrated markets, we consider a market arrangement where day ahead and ancillary services markets are executed in two subsequent sessions. A contribution to the market design considered in this work in which coupling between prices is avoided is contained in [75], where potential benefits and downsides of such a market structure are illustrated. The design strategies proposed in the cited work include the execution of iterated spot and ancillary services auctions, thus implying multiple sequential bidding sessions to ensure convergence. A kind of decoupled bidding strategy is implemented on real systems in the

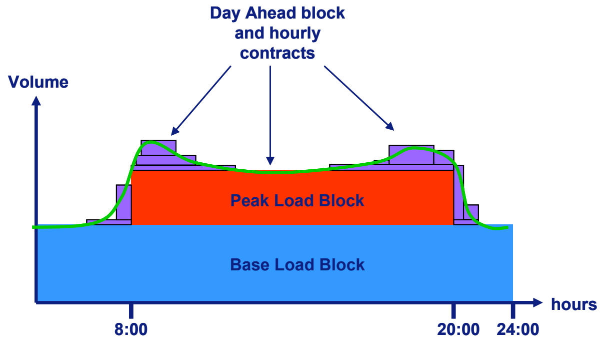
Australian Energy Market [1]. Here, generators offers to the AS market are incremental price functions of the available reserve capacity, depending on the energy dispatched at the PX level. In the remainder, we can hence neglect the coupling between prices, since the outcome of the clearing process is known when the ancillary services bids are sent to the TSO.

5.2.1 OTC contracts and day ahead market

Over the Counter (OTC) contracts are a widespread form of long-term arrangement, preventing market agents from being exposed to the highly volatile prices of energy. In this type of arrangement, the BRP can agree upon a supply contract with a retail company, for example yearly and split into base and peak load, based on a four season load profile of the retail company. The BRP is then obliged to deliver the contracted blocks of power for one year, as in Figure 5.3(a).



(a) The base and peak load contracted on OTC



(b) Realized power

Figure 5.3: Base and peak load OTC (source: APX-ENDEX)

In practice the load profile deviates from the planned load profile as given in Figure 5.3(b) from day to day.

These deviations are based on day-ahead predictions of the retail company which are sent to the BRP. Different possibilities can be chosen to handle them, namely:

1. OTC deals or contracts for the day-ahead;
2. Negotiating hourly bids and offers with an external Generator Company or BRP which may also result in OTC contracts;
3. Hourly bids and offers to the day-ahead market.

Figure 5.3(b) shows the day-ahead contracted buy volumes approaching the day-ahead load profile with the objective to reduce imbalance during the following day. As can be deduced from this figure, deviations will always arise on the day of contract execution.

One day before the delivery, the Transmission System Operator operates the clearing of the market crossing the aggregated day-ahead bid curve with the aggregated load profile (which is usually price-unelastic). The *clearing price* and *volume* for the spot market is the value detected by the intersection of the two curves, the clearing price is the price applied to every transaction on the market.

5.2.2 Ancillary Services Market

Regulating and reserve capacity can be up-regulating (involving situations of power shortages) or down-regulating (concerning situations of power overproduction). In conventional systems, every generator whose nominal capacity is greater than 60 MW is obliged to bid on the ancillary services market all the power they can increase (upward) or decrease (downward). Moreover, the market is only able to process *supply* bids from BRPs.

Supply S_i indicates the residual capacity a BRP i wants to *sell*, so that it wants to be paid for. It implies a positive cash flow, meaning that the BRP is receiving an amount of money from the TSO. In particular:

- S_i^+ stands for *positive* supply (BRP is willing to be paid for additional *production*),

- S_i^- stands for *negative* supply (BRP is willing to be paid for additional *absorption*).

In the current system BRPs can only take back their bids in the intraday market, for example in case some breakdown occurs and it becomes evident that the plant can not fulfill the submitted program. Being the intraday market closer in time to delivery, it is possible to better approximate the deviation from the scheduled E-Program.

5.3 A novel market design for ancillary services

The main element of innovation in E-Price is the double-sided nature of the AS markets. The concept, whose details are deepened later in this section, is to provide the TSO with a quite accurate estimate of the possible deviation from the E-Program, in such a way that the market can be prepared in advance, allowing to save imbalance costs. In the framework envisioned by E-Price, besides offers S^+ and S^-, ancillary services also include the request R_i, indicating all the energy BRP i wants to *buy* and can be bidirectional. It implies a negative cash flow, meaning that the BRP is willing to pay an amount of money. In particular:

- R_i^+ implies *positive* request (BRP expects to be "long" and hence is willing to pay for additional *absorption*),

- R_i^- implies *negative* request (BRP expects to be "short" and hence is willing to pay for additional *injection*).

Bids for the AS market refer to program time units (PTUs) of 15 minutes and can be sent up to one hour prior to the delivery. Let us define the following prices (in €/MWh):

- λ^{PX} is the day ahead price,

- λ^{AS-} is the price for down regulating capacity,
- λ^{AS+} is the price for up regulating capacity,
- λ^{-}_{imb} is the price for downward imbalance (power surplus),
- λ^{+}_{imb} is the price for upward imbalance (power shortage),

Sign convention is as follows: $\lambda^{AS+} \geq 0$, $\lambda^{AS-} \leq 0$, $\lambda^{+}_{imb} \geq 0$ and $\lambda^{-}_{imb} \leq 0$. A BRP can participate in both the AS- and AS+ markets. When the AS markets are cleared, the prices $\lambda^{AS+}(k)$ and $\lambda^{AS-}(k)$ are determined, as well as the the *net position* of each BRP $E_i^{AS+}(k)$ and $E_i^{AS-}(k)$. Therefore, from this process a BRP can either result as a *supplier* or a *requestor*. Payments for the only allocation of AS are proportional to the AS price $\lambda^{AS\pm}$. Trading on the AS markets can therefore lead to a positive profit (S) or to a cost (R):

$$I_{CA,i} = \sum_{k=1}^{N_{PTU}} aE_i^{AS+}(k)\lambda^{AS+}(k) + \sum_{k=1}^{N_{PTU}} aE_i^{AS-}(k)\lambda^{AS-}(k) \tag{5.1}$$

Where a is a design parameter of the market. In other words, the term a might be seen as the cost for participating to the double-sided market. For example, if a BRP results as a requestor of up-regulating power for PTU k, its cleared capacity $E_i^{AS+}(k)$ is negative (R^+) and it has to pay $aE_i^{AS+}(k)\lambda^{AS+}(k)$ to reserve the quantity $E_i^{AS+}(k)$ for regulating purposes. We use the sign convention for the cleared volumes and prices as shown in Figure 5.4.

In real time operations, the TSO sends in each T_P seconds (in the simulation framework, $T_P = 4$) a request signal called ΔP_i to BRPs, which is the request for varying the power output of controllable generators. If the need for upward regulating energy occurs, the TSO sends in positive

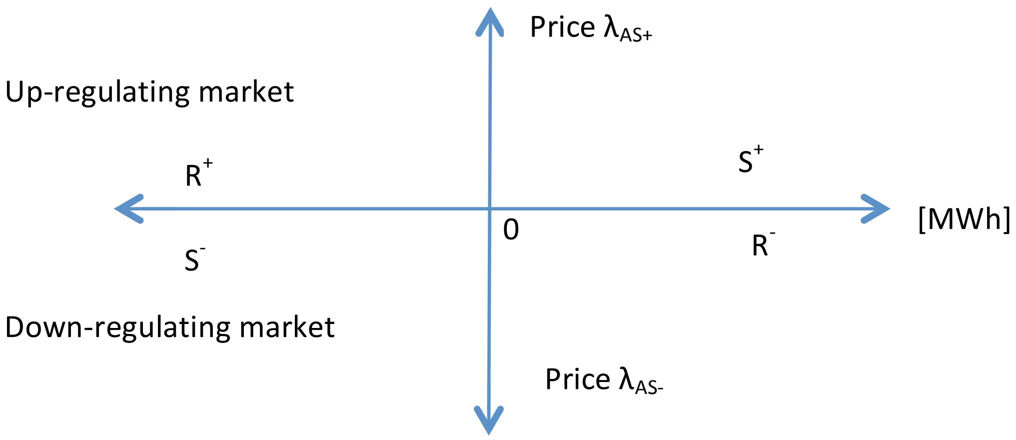


Figure 5.4: Volumes and prices cleared at the AS market: upper part up-regulating, lower part down-regulating

ΔP_i, on the contrary, if too much energy is present on the grid, the TSO transmits negative ΔP_i. The signal ΔP_i is distributed among BRPs based on their cleared capacity. The profit obtained by the supply of regulating power on the AS in PTU k is defined as follows:

$$I_{AS,i}(k) = \sum_{t=1}^{NT_p}(w\Delta P_i(t)\lambda^{AS+}(t) + (1-w)\Delta P_i(t)\lambda^{AS-}(t))\frac{T_P}{3600} \tag{5.2}$$

where

$$w = \begin{cases} 1 & \text{if } \Delta P_i(t) \geq 0, \\ 0 & \text{if } \Delta P_i(t) < 0 \end{cases} \tag{5.3}$$

where NT_p is the number of T_P periods in a PTU. Note that $I_{AS,i}$ always denotes a profit, as the signs of ΔP_i and λ^{AS} are always concordant.

One could argue that the intraday market is already in place with the aim to trade the expected deviations from E-Program arising from better forecasts of renewable sources and prices. However, intraday markets and double-sided AS differ in that the energy on the intraday market is *traded*, meaning that the transactions have to take place if one does not want to incur imbalance costs, while on the double-sided AS market reserves are only allocated, and called if the need for the reserved regulating capacity arises in real time. So, only the energy actually needed is delivered. Moreover, it has been observed that Dutch intraday markets lack liquidity, thus meaning that BRPs are not appropriately incentivized to participate. The aim of E-Price is to design a market for ancillary services where market participants receive economic incentives to bid.

5.3.1 Double-sided markets: the concept

We now recover the underlying concept of the double sided AS auction. The innovative market arrangement we refer to has been presented for the first time in [74].

All the calculations are expressed in terms of power, that is linked to energy by the relation $E = PTs$, where Ts is the sampling time (here, $Ts = 15$ min $= 1$ PTU). Each BRP has at disposal controllable (gas, coal, nuclear) and uncontrollable generators (wind, solar). Each plant is characterized by specific switch on/off costs, marginal costs and efficiency. The production of the controllable generators p_i^c range in the interval $[\underline{p}_i^c; \overline{p}_i^c]$, while for the uncontrollable generation p_i^u the BRP only has some forecast, for which a probability density function can be defined with mean $\tilde{p}_i^u$.

The mean value is used to bid on the day-ahead market. Only this value is meaningful over such a long time horizon. The expected value of uncontrollable production is then offered at the day-ahead market, or, equivalently, is withdrawn from the Unit Commitment evaluation, that is executed after the communication of the E-Program to decide which plants have to be switched on during the next day and at what power level they must be operating in order to produce the assigned energy. Combining the controllable and uncontrollable generation we obtain a total power production of $p_i = p_i^u + p_i^c$, as shown in Figure 5.5, ranging from $\underline{p}_i = \underline{p}_i^c + \tilde{p}_i^u$ to $\overline{p}_i = \overline{p}_i^c + \tilde{p}_i^u$.

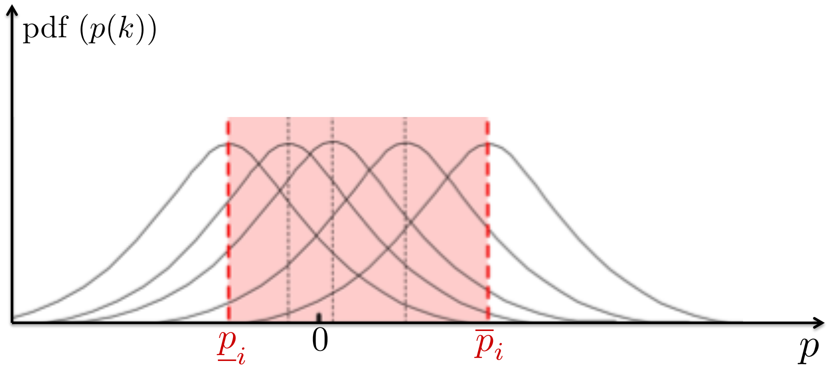


Figure 5.5: Superposition of controllable and uncontrollable power

The BRP bids the whole power from $\underline{p}_i$ to $\overline{p}_i$. After clearing, the BRP knows that the power $p_i(k)$ must be produced at PTU k of the next day. This will be the expected value of its total production, as shown in Figure 5.6. However, due to stochasticity, the production can be in any point on the curve in Figure 5.6.

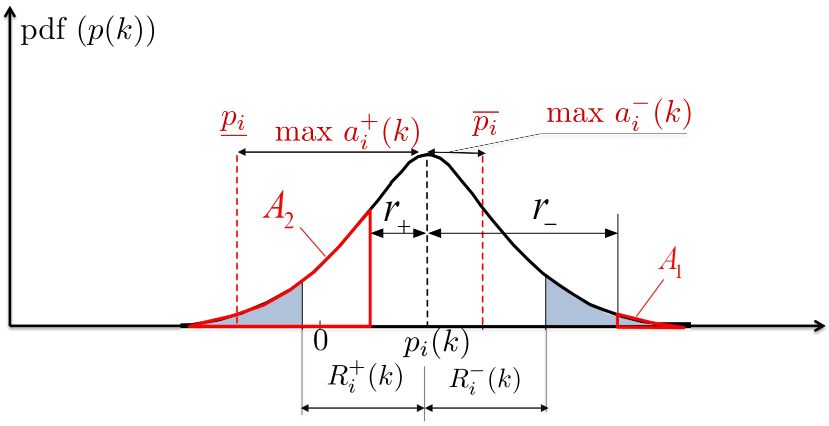


Figure 5.6: Upward and downward regulating capacity

The quantities max $a_i^+(k)$ and max $a_i^-(k)$, respectively the difference between $p_i(k)$ and the lower controllable boundary $\underline{p}_i$ and the difference between the upper saturation limit $\overline{p}_i$ and $p_i(k)$, represent the power that can be increased or decreased by controllable generation. In fact, if the actual production is higher than $p_i(k)$ (for example, if more wind is blowing), BRP i can use the controllable generators to fulfill the E-Program, up to the quantity $r_- = \max\ a_i^+(k)$. The same holds in case of under-production, where the BRP can increase its own generators up to $r_+ = \max\ a_i^-(k)$. As a result, if the BRP wants to use own controllable generation to keep up with the schedule, it faces a risk of imbalance highlighted by the red areas A_1 (over-production) and A_2 (under-production). Since using own controllable generation has costs that are sometimes higher than participating to the AS market, it might be not economically beneficial to fully employ it to fulfill the E-Program. If the BRP decides to hedge only against a certain percentage of its variability, let us say for the interval $[-R_i^+(K), R_i^-(k)]$, the risk of imbalance is reduced to the sum of the two blue areas in Figure 5.6. The BRP might be not able to cover for all the desired regulating reserve or, on the contrary, it could be sometimes able to cover for more than required. In conclusion, to cover for the interval $[-R_i^+(K), R_i^-(k)]$, the BRP can *buy* or *sell* a certain amount of reserve on the market.

The quantities r_+ and r_- are offered to the market in the form of (upward regulating and downward regulating) bids. This market design, that might seem counterintuitive because the BRP offers and requires power in the same direction is proved to be consistent and efficient for the ancillary services market (cf. [43]).

In conclusion, the bidding problem can be considered as a two-stage problem. At the first stage the BRP bids on the day-ahead market, where both day-ahead and AS prices are unknown. At the second stage AS bids have to be sent, but information on day-ahead prices have been revealed.

The resultant process is schematized in Figure 5.7.

First, based on historical market data and internal portfolio, BRPs bid

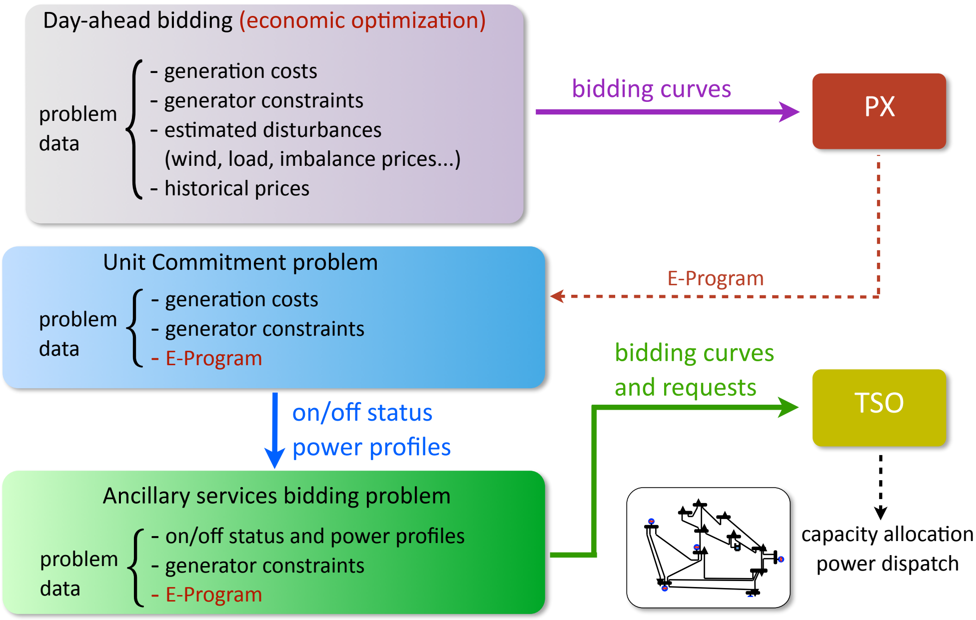


Figure 5.7: The day-ahead process

at the Power Exchange. Bids and offers are collected and the PX performs the clearing process. Public results are disclosed, and sent back to the BRPs. At this point, an Unit Commitment algorithm estimates a rough approximation of the power profile needed to comply with the committed volume, also deciding which plants are to be switched on/off during each PTU. The Unit Commitment output influences the AS Bidding Algorithm together with load and wind forecasts. AS bidding curves and requests are obtained, which again are cleared by the TSO, deciding the amount of allocated capacity based on demand and offer.

5.3.2 The imbalance system

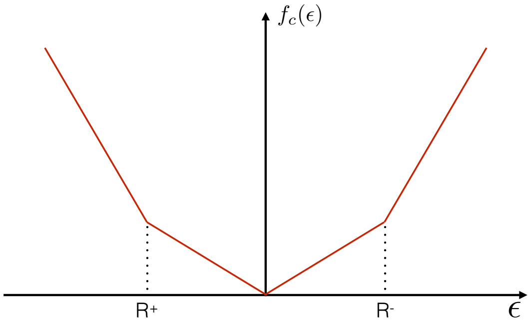
Any deviation from the scheduled E-Program is considered as *imbalance* (see [72]). Imbalances are settled by the TSO, which has previously bought capacity on the ancillary services market. In the remainder of this section, all prices and quantities are intended *per PTU*, and therefore we omit the time index k. Should the need of regulating power occur, the TSO sends a control signal to each BRP, based on the received bids. For each PTU the imbalance prices $\lambda^{\pm}_{imb}$ are calculated, which are the prices for deviations from E-Program. The price $\lambda^{\pm}_{imb}$ depends on the AS price and on the Control State (CS). This value indicates the direction of the system imbalance and can take the values $+1, -1, 0$ or 2. If only positive capacity is required (the system has a power shortage) CS has value $+1$, while if the system has a power surplus and only negative capacity is triggered the CS has value -1. If neither positive nor negative regulating capacity is required CS has value 0, while if both positive and negative capacity are required in the same PTU CS is equal to 2. Therefore the imbalance price is calculated as $\lambda^{+}_{imb} = (1 + a + \phi)\lambda^{AS+}$ in up-regulating mode ($CS = 1$), and $\lambda^{-}_{imb} = (1 + a + \phi)\lambda^{AS-}$ in down-regulating mode ($CS = -1$), and both $\lambda^{+}_{imb} = (1 + a + \phi)\lambda^{AS+}$ and $\lambda^{-}_{imb} = (1 + a + \phi)\lambda^{AS-}$ are calculated in two-sided regulating mode ($CS = 2$), where ϕ is a proportional factor. When the control state is 0, no upward nor downward regulating power is asked, and $\lambda^{\pm}_{imb} = 0$.

If the system is lacking power, up-regulating reserves are activated ($CS = 1$). BRPs causing imbalance (sourcing too much power from the system), pay their deviation at price λ^{AS+} as far as this deviation is within the cleared capacity R^{+}_{i}, the power exceeding this quantity is paid at λ^{+}_{imb}. Contrarily, if the system has a power surplus ($CS = -1$) and the BRP is injecting too much energy, it will pay the cleared capacity R^{-}_{i} at λ^{AS-} and the excess deviation at λ^{-}_{imb}. There are basically three ways the TSO can handle *passive balancing*, i.e. causing imbalance that helps the system, namely:

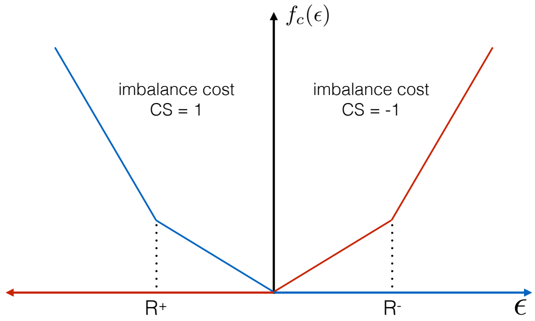
1. *Payment for passive balancing*: the BRP pays imbalance costs for deviating even though the imbalance is actually helping the system;

2. *No payment for passive balancing*: the BRP causing imbalance with opposite sign with expect to the system imbalance is not rewarded nor penalized;

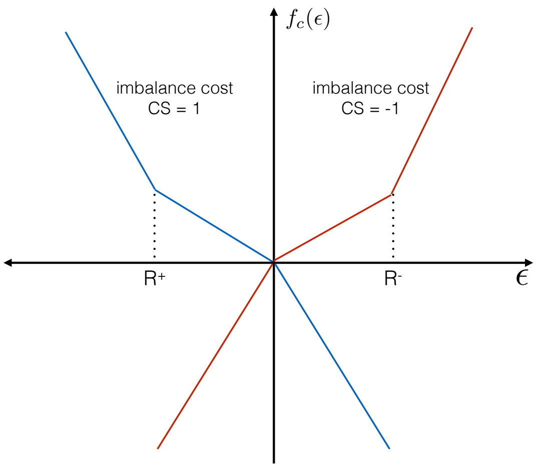
3. *Reward for passive balancing*: the BRP receives a reward for helping the system restore the balance, even though it is deviating from its schedule.



(a) Imbalance costs in case of payment for passive balancing



(b) Imbalance costs in case of no payment for passive balancing



(c) Imbalance costs in case of reward for passive balancing

Figure 5.8: Imbalance costs in the three situations

The three situations are depicted in Figure 5.8. Denoting by ϵ_i the imbalance committed by BRP i, the corresponding imbalance payment

$f_c(\epsilon_i)$ in case a is defined by the following set of equations:

$$\begin{aligned}
f_c(\epsilon_i) &= \min z_i && (5.4)\\
z_i &\geq -\lambda^{AS+}\epsilon_i && (5.5)\\
z_i &\geq -\lambda^{AS+}R_i^+ + \lambda_{imb}^+(R_i^+ - \epsilon) && (5.6)\\
z_i &\geq -\lambda^{AS-}\epsilon_i && (5.7)\\
z_i &\geq -\lambda^{AS-}R_i^- + \lambda_{imb}^-(R_i^- - \epsilon) && (5.8)
\end{aligned}$$

with the auxiliary variable z_i. In this case, irrespective of the regulation state, any imbalance is paid at $\lambda^{AS\pm}$ for the portion not exceeding the allocated capacity $R_i^\pm$, at $\lambda_{imb}^\pm$ for the remaining part. In situation b the imbalance cost function $f_c(\epsilon_i)$ depends on the regulating state. If $CS = 1$ then

$$\begin{aligned}
f_c(\epsilon_i) &= \min z_i && (5.9)\\
z_i &\geq -\lambda^{AS+}\epsilon_i && (5.10)\\
z_i &\geq -\lambda^{AS+}R_i^+ + \lambda_{imb}^+(R_i^+ - \epsilon_i) && (5.11)\\
z_i &\geq 0 && (5.12)\\
& && (5.13)
\end{aligned}$$

while if $CS = -1$

$$\begin{aligned}
f_c(\epsilon_i) &= \min z_i && (5.14)\\
z_i &\geq -\lambda^{AS-}\epsilon_i && (5.15)\\
z_i &\geq -\lambda^{AS-}R_i^- + \lambda_{imb}^-(R_i^- - \epsilon_i) && (5.16)\\
z_i &\geq 0 && (5.17)\\
& && (5.18)
\end{aligned}$$

Finally, in case c the imbalance cost function reads as:

$$f_c(\epsilon_i) = \max\{-\lambda^{AS+}R_i^+ + \lambda_{imb}^+(R_i^+ - \epsilon_i), \min\{-\lambda^{AS+}\epsilon_i, -\lambda_{imb}^+\epsilon_i\}\} \quad (5.19)$$

with $CS = 1$ and

$$f_c(\epsilon_i) = \max\{-\lambda^{AS-}R^- + \lambda_{imb}^-(R_i^- - \epsilon), \min\{-\lambda^{AS-}\epsilon_i, -\lambda_{imb}^-\epsilon_i\}\}. \quad (5.20)$$

Of course, in any case prices have to be adjusted in order to guarantee financial neutrality for the TSO: no profit or loss should come from the system balancing. This means for example that in case of no payment for passive balancing, active imbalance must be lower than the total supplied regulating power because there were BRPs actually helping the system without being paid for that. Imbalance prices should therefore be lower because not all the needed regulating capacity has been supplied via AS reserves. This is achieved by solving *a posteriori* the equation (at each PTU k)

$$\sum_{i=1}^{N_{BRP}} I_{AS,i} + \sum_{i=1}^{N_{BRP}} I_{imb,i}(\gamma) = 0 \quad (5.21)$$

where N_{BRP} is the number of BRPs, $I_{AS,i} = \sum_{k=1}^{N_{PTU}I_{AS,i}(k)}$ is the total income for selling regulating power in real-time, $I_{imb,i}$ is the total imbalance cost and γ is a correcting factor. Note that $I_{imb,i} = \sum_{k=1}^{N_{PTU}} f_c(\epsilon_i, k)$. In our framework we choose a combination of situation a and b: neither penalty nor reward is given for passive balancing in up-regulating mode $CS = 1$, while payment for both active and passive balancing is applied in down-regulating mode $CS = -1$. This is justified by the considered affine cost structure: if negative imbalance is not penalized in down-regulating mode, every BRP will be automatically incentivized to set each generator to the minimum to save production costs.

5.3.3 Day-ahead operations

In conclusion, on the day before the execution the BRP has to make decisions on:

1. The bidding curve for the day-ahead auction, intended as a piece-wise constant curve expressing the minimum required price (€$/MWh$) for producing a given power at each hour of the following day, taking into account uncertainty about AS prices,

2. The AS bidding curves, offering up-regulating and down-regulating capacity based on the residual capacity allowed by the Unit Commitment and accounting for wind and load stochasticity. This curves are also formulated as piece-wise constant curves expressing the minimum expected reward for increasing/decreasing the power set point by a MWh.

3. The quantities R_i^+ and R_i^- to request to the market in order to hedge against imbalances. In our framework the costs for participating to the double-sided market are set as $\lambda^{R+} = a\lambda^{AS+}$ and $\lambda^{R-} = a\lambda^{AS-}$, where $a \in [0.05, 0.15]$ is a constant deemed as the opinion that the system has about the possibility of doing imbalance. In other words, the BRP resulting as *requestor* pays a fee for allocating AS services.

Chapter 6

Proposed solutions for bidding on DA and AS markets

In Section 5.3 we have formalized the main problems to be solved by BRPs for trading on the energy and AS markets. In this chapter we present the proposed solutions to cope with those problems. Specifically, in Section 6.1 we present some previous work on optimal bidding, in Section 6.2 we describe the day ahead strategy, in Section 6.4 the AS bidding approach is presented. In Section 6.5 experimental results obtained from testing the proposed market strategies in a simulation environment are reported and in Section 6.6 some conclusions are drawn with respect to the proposed approach.

6.1 State of the art

Developing a day-ahead bidding strategy is a complex task, since it requires a careful evaluation of a wide set of variables, both external (prices, weather forecast), and internal (portfolio,cost structure, risk attitude). It is not easy to compare the proposed solutions to the current standard practice. Generally speaking, BRPs with little installed capacity tend to bid at marginal costs to be competitive, while non price-taker companies have own internal bidding strategies that cannot be disclosed. The problem of composing optimal bidding strategies at the power exchange and capacity reserves market has been extensively addressed in the literature especially since the electricity market deregulation. The first works after the market liberalization typically accounted for two or three predetermined bidding levels among which to choose (e.g. bid low or bid high) [62, 69]. In [69] the authors, by means of illustrative examples, apply dynamic programming to formulate optimal bidding problems. The example refer to simple generating units (hydro, thermal) and show how restrictions on the form of the curves (for example, non-decreasing conditions) affect the bidding strategy. Dynamic programming is used to tackle problems where the state and the dispatch at period k are correlated to the ones at period $k + 1$, for example in the case of hydro plants where the level of the reservoir depends on the dispatch strategy adopted in the previous period and final conditions are imposed (example, the reservoir must be empty at the final period). Numerous works relate to optimization of hydro-power production [26, 27, 44]; this is an insightful and interesting problem, where decisions at each stage heavily affect subsequent steps. The paper [16] addresses the bidding problem from the perspective of a price-taker thermal producer bidding in the day-ahead market under price uncertainty. Of course, game theoretical approaches and the determination of the Nash equilibrium among competing players has always played a role [45, 46]. An integrated bidding and scheduling algorithm with risk management under a deregulated market using a combination of Lagrangian relaxation and stochastic dynamic programming is proposed in [49]. The requested form of the bidding curves also influences the market strategy. An approach based on Monte Carlo simulation and Genetic Algorithms where player bid linear supply curves taking into account rival behavior is shown in [32]. In [35] the authors propose an optimal bidding strategy for thermal and generic programming units in the day-ahead market in presence of Virtual Power Plants (VPP) and bilateral contracts. An heuristic bidding strategy for buyers and sellers in a continuos double auctions is proposed in [52]. The beneficial effect of having delay tolerant consumers in the portfolio to cope with renewables uncertainty is described in [61].

An approach similar to the one described in this thesis is used in [29]. The authors propose two different bidding strategies for the PX and the AS market for suppliers with marginal generating units. At the PX level the goal is to maximize benefit from trading on both energy markets. If a unit is dispatched then the supplier can proceed with the selected strategy, otherwise either the generating unit must be shut down leading to shut-down/switch-on costs, or another bidding strategy for the AS reserve market should be operated, in order to guarantee the minimum stable output for the unit. An unit commitment problem is then solved in order to minimize the number of shut-down/switch-on cycles. The framework of the cited paper is the California market, where the AS auction is cleared after the PX and suppliers submit *linear* bid curves. Hence, BRPs must determine the coefficients of two linear functions (PX and AS) for each hour of the day.

The BRP day-ahead and AS bidding problems present different conditions. In fact, energy cannot be stored and there is not any particular restriction imposed to the generators. That is the reason why we chose to formulate independent stochastic optimization problems for each hour (or PTU) of the day.

As in [16] the proposed BRP day-ahead bidding strategy selects the optimal dispatch corresponding to a price level, and composes the bid curves interpolating the obtained optimal pairs. The idea behind is the following: a BRP with a portfolio of generating units of various types (coal, thermal, nuclear) aims at maximizing their efficiency, operating them at a non-minimum power level. Hence at the day ahead level, when the optimization is still coarse, one plans to keep the generators status steady, in order to operate them efficiently and to save cycle costs. This is the reason why the day-ahead algorithm considers the operating status ON/OFF of the plants as fixed. After the definition of the E-Program, the unit commitment is executed. If the energy assigned during several subsequent PTUs is near the minimum power level of the plants, then the UC can decide to shut down one or more costly plants, distributing their scheduled production among the other generators. Contrarily, if the assigned E-Program is close to the upper bound, the UC algorithm could decide to switch on some plants in order to have them operating for offers to the AS market. Finally, one hour ahead, when the generator status is fixed (it is late to change it, since switch on/shut down times can take up to 6 hours) and uncontrollable production and load is highly predictable, AS bids are submitted.

6.2 Day-ahead bidding strategy

The first high-level task of a BRP consists of composing offers for the day ahead market (DAM). Each day BRPs compose and submit offers to the PX. This activity is crucial for the BRP's economic equilibrium, and involves the analysis of several sources of uncertainty. The question a BRP has to answer to, in order to take the optimal decision, is what amount of the total production capacity of the generators should be offered to the PX market, and how much energy, if any, should instead be reserved for the control reserves market, in order to optimize an economic objective that can be the pure expected profit or a risk-based index. The bid/offers submitted by BRPs for the DAM and for the control reserves are in the form of piecewise constant curves, one for each hour of the following day.

A BRP owning one or several generators is in general a producer and hence submits sell orders or shortly called offers (ask). However, it can also submit buy orders or shortly called bids when market clearing prices are expected to be so low that it is convenient to buy energy, because buying is cheaper than producing. Regulating and reserve capacity can be directed both upwards (feeding energy into the grid) and downwards (source energy from the grid). Aggregated offers for the PX from all BRPs constitute the basis for the auction process, whose result is the E-Program, the bulk schedule for the following day.

The DAM bidding problem can be seen as a two-stage optimization problem: At the first stage the BRP has to make offers on the day ahead Market without knowing the ancillary services prices; at the second stage, once the quantity of energy allocated to the DAM is known, BRP has to make bids/offers on the ancillary services Market.

The problem of deciding both the energy volume to offer on the market and the corresponding price is clearly bilinear[1]. In order to keep the problem linear, prices are fixed to some user-defined values and energy volumes are computed such that they are optimal for the chosen prices (a similar approach has been taken, e.g., in [26]). Let $\lambda_1^{PX}, \lambda_2^{PX}, \ldots, \lambda_N^{PX}$ be the sequence of fixed PX prices. For each λ_p^{PX}, we generate $2L$ scenarios of possible prices for the ancillary services Market λ_s^{AS}, $s = 1, 2, \ldots, 2L$, constituting the second stage of the optimization problem. Then, the problem is solved for each of the generated prices λ_p^{PX}, obtaining the energy volumes E_p^{PX} to offer on the market, with $p \in 1, 2, \ldots, N$. Finally, the

[1] A function is bilinear when it is linear in each of its variables. The simplest example is $f(x, y) = xy$.

piecewise constant bid curve is constructed by interpolation as

$$\lambda(E) = \begin{cases} \lambda_1 & \text{if} \quad E_1 \leq E < E_2, \\ \lambda_2 & \text{if} \quad E_2 \leq E < E_3, \\ \vdots & \vdots \\ \lambda_{N-1} & \text{if} \quad E_{N-1} \leq E < E_N, \\ \lambda_N & \text{if} \quad E = E_N. \end{cases} \tag{6.1}$$

In order to solve the problem for the PX we assume a *price taker* point of view, that is, the considered BRP's offers do not influence the market and the BRP accepts any couple (energy, price) decided by the market on the proposed bidding curve. In the following section the procedure to generate DAM and ASM price scenarios is described.

6.2.1 Scenario Generation for DAM and ASM prices

A set of N values of expected DAM prices $\lambda_1^{PX}, \lambda_2^{PX}, \ldots, \lambda_N^{PX}$ are determined by uniform sampling of the interval of possible prices $[0, \lambda_{max}^{PX}]$, with user-defined step size λ_{step}, where λ_{max}^{PX} is the maximum price considered in the DAM supply curve. Note that λ_{max}^{PX} is taken so that the corresponding energy amount offered on the market is equal to the maximum production capacity of the BRP.

ancillary services prices are generated based on historical data relating control energy prices (as differential to the day ahead Market price) with the system imbalance, provided by TenneT[2]. This relation, shown in Figure 6.1, is market driven and appears quite stable and robust over the years, hence being suitable to be used for modeling purposes. Any other empirical distribution can be used in the optimization to generate AS prices. The left side of the graph refers to the case where the mar-

[2]TenneT is the Dutch TSO since1998

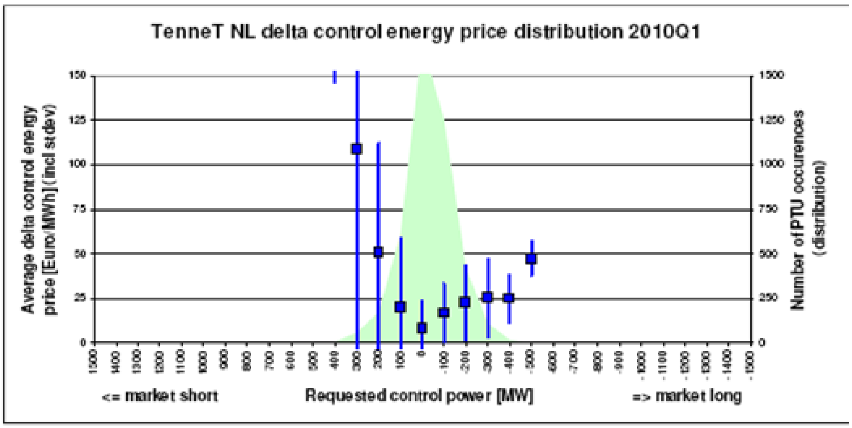


Figure 29 Control energy price distribution TenneT NL Q1

Figure 6.1: Relation between PX and Control prices (source: TenneT)

ket is short, BRPs sell Energy to the TSO, and the ASM price is higher than the DAM price. The right side is related to the case where there is surplus of energy, BRPs buy energy from the TSO, and the ASM price is below the DAM price. Given an expected DAM price λ_i^{PX}, a discrete set of possible ASM prices is inferred from the graph and used in the optimization problem. ASM scenario generation yields a set of L *downward* ASM prices $\lambda_1^{AS-}, \lambda_2^{AS-}, \ldots, \lambda_L^{AS-}$ related to the case where there is surplus of energy so ASM price is lower than DAM price, and a set of L *upward* prices $\lambda_1^{AS+}, \lambda_2^{AS+}, \ldots, \lambda_L^{AS+}$ concerning the case where we expect the market to be short, so the ASM price is greater than the DAM price. Probabilities of each considered ASM price, denoted by $\pi_1^{AS-}, \ldots, \pi_L^{AS-}$, $\pi_1^{AS+}, \ldots, \pi_L^{AS+}$, are also empirically inferred from the graph.

6.2.2 Generators model

In order to construct the bid curve the BRP has to take into account production costs and efficiency of generators. Since unit commitment is decided only after that the E-Program has been defined, here we use an approximate model of generators where the current plants ON/OFF status is assumed to be constant for the following day and no start-up and shut-down costs are considered.

The BRP model includes n controllable generators $\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_n$. Maximum and minimum power of each generator are defined by $P_{max,j}$ and $P_{min,j}$, respectively, $j = 1, 2, \ldots, n$. Efficiency of each generator $\mathcal{E}_j$ is defined as

$$\mathcal{E}_j = M_j \left(a_2 \left(\frac{P_j}{P_{max,j}} \right)^2 + a_1 \frac{P_j}{P_{max,j}} + a_0 \right) \tag{6.2}$$

where a_0, a_1, a_2 are given coefficients, that are dependent on the type of generator, M_j is the maximum efficiency achievable by the generator, and P_j is the power set-point. Minimum and maximum energy that can be produced by each generator $\mathcal{G}_j$ in an hour are denoted, respectively, by $E_{min,j}$ and $E_{max,j}$, $j = 1, 2, \ldots, n$, and are computed as a function of the power level at the previous PTU and of the ramp rate specifications of each generator. The cost of producing 1 MW by generator $\mathcal{G}_j$ is defined using (6.2) as

$$\mathcal{C}_j = \frac{T C_j P_j}{M_j \left(a_2 \left(\frac{P_j}{P_{max,j}} \right)^2 + a_1 \frac{P_j}{P_{max,j}} + a_0 \right)}, \tag{6.3}$$

where C_j is the fuel price in €/MWh and P_j is assumed constant over the time interval $T = 1$ [h], $j = 1, 2, \ldots, n$.

6.2.3 Optimization problem formulation

The optimization problem to be solved for each hour $h = 0, 1, \ldots, 23$, and for each of the N fixed DAM prices λ_p^{PX}, $p = 1, 2, \ldots, N$, is formulated as follows

$$\min \quad \ell + \frac{1}{1-\beta}\left(\sum_{s=1}^{L} \pi_s^{AS+}[f_s - \ell]^+ + \sum_{s=L+1}^{2L} \pi_{s-L}^{AS-}[f_i - \ell]^+\right) \tag{6.4a}$$

$$\text{s.t.} \quad f_s = \sum_{j=1}^{n} \frac{C_j u_{sj}}{M_j\left(a_{2,j}\left(\frac{u_{sj}}{TP_{max,j}}\right)^2 + a_{1,j}\frac{u_{sj}}{TP_{max,j}} + a_{0,j}\right)} + [\lambda_{imb,s}x_{imb,s}]^- - \lambda_p^{PX}x_p^{PX} - \lambda_s^{AS+}x_{up,s}^{AS},\ s = 1, \ldots, L, \tag{6.4b}$$

$$f_s = \sum_{j=1}^{n} \frac{C_j u_{sj}}{M_j\left(a_{2,j}\left(\frac{u_{sj}}{TP_{max,j}}\right)^2 + a_{1,j}\frac{u_{sj}}{TP_{max,j}} + a_{0,j}\right)} + |\lambda_{imb,s}x_{imb,s}| - \lambda_p^{PX}x_p^{PX} + \lambda_{s-L}^{AS-}x_{do,s-L}^{AS},\ s = L+1, \ldots, 2L, \tag{6.4c}$$

$$\underline{u_j} \leq u_{sj} \leq \overline{u_j},\ j = 1, \ldots, n,\ s = 1, \ldots, 2L, \tag{6.4d}$$

$$\sum_{j=1}^{n} u_{sj} - x_p^{PX} - x_{up,s}^{AS} = x_{imb,s},\ s = 1, \ldots, L, \tag{6.4e}$$

$$\sum_{j=1}^{n} u_{sj} - x_p^{PX} + x_{do,s-L}^{AS} = x_{imb,s},\ s = L+1, \ldots, 2L, \tag{6.4f}$$

$$x_{do,s}^{AS} \leq x_p^{PX},\ s = 1, \ldots, L, \tag{6.4g}$$

$$x_p^{PX} \geq 0, \tag{6.4h}$$

$$x_{up,s}^{AS} \geq 0,\ s = 1, \ldots, L, \tag{6.4i}$$

$$x_{do,s}^{AS} \geq 0,\ s = 1, \ldots, L. \tag{6.4j}$$

where $T = 1$ [h]. The decision variables of the optimization problem are:

- $\ell \in \mathbb{R}$: variable for CVaR approximation (see Section 2.1.3),
- $x_p^{PX} \in \mathbb{R}$: energy offered on the day ahead market at price λ_p^{PX},

- $x_{up,s}^{AS} \in \mathbb{R}$: energy reserved for the ancillary services Market for upward direction at price $\lambda_{up,s}^{AS}$, $s = 1, 2, \dots, L$,
- $x_{do,s}^{AS} \in \mathbb{R}$: energy reserved on the ancillary services Market for downward direction at price $\lambda_{do,s}^{AS}$, $s = 1, 2, \dots, L$,
- $x_{imb,s} \in \mathbb{R}$: estimated energy imbalance for sth scenario, $s = 1, 2, \dots, 2L$,
- $u_{sj} \in \mathbb{R}$: energy produced by the jth generator, $j = 1, 2, \dots, n$, related to the case where the energy bid on the ASM is $x_{up,s}^{AS}$, if $s = 1, 2, \dots L$, or $x_{do,s-L}^{AS}$, if $s = L+1, L+2, \dots, 2L$.

The control parameters that can be used to tune the bidding algorithm and differentiate the behavior of the BRPs are:

- $\beta \in \mathbb{R}_+$: a coefficient modeling BRP's attitude towards risk. Common values for β are in the range [0.9, 0.99], where higher values imply a more conservative approach.
- $\phi \in \mathbb{R}_+$: a penalty term used to derive λ_{imb} as $\lambda_{imb} = \phi\lambda^{AS\pm}$. Common values for ϕ are in the range [0.05, 0.1], where higher values make the BRP less likely to generate imbalance.
- $\lambda_{step} \in \mathbb{R}_+$: the step size used to generate N DAM energy prices and construct the piecewise constant bid curve. Common values for λ_{step} are in the range $[1, 20]$, where smaller values allow for a finer bid curve, but require more computational load.

Constraints (6.4b)-(6.4c) define the loss function as the difference between expected production and imbalance costs, and expected profits. Imbalance costs are modeled as $[\lambda_{imb,s}x_{imb,s}]^-$, for $s \in \{1, 2, \dots, L\}$ and as $|\lambda_{imb,s}x_{imb,s}|$, for $s \in \{L+1, L+2, \dots, 2L\}$, hence passive balancing is not rewarded nor penalized in up regulating mode $CS = 1$, while both passive and active imbalance are penalized when $CS = -1$. The estimated imbalance price is taken here as a function of the ASM expected price, namely

$$\lambda_{imb,s} = \begin{cases} (1+a+\phi)\lambda^{AS}_{up,s} & \text{if} \quad s \in \{1,\ldots,L\}, \\ (1+a+\phi)\lambda^{AS}_{do,s-L} & \text{if} \quad s \in \{L+1,\ldots,2L\}, \end{cases} \tag{6.5}$$

Constraints (6.4d) impose boundaries on the minimum and maximum energy production of generators. Constraints (6.4e) and (6.4f) define the expected energy imbalance, considering that no energy can be stored. Constraint (6.4g) prevents the BRP from offering a negative downward capacity.

Problem (6.4) is a nonconvex optimization problem, due to constraints (6.4b) and (6.4c). In order to solve it, production costs can be approximated by affine or quadratic curves, yielding a convex problem (respectively, a QP or a QCQP). Production costs for generators of BRP #1 and their affine approximations are shown in Figure 6.2. The overall BRP day ahead bidding procedure is listed in Algorithm 2. An example of bidding curve is shown in Figure 6.3.

Algorithm 2 BRP day ahead bidding algorithm

For each hour $h \in \{0,1,\ldots,23\}$:

1. Update the estimation of initial power set-points P_0 for hour h;
2. Compute minimum and maximum energy $\underline{u_j}$, $\overline{u_j}$, $j = 1,2,\ldots,n$, given the estimated power set-point P_0, the maximum ramp rate and the efficiency curves of each generator;
3. Generate a set of N possible DAM prices $\lambda^{PX}_1, \lambda^{PX}_2, \ldots, \lambda^{PX}_N$;
4. For each fixed DAM price, generate L ASM downward prices $\lambda^{AS-}_1, \lambda^{AS-}_2, \ldots, \lambda^{AS-}_L$ and L ASM upward prices $\lambda^{AS+}_1, \lambda^{AS+}_2, \ldots, \lambda^{AS+}_L$ together with their corresponding probabilities $\pi^{AS-}_1, \ldots, \pi^{AS-}_L$, $\pi^{AS}_{+1}, \ldots, \pi^{AS+}_L$, and solve the optimization problem (6.4);
5. Build the bidding curve for day d and hour h, according to (6.1).

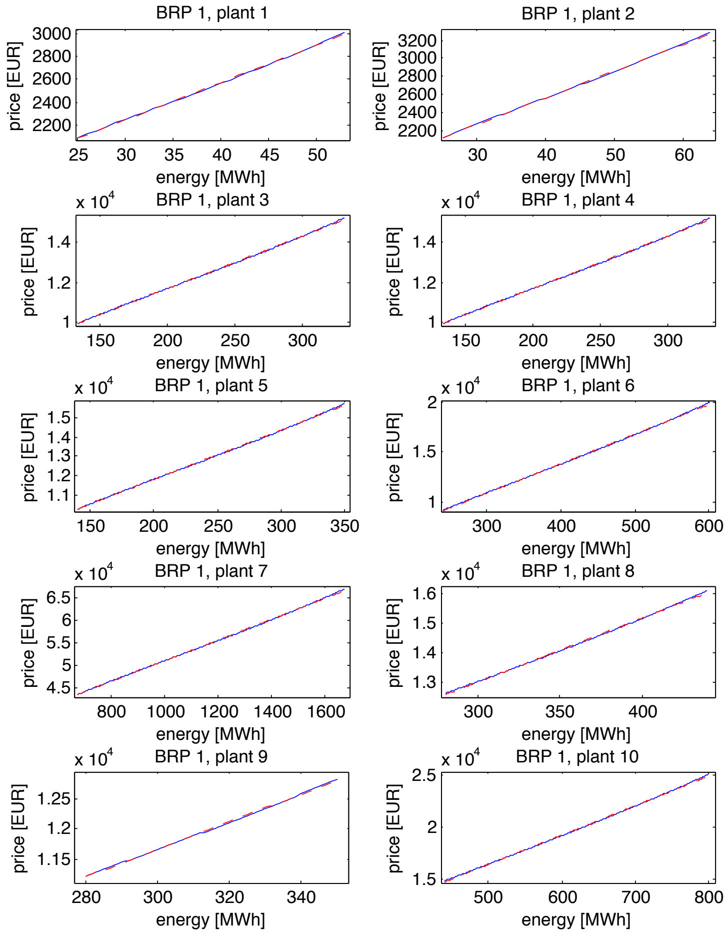


Figure 6.2: Production costs (blue, solid line) and affine approximations (red, dashed line) for BRP #1

6.2.4 Formulation of buy curves on the day ahead market

The algorithm described in the previous section deals with the formulation of offers to sell the energy produced by the BRP on the DAM. However, a BRP could also have internal loads that need to be satisfied in order to fulfill OTC contracts. In this framework, the energy needed to satisfy such loads, minus possibly available energy from intermittent generation, is assumed to be bought on the market. Namely, let l_h and $\hat{r}_h$ be the internal load and the expected energy production from uncontrollable generation plants, respectively, at hour h of the considered day. Then, for each hour

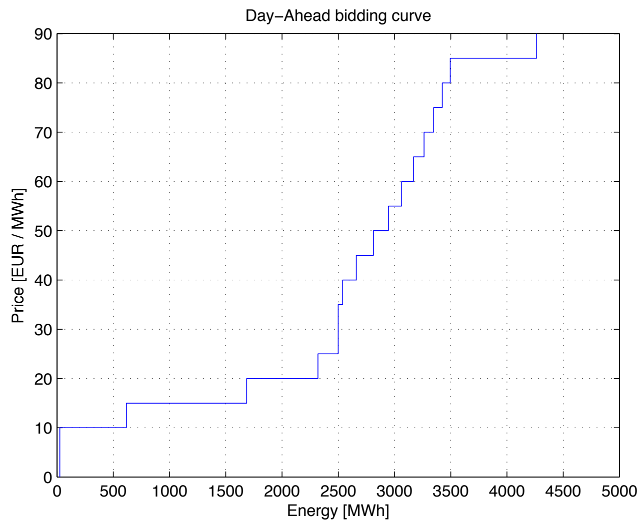


Figure 6.3: day ahead bidding curve obtained from Algorithm 2

of the day $h = 0, 1, \ldots, 23$, a bid is submitted to the DAM in the form of a price-inelastic curve, where the energy requested is equal to $l_h - \hat{r}_h$, at any price set by the market clearing process. In other words, generation from renewables is treated as a negative load. Possible imbalances due to the difference between the expected intermittent generation $\hat{r}_h$ and the real value r_h observed at hour h are compensated by trading energy on the ancillary services Market.

6.3 Unit Commitment

After the market is cleared and E-Program is communicated to every BRP, a unit commitment algorithm calculates the power schedule for the following day. Unit commitment aims to find a cost-minimal schedule and a production level for each generating unit over time, minimizing operating and cycle costs while satisfying production constraints and it is usually formulated as a mixed-integer problem. In this framework unit commitment is solved with well known standard techniques as in [76]. Given n

thermal (oil, gas, coal) units, let z_{jh} be unit states, i.e. binary decision variables such that $z_{jh} = 1$ if the generator j is running at time h, $z_{jh} = 0$ if the generator is down, and let u_{jh} be the production of unit j at h. Operating costs consist in fuel costs as described in 6.2.2, we call them $FC_j(z_{jh}, u_{jh})$, and start-up costs $SC_{jh}(u_{jh})$. Let T be the time horizon (UC is often calculated hourly, so $T = 24$). Denoting by $u_j = [u_{j1} \dots u_{jT}]^T$ and $u_j = [z_{j1} \dots z_{jT}]^T$ respectively the power output and unit status of each generator over time and defining $U = [u_1 \dots u_n]$ and $Z = [z_1, \dots, z_n]$, the objective function reads:

$$\min_{U,Z} \sum_{h=1}^{T} \sum_{j=1}^{n} FC_j(z_{jh}, u_{jh}) + SC_{jh}(u_{jh}) \tag{6.6}$$

The demand p_{0h} has to be satisfied:

$$\sum_{j=1}^{n} u_{jh} \geq p_{0h}, \forall j,\ h \tag{6.7}$$

The output of a unit must be zero if the unit is shut down:

$$z_{jh}\underline{u_j} \leq u_{jh} \leq z_{jh}\overline{u_j}, \forall j,\ h \tag{6.8}$$

Minimum up and down time constraints are imposed to prevent thermal stress and high maintenance costs. Denoting by $\underline{\tau}_j$ the minimum down time of unit j (in hours),

$$z_{jh+\tau-1} + z_{jh-1} - z_{jh} \leq 1 \tag{6.9}$$

for all $\tau = 1, \dots, \min\{T - t, \underline{\tau}_j - 1\}$. Finally, a reserve margin $r_h \geq 0$ can be imposed, to ensure that there is spare production capacity for regulating purposes:

$$\sum_{j=1}^{N}(z_{jh}\overline{u_j} - u_{jh}) \geq r_h, \forall j \tag{6.10}$$

6.4 Ancillary services bidding strategy

The approach to the AS bidding problem is similar to the day-ahead strategy. A set of AS prices is fixed and the corresponding optimal dispatch is calculated by solving a scenario based stochastic optimization problem. The curve is finally interpolated from the resulting points as done in [50]. The main differences with respect to the day-ahead problem stand in the shorter time horizon: on the AS trade exchange bids can be submitted up to one hour prior to delivery. This originates some consequences:

- uncertainty on load and wind is reduced,
- the unit commitment is set and cannot be modified: shut down - switch on times prevent the BRP from taking decision on a short time horizon,
- the BRP also sends requests for AS.

The stochastic scenarios here model the uncertainty on the intermittent production profile. Specifically, wind and load are aggregated into a single curve, where load is considered as a negative uncertain production. At PTU k the most likely wind scenario is the wind blowing at PTU $k-1$, as suggested by the persistence method, that will be illustrated in section 6.4.2. Based on historical data we generate a set of *prosumption* scenarios $\xi_1, \ldots, \xi_S$ with corresponding probabilities $\pi_1, \ldots, \pi_S$.

The AS problem formulation also accounts for price-elastic load, consisting of independent *prosumers* free to adjust their consumption based on energy prices. The behavior of this kind of prosumers is described in

[10]. The load scenarios are modified according to generated prices λ_p^{AS}. If prosumers are active, they are going to switch off their loads as the price for ancillary services increases over a certain threshold λ_{load}, namely the contract price. The relation between $\lambda_{diff} = [\lambda^{AS} - \lambda_{load}]^+$ and the load decrease is approximately linear. For prices $\lambda_p^{AS} > \lambda_{load}$, the load decrement is subtracted from the stochastic scenarios.

In the following sections the scenario generating methodology is described more in detail.

6.4.1 Scenario generation

While computing the AS bidding curve, a series of *prosumption* scenarios is generated at each step, built from historical data and determined as the difference between forecast wind and expected consumption. These two quantities are non-controllable variables, over which we only have predictions and wind is considered as "negative load". Assuming that the two variables are independent, combining the two kinds of load in a single grid allows us to build a wide range of possible scenarios. So, if $\mathcal{L}^c = \{\xi_1^c, \ldots, \xi_L^c\}$, is the set of L possible consumption scenarios, with corresponding probability vector $\Pi_c = \{\pi_1^c, \ldots, \pi_L^c\}$, and $\mathcal{L}^w = \{\xi_1^w, \ldots, \xi_W^w\}$ is a set of W likely wind production scenarios, with corresponding probability vector $\Pi_w = \{\pi_1^w, \ldots, \pi_W^w\}$, the set of the possible prosumption scenarios is given by the cartesian product $\mathcal{L}^C \times \mathcal{L}^W = \{\xi_{11}, \ldots, \xi_{LW}\}$, consisting of $L \times W$ elements, and with probability vector $\Pi = \{\pi_{11}, \ldots, \pi_{LW}\}$. The generic element ξ_{ij}, $i = 1, \ldots, L, j = 1, \ldots, W$ is calculated as:

$$\xi_{ij} = \xi_i^w - \xi_j^c \tag{6.11}$$

while the element π_{ij} is:

$$\pi_{ij} = \pi_i^c \pi_j^w \tag{6.12}$$

assuming that the wind and consumption processes are independent. In the remainder we define $S = LW$ and we use the row-wise linear indexing so that ξ_{ij} simply reduces to ξ_i, $i = 1, \ldots, S$.

6.4.2 Wind

In our problem setting, the method for generating wind scenarios tracks a technique currently used in practice, called *persistence method*. The idea behind is simple: since the prediction horizon is relatively small, we assume that the wind that is blowing in the current instant will be the same in one hour. To do so, average wind power data relative to a period of one year has been collected. The sample time is one PTU. The forecast error is computed as $e(t) = w(t+4) - w(t)$, for every PTU of the year, where $w(k)$ is the average wind power in PTU k. A histogram of the forecast error collected over one year is shown in Figure 6.4.

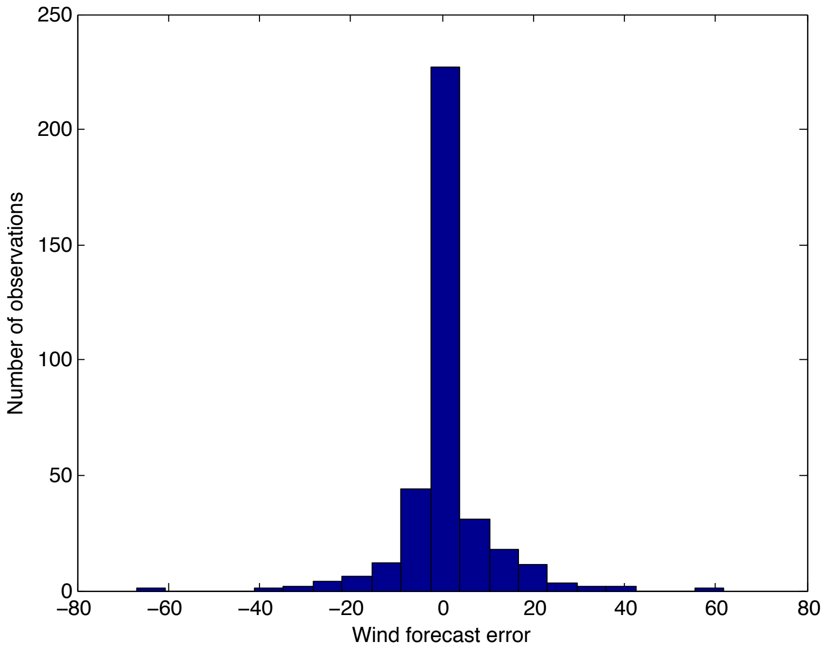


Figure 6.4: Empirical distribution of one-hour-ahead wind forecast

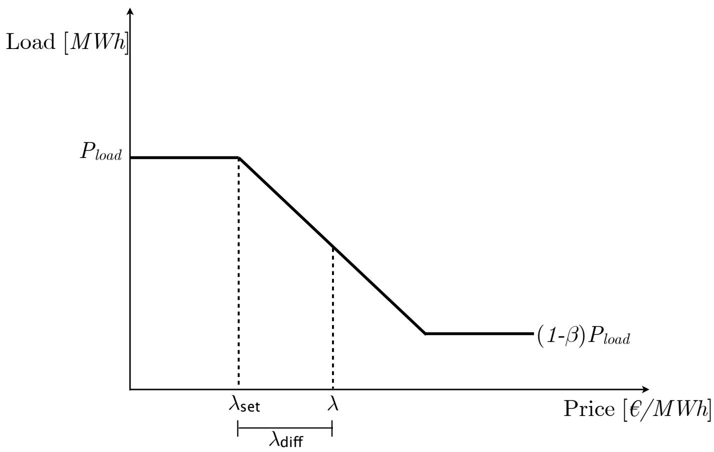


Figure 6.5: The price-elastic load

It can be observed that the predominance of the one-hour-ahead forecast error (more than 95% cases) is contained between −28 and +24 MW in the whole network.

6.4.3 Price elastic load

The other element forming stochastic scenarios of the AS bidding problem is the price-elastic load of prosumers. Price-elastic loads are a device that can be used by BRPs to balance their production internally. When for example the price for power shortage is very high, a BRP can activate its prosumers by forwarding them a price signal λ_{load} communicating them to decrease their consumption. Prosumers are characterized by a *price elasticity* [€/MWh], that is the parameter by which they regulate their load, assumed to be known to the BRP. Price elasticity generate short-term demand response (elastic) function for each PTU, thus implying an active and indirect participation of prosumers in the AS markets.

The following inputs and parameters are needed to define the prosumers:

1. $P_{nom,i}$: the nominal load assigned to each BRP i, this is the maximum load that BRP i will supply.

2. λ_{load}: a price signal communicated by the BRP to its price-elastic load to regulate consumption, in our case we assume that this signal is simply the upward AS market outcome λ^{AS+}.

3. λ_{set}: the price above which prosumers start to be price sensitive.

4. α: this parameter is expressed in [MW/€] and indicates the sensitivity of the load with respect to the price.

5. β_p: the participation rate of price-elastic prosumers. This parameter can be set at $\beta_p = 1$, and incorporated into the price sensitivity.

The load of BRP i in PTU t is calculated by:

$$P_{load,i} = \max\left\{\min\{P_{nom,i}, P_{nom,i} - \alpha(\lambda_{load} - \lambda_{set})\}, (1-\beta_p)P_{nom,i}\right\} \tag{6.13}$$

The function $P_{load,i}$ is illustrated in Figure 6.5. In order to introduce stochasticity in the load we model an additive noise as a gaussian noise with known mean and variance $\mu_{load}, \sigma_{load}$.

6.4.4 Optimization problem formulation

The resulting optimization problem is as shown in (6.14).

$$\min \quad \ell + \frac{1}{1-\beta}\left(\sum_{s=1}^{S} \pi_s [f_{sp} - \ell]^+\right) \tag{6.14a}$$

$$\text{s.t.} \quad f_{sp} = f_{sp}^1 + f_{sp}^2 + f_{sp}^3 + f_{sp}^4 \qquad s = 1,\dots,S, \tag{6.14b}$$

$$f_{sp}^1 = \sum_{j=1}^{n} \frac{C_j u_{sjp}}{M_j\left(a_{2,j}\left(\frac{u_{sjp}}{TP_{max,j}}\right)^2 + a_{1,j}\frac{u_{sjp}}{TP_{max,j}} + a_{0,j}\right)} \qquad s = 1,\dots,S, \tag{6.14c}$$

$$f_{sp}^2 = -\lambda_p^{AS} x_p^{AS}, \quad s = 1,\dots,S, \tag{6.14d}$$

$$f_{sp}^3 = -a\lambda_p^{AS} x_p^{R}, \quad s = 1,\dots,S, \tag{6.14e}$$

$$f_{sp}^4 \geq -\lambda_p^{AS} x_{sp}^{imb}, \quad s = 1,\dots,S, \tag{6.14f}$$

$$f_{sp}^4 \geq -\lambda_{imb,p} x_{sp}^{imb} + (\lambda_{imb,p} - \lambda_p^{AS}) x_p^{R}, \; s = 1,\dots,S, \tag{6.14g}$$

$$f_{sp}^4 \geq 0, \quad s = 1,\dots,S, \tag{6.14h}$$

$$\sum_{j=1}^{n} u_{sjp} - x_p^{AS} - p^{PX} + \xi_s - x_{sp}^{imb} = 0, \quad s = 1,\dots,S, \tag{6.14i}$$

$$u_{sjp} \geq P_{min,j} z_{jk}, \; j = 1,\dots,n, \; s = 1,\dots,S, \tag{6.14j}$$

$$u_{sjp} \leq P_{max,j} z_{jk}, \; j = 1,\dots,n, \; s = 1,\dots,S, \tag{6.14k}$$

$$\lambda_p^{AS} x_p^{AS} \geq 0 \tag{6.14l}$$

$$\lambda_p^{AS} x_p^{R} \leq 0 \tag{6.14m}$$

$$x_p^{AS} \geq x_{p-1}^{AS}. \tag{6.14n}$$

With $x_{p0}^{AS} = -\infty$. The optimization problem is solved at every PTU k, $k = 1,\dots,96$, once defined the DAM price and E-Program for PTU k and for a range of P prices $\lambda_p^{AS}(k)$, $p = 1,\dots,P$. The value z_{jk} is the unit commitment status of unit j, $z_{jk} = 1$ if the unit is on, $z_{jk} = 0$ if unit j is off at PTU k. If $\lambda_p^{AS}(k) \geq 0$ then it is explicitly λ^{AS+}, i.e. the system is lacking energy, it is therefore in up-regulating mode and upward regulating capacity is required. If $\lambda_p^{AS}(k) < 0$ then we are in down regulating mode (λ^{AS-}), downward regulating capacity is required.

The decision variables of the optimization problem at stage k are:

- $x_p^{AS} \in \mathbb{R}$: power bid on the ancillary services market for upward or downward regulating capacity at price λ_p^{AS}, $p = 1, 2, \ldots, P$. This variable can be either positive or negative,
- $u_{sjp} \in \mathbb{R}$: energy produced by the jth generator, $j = 1, 2, \ldots, n$ in scenario i, $s = 1, \ldots, S$ at price p, $p = 1, \ldots, P$.
- x_p^R is the amount of activated external AS capacity with price p,
- x_{sp}^{imb} is the imbalance done in scenario s with price p.

Therefore, each problem has $S(n+1)+2$ decision variables. At the end, the P x_p^{AS} decision variables are aggregated into one single bid curve for PTU k and the P x_p^R into two values: upward and downward capacity requests R_i^+ and R_i^-. Those values are obtained taking the average value over positive (for down-regulating request R_i^-) and negative (for up-regulating request R_i^+) values. Constraints (6.14b) define the loss function as the sum of four components: f^1 are production costs ((6.14c)), f^2 is the revenue from AS market ((6.14d)), f^3 represent costs for allocating capacity ((6.14e)), f^4 are imbalance costs, defined by constraints (6.14f), (6.14g) and (6.14h). Constraints (6.14k) and (6.14j) impose boundaries on the minimum and maximum power production of generators. Constraints (6.14i) enforce internal balance and avoid energy storage. Constraints (6.14l)-(6.14m) bind the algorithm to bid a positive amount of energy and ask a negative one if the price is positive, and vice versa. Constraint (6.14n) ensures non decreasing condition.

A bid curve for upward regulating capacity obtained with this method is depicted in Figure 6.6. The blue line represents the offer curve, the green dotted line is the request of approximately 40 MWh.

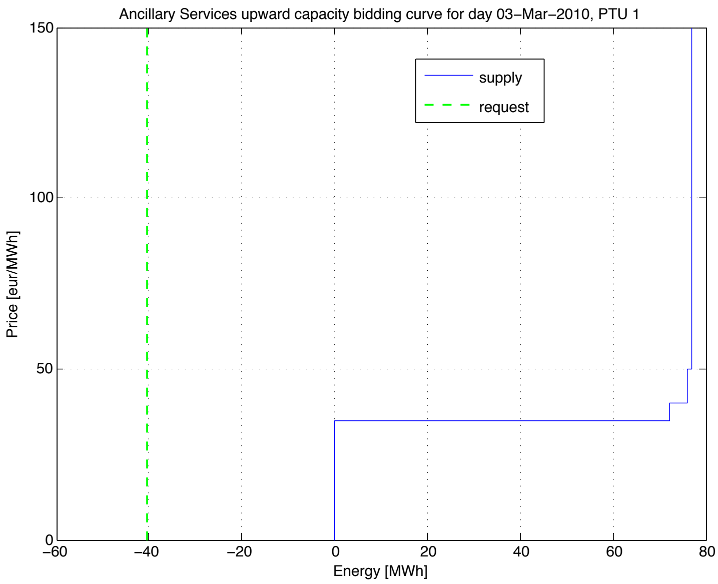


Figure 6.6: Upward regulating capacity bid of BRP 1 in PTU 1

6.5 Case study

In E-Price a simulation environment has been developed to test the devised algorithms. The simulation framework is used to verify theoretical options for the future operation of the power system. These theoretical options have been implemented as algorithms describing market operation, BRP operation and TSO operation. The model framework is such that it can incorporate the algorithms and test them based on different input data sets. As the multiple algorithms have parameters for tuning their behavior, combinations of algorithms and parameters have been chosen to reduce the number of required experiments while covering a variety of realistic options. The BRP DA and AS bidding algorithms have been implemented and tested in MATLAB R2009b on a MacBook Pro 2.4 GHz and 4 GB RAM using Yalmip and CPLEX 12.4. In order to assess the validity and usefulness of the results the simulation framework has to represent the current system as accurately as possible. However, there are many limitations:

- Model validity: as much accurate a model can be, it will never be able to include all the complex set of variables, parameters and human decisions forming a power grid. In particular, the BRP behavior is not always well known. Real bidding is often a mix of human actions and several combined algorithms, therefore a comparison between the devised BRP algorithms and real-life practice is not realistically possible.

- Model complexity: some simplifying assumptions have to be made in order to limit the software complexity. In particular, storage is not contemplated and congestion management (i.e. behavior in presence of a tie-line overflow) is treated in another dedicated simulation framework.

	Unit Cost (€/GJ)	Unit Cost (€/MWh)
Gas	$5,36$	$19,29$
Coal	$2,65$	$9,4$
Nuclear	$0,83$	$2,98$
Furnace Gas	$1,00$	$3,6$
Biomass	$3,32$	$11,95$

Table 6.1: Fuel costs

	$a2$	$a1$	$a0$
Gas turbine	$-0,5128$	$1,359$	$0,1538$
Combine cycle gas turbine	$-0,4817$	$0,991$	$0,4907$
Other (Coal, Nuclear, PCP)	$-0,3952$	$0,7622$	$0,6325$
Integrated gasification combined cycle	$-0,7901$	$1,3902$	$0,3885$

Table 6.2: Efficiency curves coefficients

- Time frame: still for complexity issues the simulation period is set at 24 hours. This time horizon allows us to assess the main performance indexes of the implemented models and algorithms and to test the correct functioning of the virtual power grid, but a more detailed analysis about the economic long-run performance of BRPs is not possible.

The parameters composing the physical model of the power network have been provided by KEMA. With reference to the Dutch electricity market, we consider a system consisting of 7 BRPs, with different generating assets and loads. Thermal plants are fueled with gas, coal, nuclear energy, furnace gas and biomass. Unit production costs in €/GJ and €/MWh and efficiency curve parameters as described in 6.2.2 are reported in Tables 6.1 and 6.2.

In this thesis, we will focus on the main simulation results related to BRPs management. A complete analysis of the results obtained from the simulations are the scope of the project report [21]. Although as stated above, an objective comparison of the E-Price BRP behavior and real-life management is not possible, some observations and remarks are done with respect to the double-sided market arrangement and the proposed algorithms.

6.5.1 Input data

The E-Price project has a strong focus on ancillary services and particularly on primary and secondary reserves. One of the main goals of the project is to be able to efficiently deal with imbalances in real time. Different data sets were designed to study:

- the impact of a plant trip;
- the effect of wind and load uncertainty;
- the impact of a bigger penetration of renewable sources.

To this aim, 5 input data sets have been proposed, each differing from the other for more or less dramatic wind fluctuations, presence of elastic prosumers, plant trips. In this thesis we neglect the data set with plant trips, as it mostly concerns the ability of a BRP to recover from a sudden power shortage, and is therefore beyond the scope of this thesis.

For each of the 23 wind farm locations, wind power measurements were selected and addressed to the individual locations. The wind power values originate from wind speed measurements available within KEMA. The wind speed data consist of hourly values for one year (from June 1st 2004 until May 31st 2005) and correspond to 24h ahead prediction errors for multiple locations in the Netherlands. For each of the locations, the wind speed time series have been transformed into power time series using a power curve and consequently this has been interpolated to obtain second based values. Two different power curves are generated, with different nominal capacities of 2230MW and 10000MW respectively, to test different amount of wind penetration. For the selected day the wind profiles for the multiple locations look as in Figure 6.7.

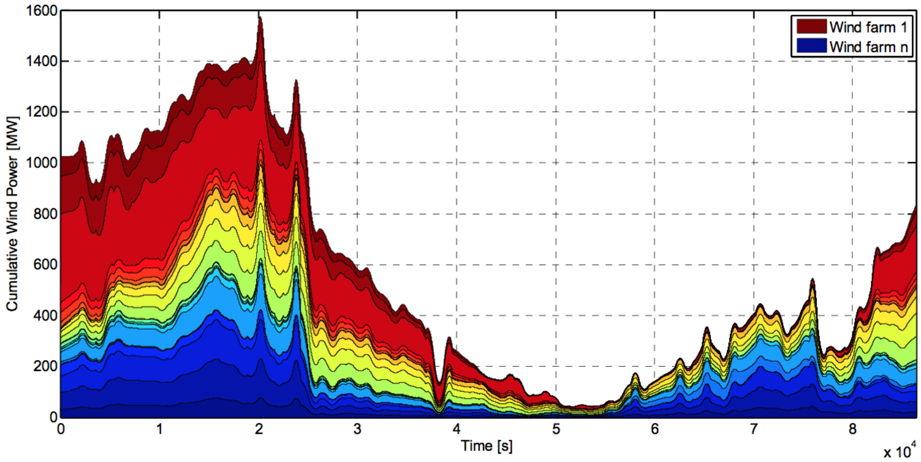


Figure 6.7: Generated wind power for the 23 wind locations in the Netherlands

Other input data relevant for the simulation are:

- Load signal: a 15 minute load signal is obtained from measurements of a selected day in 2010;
- Load prediction: a simple load prediction is performed to create an expected load signal to use for the day ahead market clearing. Deviations between prediction and actual signal therefore results in system imbalance.
- Market data: data consisting of clearing volumes and prices of energy exchanged on the DAM for each hour of the day and each day of the year 2010 were provided by APX[3]. APX also provided historical delta prices for 2010 Dutch market, i.e., time series of the difference between energy market clearing price on the Exchange market and energy price on the corresponding ancillary services market operated by the TSO.

[3]APX is an independent fully electronic exchange for anonymous trading on the spot market `www.apxgroup.com`

- Wind realized and forecast power: the realized wind power over the year, sampled at a frequency of one minute, serves as a basis for the scenario generation in the day-ahead and ancillary services algorithms.

The input data sets relevant for the simulations differ for:

- **perfect/imperfect wind forecast**: to test the impact of wind uncertainty, see Figure 6.8. The red line represents the forecast, the blue line is the realized wind power. In the simulated case studies, when forecast is realistic (i.e. imperfect) the wind power realization is higher than expected during the whole time horizon;

- **large/medium wind power production**: to verify the influence of higher renewables penetration.

Parameters and market design at the system level differ for:

- **single sided/double sided ancillary services market** (current vs E-Price situation);

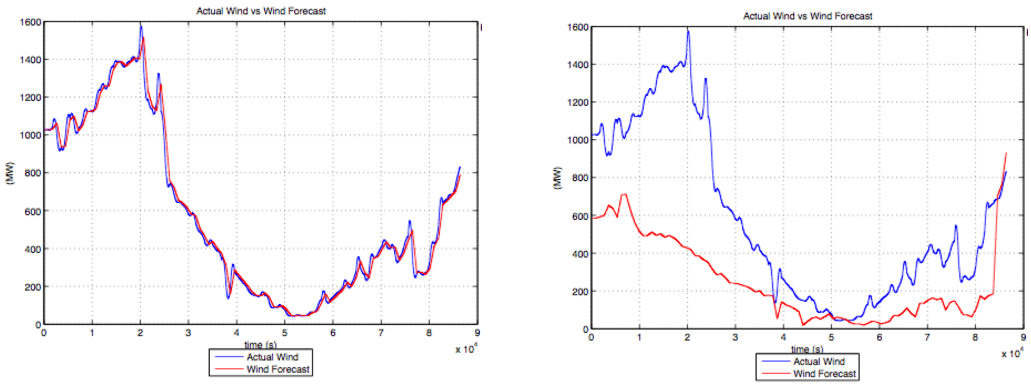
- **low/high system risk attitude** a.

Tunable parameters at individual (BRP) level are:

- **internal risk attitude** β;

- **step size** λ_{step}.

For simplicity, a fixed value for the parameter λ_{step} is set for each BRP, so this is not considered as a tunable parameter. The only requirement is that λ_{step} is small enough to guarantee sufficient granularity to the bidding curve, so $\lambda_{step} = 5$ € or $\lambda_{step} = 10$ €. Performance metrics used to evaluate the algorithms in the various case studies are production costs, net profit and price volatility. In the next sections we report the main



(a) Forecast and realized wind power with perfect forecast

(b) Forecast and realized wind power with realistic forecast

Figure 6.8: Wind power realization and forecast

simulation results in relation to the different data sets and tunable parameters. Since the applied day-ahead strategy is the same in both the E-Price and benchmark solution, only the results related to ancillary services are reported.

6.5.2 Risk attitude β

We start our analysis by observing how the bidding behavior of a single BRP changes in relation to the internal risk attitude β in the double-sided situation. The value *beta* is a CVaR related parameter defined in section 2.1.3, which is used in the bidding problem formulations (6.4) and (6.14). This parameter is generally set at standard values 90%, 95% or 99%. In general, any value from 50% to 99% can be used. If $\beta = 90\%$ then VaR is the lowest value l_β such that $P[f(u,s) \leq l] \geq 90\%$, and CVaR is the expectation over that interval. An increase in β to, for example, 99% should lead to a more conservative behavior since now it is asked $P[f(u,s) \leq l] \geq 99\%$. Table 6.3 confirms this expectation. The table reports the couples *price-volume* composing the upward regulating AS bidding curves of BRP1 in the first PTU. BRP 1 consists of 8 CCGT plants fueled with gas, 1 coal plants and 1 biomass generator. The first column contains the fixed grid of prices, the other two columns the optimal dispatched volume at that price respectively with $\beta = 0.92$ and $\beta = 0.99$. Bid curves are expressed in *energy* (MWh) and we have chosen $\lambda_{step} = 5$ €.

Price [€]	Offered volume $\beta = 0.92$ [MWh]	Offered volume $\beta = 0.99$ [MWh]
5	0	0
10	0	0
15	0	0
20	0	0
25	0	0
30	0	0
35	0	0
40	74.7802	72.2767
45	78.3879	75.8844
50	79.6396	75.8844
55	79.6396	77.1362
60	79.6396	77.1362
65	79.6396	77.1362
70	79.6396	77.1362
75	79.6396	77.1362
80	80.8913	77.1362
85	80.8913	77.1362
90	80.8913	77.1362
95	80.8913	77.1362
100	80.8913	77.1362
Request	-38.3751	-40.3636

Table 6.3: AS up-regulating bidding curve with different risk attitude β

We can observe that with a higher value of β, a lower volume is offered at the same price. The requested volume is higher, confirming the expectation of more conservative behavior with high risk parameter β: the BRP offers less energy and requires for the allocation of more regulating capacity to avoid real time imbalances.

When the parameter a, modeling the system risk attitude is varying, the effect can be seen mostly on the request value, while the supply curve is basically unvaried: it is natural that a low price for capacity allocation leads a BRP to request higher amounts of reserves. Optimal couples of the supply curve and request values are shown in Table 6.4.

Price [€]	Offered volume $a = 0.1$ [MWh]	Offered volume $a = 0.2$ [MWh]
5	0	0
10	0	0
15	0	0
20	0	0
25	0	0
30	0	0
35	0	0
40	73.6817	73.6817
45	77.6215	77.1617
50	78.5411	78.3112
55	79.0010	78.5411
60	79.0010	78.7711
65	79.2309	79.0010
70	79.2309	79.0010
75	79.4608	79.0010
80	79.4608	79.0010
85	79.4608	79.2309
90	79.4608	79.2309
95	79.4608	79.2309
100	79.4608	79.2309
Request	−47.7236	−38.6379

Table 6.4: AS up-regulating bidding curve with different system risk attitude a and $\beta = 0.95$

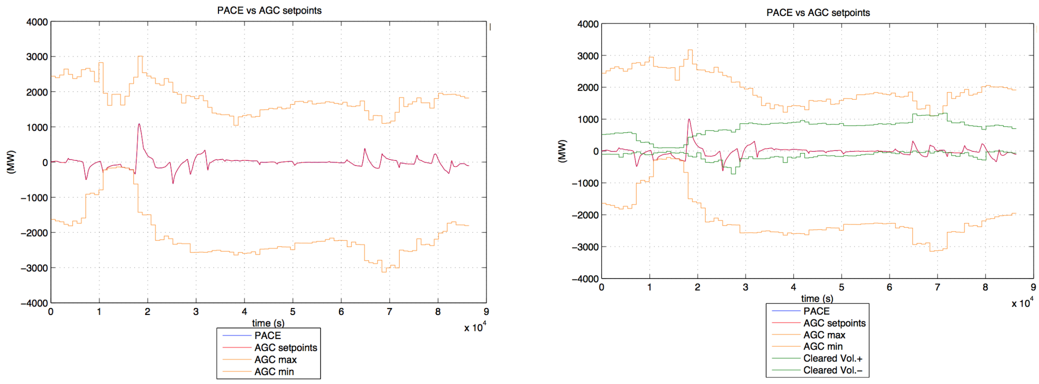
In order to limit the number of tested case studies, in the simulation environment the BRP internal risk attitude has been determined at the beginning, and remains constant for the whole set of case studies. The BRP risk parameters β are reported in Table 6.5.

6.5.3 The single-sided and double-sided markets

In this section we compare the devised algorithms with the identified benchmark, that is, when only supply curves can be offered to the AS market. The economic social benefit obtained with the introduction of double sided

	BRP 1	BRP 2	BRP 3	BRP 4	BRP 5	BRP 6	BRP 7
β	0.95	0.95	0.925	0.95	0.9	0.9	0.9

Table 6.5: Risk attitude β for the 7 BRPs



(a) AGC set-points and boundaries for the single-sided market

(b) AGC set-points and boundaries for the double-sided market

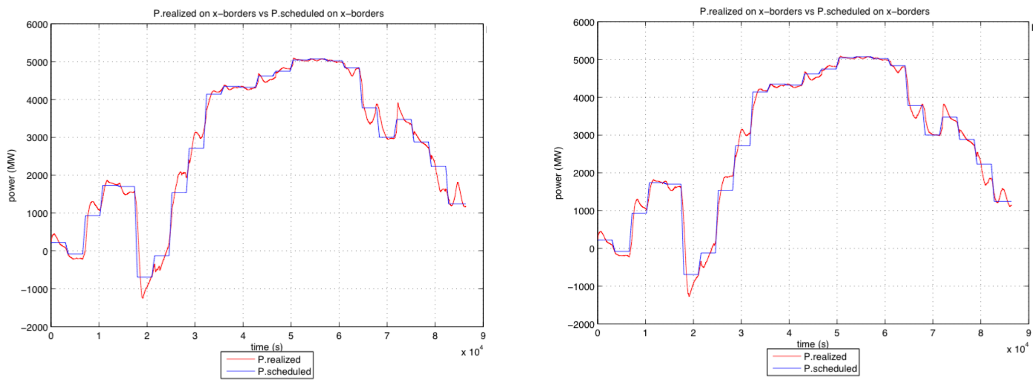
Figure 6.9: AGC set-points for the benchmark and E-Price situations, with perfect wind forecast

ancillary services is investigated.

Let us look at Figure 6.9, referring to perfect wind forecast conditions (i.e. there is perfect match between the 24 hours ahead wind forecast and the realized wind production) and $a = 0.1$. The x-axis indicates time (in seconds), the y-axis power (in MW). The red line represents the AGC signal (i.e. the request for secondary control) transmitted in real time by the TSO to BRPs. It matches the PACE (Processed Area Control Error), that is proportional to the difference between the scheduled E-Program and the energy actually present on the grid. The orange lines (in both subfigures) are the total accumulated energy *bids*. In Figure 6.9(b) the green lines are the *cleared* AS quantities. Note that in the interval $[1 \times 10^4, 2 \times 10^4]$ cumulative downward bids are barely higher than 500 MW, this indicates that all BRP are approaching their minimum power level and that they cannot offer downward regulating capacity to the market.

We can observe that, being the compared data sets the same, the AGC requests are very similar, as the imbalances derive from the same wind and load realizations and they have the same magnitude.

It should be observed that in perfect forecast conditions, the Processed Area Control Error (PACE) is mainly due to the fact that the Netherlands are not an isolated power system, but they exchange power with neighboring countries. In this particular setting, the large amount of wind realized allows to export several MWs of active power. The sharp steps in the X-border schedule (energy exchanged with neighboring countries), can be observed in Figure 6.10. In particular, before 2×10^4 s the schedule presents



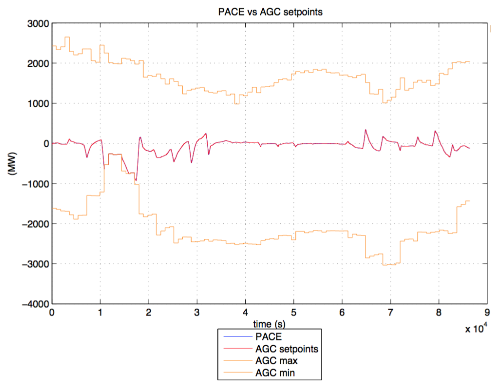
(a) X-border trade in the single-sided situation

(b) X-border trade in the E-Price situation

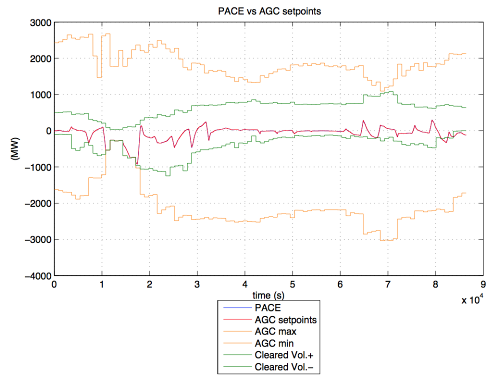
Figure 6.10: X-border exchange schedule and realized production with neighboring countries

a sharp droop that might cause imbalances and frequency deviations.

The most important aspect to note here is that, except for unexpected big fluctuations (highlighted by the steep jumps in the AGC curve) almost all deviations are covered by the negotiated AS power. This energy has already been reserved and there is no need to call for the imbalance settlement. BRPs deviating from their contracted E-Program (even though slightly since we are in perfect forecast conditions) are able to cover for most imbalances by means of their own internal regulating capacity or of



(a) AGC set-points and boundaries for the single-sided market



(b) AGC set-points and boundaries for the double-sided market

Figure 6.11: AGC set-points for the benchmark and E-Price situations, with imperfect wind forecast

their purchased reserve, that has a lower value than the imbalance price. Note that cleared downward reserves (R^-) in Figure 6.9(b) are generally lower than upward cleared reserves (R^+). This phenomenon is explained by the affine cost structure used in this framework. In fact a BRP will always prefer recurring (if available) to its own downward controllable production, implying saving costs, than buying that capacity on the market. This makes the cumulative downward capacity request R^- lower than the upward request R^+.

Analogous remarks can be made in case of imperfect forecast, as shown in Figure 6.11. In this case, the AGC request highlighted by the red line tends towards negative values, this is explained by the sign of the mismatch between forecast and realized wind, which is always negative over the simulation period (more wind than expected blows in the system). As a consequence, also negative requests R^- are more relevant, BRPs are all experiencing a power surplus and require downward regulating capacity on the AS market to avoid imbalances.

Production costs

Production costs reflect overall efficiency: day ahead energy market strategies, AS market architecture and BRP AS market bidding, and real time BRP control. An efficient market architecture should select the cheapest assets to produce energy and regulating power, that is, production costs should be minimized. Therefore, production costs reflect the social welfare of the system. Table 6.6 reports production costs for the examined data sets in single-sided and double sided situation. System risk attitude is set $a = 0.1$. The first column describes the input data set, more specifically

Data Set / market	Single-sided (€)	Double-sided (€)	Δ_{DS-SS}
perfect + medium	1.829.144	1.829.245	101
perfect + large	1.651.814	1.642.345	−9.469
imperfect + medium	1.758.381	1.757.093	−1.288
imperfect + large	1.601.725	1.601.053	−672

Table 6.6: Production costs in SS and DS with system risk attitude $a = 0.1$

the first term indicates the type of forecast (perfect vs imperfect) the second refers to the wind amount (medium vs large). We can observe that costs have basically not changed. The same can be said when the system risk attitude a is increased from 0.1 to 0.2 (see Table 6.7).

Data Set / market	Single-sided (€)	Double-sided (€)	Δ_{DS-SS}
perfect + medium	1.829.144	1.828.477	−667
perfect + large	1.651.814	1.640.669	−11.145
imperfect + medium	1.758.381	1.756.676	−1.705
imperfect + large	1.601.725	1.600.652	−1.073

Table 6.7: Production costs in SS and DS with system risk attitude $a = 0.2$

The simulation results suggest that the introduction of the double sided AS markets only affects approximately 0.6% of the total costs (around 10.000 Euros per day for the whole NL). The fact that the total social welfare of the system has almost not changed indicates that all the considered solutions are comparably efficient with respect to this performance metrics. The bidding strategies used in the simulations were cost reflective (no market power consideration has been taken into account), and the obtained solutions have converged to (or close to) the optimal working point in terms of the social welfare. It is interesting and important to observe that the benefit of hedging against the risk were not paid by an increase in the total system costs. One would expect that the forward risk hedging would imply some additional costs that however did not show up in the simulations. Increasing the system risk attitude leads in each data set to a slight (almost negligible) increment in production costs, suggesting that with higher imbalance costs BRPs lean more on their controllable production than on the more expensive external capacity allocation.

Net profit

The net profit of a BRP i is given by:

$$\Pi_i = I_{PX,i} + I_{AS,i} - I_{imb,i} + I_{load,i} - I_{fuel,i} \tag{6.15}$$

The term $I_{PX,i}$ is the income from PX trade, i.e. earnings on the day ahead market, $I_{AS,i}$ is the income/cost deriving from AS trade (it can comprise allocating capacity and activation costs in double-sided markets), $I_{imb,i}$ are imbalance costs and $I_{fuel,i}$ are fuel costs. Finally, $I_{load,i}$ is the revenue coming from load contracts with external consumers. In the devised framework, BRPs only own production assets and no consumption units. So, they get earnings from trading energy on the spot and AS markets, while the demand is simply forwarded from external loads to the markets via BRP, who have no direct cost or earning from this transaction.

The asymmetry between production and load, together with the signif-

Data Set	Market	BRP 1	BRP 2	BRP 3	BRP 4	BRP 5	BRP 6	BRP 7	SUM
perf/med	SS	2.080.836	1.897.928	1.033.399	1.948.894	744.275	210.003	658.544	8.573.879
	DS $a = 0.1$	2.051.799	1.871.987	1.034.514	1.963.184	746.662	217.103	690.180	8.575.428
	DS $a = 0.2$	2.028.659	1.853.546	1.040.602	1.972.723	743.700	218.626	714.977	8.572.834
perf/lar	SS	1.934.527	2.157.764	1.217.031	2.138.469	757.369	313.767	663.531	9.182.460
	DS $a = 0.1$	1.947.628	2.115.706	1.227.036	2.148.042	772.623	309.569	696.071	9.216.676
	DS $a = 0.2$	1.943.257	2.081.741	1.228.518	2.146.340	790.270	320.975	716.395	9.227.496
imp/med	SS	2.143.775	1.912.602	962.444	1.927.196	751.416	177.970	665.872	8.541.276
	DS $a = 0.1$	2.134.444	1.888.556	968.782	1.936.227	751.621	175.433	691.706	8.546.768
	DS $a = 0.2$	2.123.283	1.871.619	974.708	1.940.503	749.915	174.054	710.326	8.544.408
imp/lar	SS	2.309.520	1.890.993	942.587	2.148.908	812.897	127.938	712.383	8.945.226
	DS $a = 0.1$	2.253.194	1.870.887	964.994	2.174.450	821.889	149.201	751.821	8.986.438
	DS $a = 0.2$	2.291.429	1.833.081	936.824	2.156.387	835.350	138.214	789.845	8.981.129

Table 6.8: Net profit of BRPs, all values expressed in €

icant X-border trade depicted in Figure 6.10 explains why the net profit of BRPs does not sum up to zero. In fact, one could expect that the overall net profit of BRPs is approximately zero, i.e. there are some BRPs selling energy and others buying. However, since BRPs do not actually own consumption units, they only gain a profit from the sales on the power exchange. We can imagine of a big additional BRP comprising all the national load who collects the day-ahead bulk demand and forwards it to the market. From Table 6.8, we observe in all case studies a slight increase of the overall net profit when passing to the double-sided market, even though some BRPs earn less, like BRPs 1 and 2. An increase in the system risk attitude a leads to a slight reduction in the profit, as BRPs act in a more conservative way.

Price volatility

Price volatility is an important indicator of the market design quality. In this section, the final AS prices are qualitatively compared in the single-sided and in the double-sided market design. We compare AS prices in perfect forecast conditions and medium wind power size, respectively in single-sided and double-sided market. AS prices per PTU are shown in

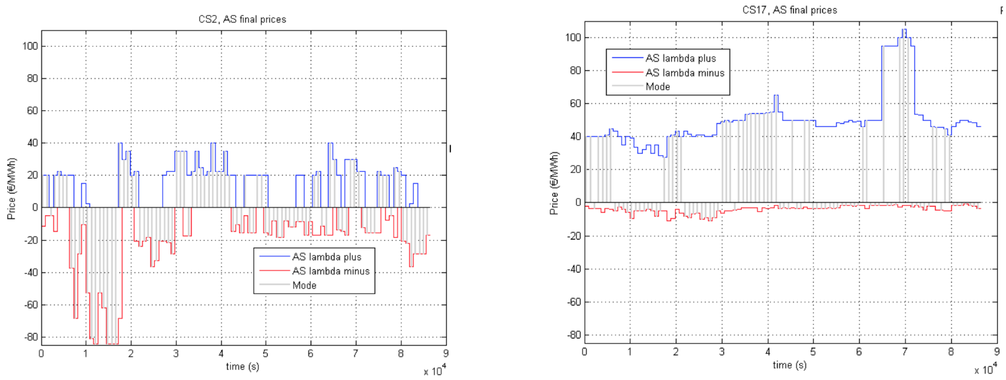
Figure 6.12(a)-(b). The conclusions are the following:

- Prices in the double-sided markets are in general less volatile , with less sudden sharp peaks;
- Prices for the upward AS provision seem to be higher in the double-sided AS;
- Prices for the downward AS provision are much smaller in the double-sided AS.

The first statement is easily explained by the clearing process of double-sided markets, operated one hour prior to delivery. While in the single-sided market the price is determined by the last activated bid in real time, in the double-sided situation the price is a function of the total allocated capacity, which is more or less constant over one hour. The same fact also explains the higher prices in the double-sided market: not all the allocated capacity will be used in real time, so it is natural that the clearing point is higher than in the corresponding single-sided situation, where only the needed energy is called in real-time. Moreover, the correcting factor a and the imbalance penalty ϕ makes this difference even stronger. The latter point can be explained by the BRP strategies, i.e., by the fact that BRPs assume that it is always cheaper to reduce own power set-points in case of overproduction than to have other BRPs reducing production in their place. Therefore, the requests (volumes) for downward regulation on the double-sided markets are small, and consequently, the clearing AS downward price is also small.

6.6 Conclusions

In this chapter, the day ahead and ancillary services bidding problems on double-sided markets have been addressed. These are delicate and risky



(a) Price development in the single-sided market

(b) Price development in the double-sided market

Figure 6.12: Price development

tasks, because energy prices are extremely volatile and no energy storage device is present in the Dutch grid to absorb price shocks. Moreover, the coupling between markets requires an even more attentive operation planning, since the energy sold on the day ahead market cannot be used for other purposes.

The E-Price solution is thoroughly designed to tackle the risk borne by BRPs and to support them in the decision making process. A scenario-based algorithm is devised in both cases where a linear stochastic problem is solved at every time unit (hour or PTU), to build the piecewise constant bid curve. A risk measure $CVaR$ is minimized, in a way that reduces the risk of the BRP incurring great losses. Operating constraints such as minimum and maximum power set-points, ramp rate limits and internal balancing are satisfied.

The most important feature of E-Price bidding curves is the introduction of AS *request* quantities. These confidence intervals allow the system to be prepared to possible imbalances and react promptly.

The main concluding remarks can be synthesized as follows:

- The proposed scenario based optimization approach is suitable for the optimal bidding problems, it helps the BRP hedge against the risk and it reflects the BRP risk attitude, modulated through the parameter β;

- Applying double-sided AS has comparable fuel usage compared with single-sided AS, while it still allows BRPs to hedge their risks for being in imbalance;
- Applying double-sided AS slightly increases BRP income compared with SSAS (in most cases);
- Applying a larger value for the system risk a slightly decreases the BRP profit (in most cases);
- Applying double-sided AS leads to less volatile prices and to the allocation of the necessary regulating capacity beforehand.

Chapter 7

Conclusions and future developments

After the deregulation of electricity supply, financial and electricity contracts can be traded in a similar way. Nevertheless, there are still major differences between the traditional stock market and the electricity market like lacking liquidity, high volatility and jumps, non-normal distributions; deriving from special characteristics of electricity like the non-storability and the transmission constraints.

In this thesis we have provided novel stochastic optimization approaches to trade assets on financial and energy markets. In the first part we have formulated the dynamic option hedging problem under a stochastic model predictive control viewpoint. A minimum variance criterion is used to minimize expected hedging error at each trading instant, where a pricing engine (generally based on Monte Carlo simulations) is available to predict future option prices. Numerical tests are carried out on a European call, a Napoleon cliquet and a barrier call option. The proposed tool can handle a wide variety of payoff functions and multiple assets and showed good performance compared to the traditional delta hedging especially when applied to exotic options. The SMPC approach has been then extended to the case of transaction costs. The minimum variance criterion has been proven to be not suitable to handle this type of market, therefore three alternative risk measures have been introduced respectively based on a combination of quadratic error and variance, a CVaR related index and a min-max approach. The SMPC algorithm has been proven to be effective when applied to exotic options (barrier). It can be used *online* by financial institutions to define an hedging strategy as a support system for traders, or for the *offline* quantification of the expected hedging error given some market hypothesis. The work can be extended to other types of exotic options and payoffs, and to those options which are not *at-the-money*, that is, when the market does not deem the two future opposite situations of the price going up and the price going down as equally likely and as a consequence, the strike price is fixed below or above the current market price.

In the second part of this thesis, the day ahead and ancillary services bidding on electricity markets have been addressed. We have first described the current market design and the proposed alternative architecture devised in the E-Price project, based on the concept of double-sided auction for ancillary services. Unlike stock markets, uncertain variables in the electricity markets can not be approximated by Gaussian curves, and one has often to resort to empirical distribution functions.

A two-stage stochastic optimization approach based on CVaR minimization has then been presented for the optimal bidding on sequential energy markets. The first stage decision variables are given by the bidding curves to be submitted on the day ahead auction, at 12.00 of the day before delivery in the form of non-decreasing piecewise constant curves. Stochastic variables at this stage are the prices for secondary control which will be public only a few minutes before delivery (or weeks later in some current market arrangements). The second stage decision variables, to be taken after the disclosure of spot prices, are the bidding curves for the double-sided ancillary services market. Here the uncertain variable is the amount of energy produced by renewable sources with low marginal costs but high volatility and thus liable to cause imbalance costs. The right balance between request and supply has to be chosen, taking into account unit commitment constraints and price-elastic loads. The E-Price solution is thoroughly designed to tackle the risk borne by BRPs and to support them in the decision making process. A scenario-based algorithm is devised in both cases where a linear stochastic problem is solved at every time unit (hour or PTU), to build the piecewise constant bid curve. A risk measure $CVaR$ is minimized, in a way that reduces the risk of the BRP incurring great losses. Operating constraints such as minimum and maximum power set-points, ramp rate limits and internal balancing are satisfied.

The algorithms developed in E-Price have been tested in a simulation environment. The double-sided market architecture for ancillary services has proven to be a valid alternative to the single-sided current benchmark, as it decreases control effort by the TSO and allows the system to react promptly to power imbalances without increasing production costs due to the hedging option. The BRP algorithms presented in this thesis help the BRP hedge against the risk and reflect the BRP risk attitude.

Of course, there are many directions for the extension and future development of this work. First, many simplifying hypothesis have been made in order to improve tractability, like sequential markets and the hypothesis of independence between the load and wind production. If extended to integrated markets, the bidding problems on day-ahead and ancillary services auctions should be treated jointly also with unit commitment as a three-stage stochastic optimization problem. The size of such problem would rise consistently and decomposition techniques would be needed. Also, the spatial distribution of the demand and the related congestion problems have been disregarded. In reality, the market behavior of a BRP can change consistently when it can anticipate congestion of some lines of the grid. Generating companies operating in highly congested systems can exercise a strong market power. A game theoretical approach could be used to find the equilibrium conditions in such market, where BRPs make assumptions on their rivals reaction to price or power set-points changes.

Appendix A

Dynamic hedging based on mixed-integer programming

Piecewise affine transaction costs as in (4.19) can be also handled by introducing binary variables. Let $x^u(t) \triangleq u(t-1) \in \mathbb{R}^n$ be the composition of the portfolio immediately before trading at time t and introduce auxiliary variables $\delta_i(t) \in \{0,1\}$

$$[\delta_i(t) = 1] \leftrightarrow [u_i(t) - x_i^u(t) \geq 0] \tag{A.1}$$

and $q_i(t) \in \mathbb{R}$

$$q_i(t) = \begin{cases} u_i(t) - x_i^u(t) & \text{if } \delta_i(t) = 1 \\ 0 & \text{otherwise} \end{cases} \tag{A.2}$$

By using the so-called "big-M" technique, (A.1) can be translated into the mixed-integer linear inequalities

$$u_i(t) - x_i^u(t) \;\geq\; -M_i(1 - \delta_i(t)) \tag{A.3a}$$

$$u_i(t) - x_i^u(t) \;\leq\; M_i\delta_i(t) - \epsilon \tag{A.3b}$$

and (A.2) into

$$\begin{aligned}
q_i(t) &\leq u_i(t) - x_i^u(t) + M_i(1-\delta_i(t)) && \text{(A.4a)}\\
q_i(t) &\geq u_i(t) - x_i^u(t) - M_i(1-\delta_i(t)) && \text{(A.4b)}\\
q_i(t) &\leq M_i\delta_i(t) && \text{(A.4c)}\\
q_i(t) &\geq -M_i\delta_i(t) && \text{(A.4d)}
\end{aligned}$$

where M_i is an upperbound on $|u_i(t)-x_i^u(t)|$, that is the maximum allowed asset reallocation, and $\epsilon > 0$ is a small scalar (e.g., the machine precision). Eq. (4.18) can be therefore interpreted as the evolution of a *hybrid dynamical system*, that is expressed in the following *mixed logical dynamical* (MLD) form [7]

$$\begin{aligned}
w(t+1) &= (1+r)\left(u_0(t) - \sum_{i=1}^{n} q_i(t) - 2(u_i(t) - x_i^u(t))\right) + \\
&\quad \sum_{i=1}^{n} s_i(t+1)u_i(t) && \text{(A.5a)}\\
x^u(t+1) &= u(t) && \text{(A.5b)}\\
\text{s.t.} &\quad \text{(A.3), (A.4)} && \text{(A.5c)}
\end{aligned}$$

with states $w(t), x^u(t)$, input $u(t)$, auxiliary vector $\delta(t) = [\delta_1(t)\ \ldots\ \delta_n(t)]' \in \{0,1\}^n$ of binary variables, and vector $q(t) = [q_1(t)\ \ldots\ q_n(t)]' \in \mathbb{R}^n$ of auxiliary continuous variables. Note that, from a system theoretical viewpoint, transaction costs introduce a unit delay (A.5b) in the dynamics, due to the additional state variable $x^u(t)$.

By using the stochastic hybrid dynamical model (A.5c), problem (4.26) can be recast as a mixed-integer quadratic programming (MIQP) problem (see [7] for details) to be minimized with respect to vector $u(t) \in \mathbb{R}^n$, for which very efficient solvers are available [34, 40]. See also [30] for a related approach. For options involving a single stock, the number n of assets is usually very small ($n = 1$ or $n = 2$), so that the minimum variance problem with transaction costs can be solved also by enumerating the possible 2^n instances of vector $\delta(t)$ (that is, in system theoretical terms, by transforming the MLD dynamics (A.5c) into an equivalent piecewise affine (PWA) form [4] and enumerating the "modes" of the resulting PWA dynamics) and by solving the corresponding quadratic programs (QP) (4.17) subject to $u_i(t) \geq x_i^u(t)$ if the corresponding $\delta_i(t) = 1$, or $u_i(t) \leq x_i^u(t)$ if $\delta_i(t) = 0$, for all $i = 1, \ldots, n$.

While the method of Section 4.5.1 is in general more efficient from a numerical viewpoint, in that it completely avoids introducing integer variables to handle proportional transaction costs, the MIQP method of this section is more general, for example it can be easily extended to handle transaction costs of the form $h_i(u_i(t) - u_i(t-1)) = \min\{c_0, \epsilon^i s_i(t)|u_i(t) - u_i(t-1)|\}$, where c_0 is a given minimum fixed cost to be paid in each transaction.

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