School of Social Sciences

Labor Market Effects of Migration: Evidence from EU Enlargement and Application of Search-and-Matching Framework

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Labor Market Effects of Migration: Evidence from EU Enlargement and Application of Search-and-Matching Framework

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To my family
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Chapter 1

Introduction

The existence and the direction of labor market effects of migration has been subject to long and extensive debates among both economists and policy makers. Many studies have been carried out that try to establish the direction and magnitude of the migration effects on the labor markets of migrant-sending and receiving countries (Borjas, 2005; Brucker et al., 2014; Basten and Siegenthaler, 2013; Card, 2005). The results of these empirical findings indicate that migration effects can substantially differ depending on the location and time considered.

This study has been motivated by two simple observations. First, not all workers in migrant-sending countries and not always gain from the decrease of labor supply caused by emigration in terms of wages and employment prospects. Second, certain migrant-receiving countries perform on average better then others while facing large migration waves. Among the most successful examples of migrant-receiving countries, one can think of Canada, Australia and Switzerland. While the first two countries have historically been “the nations of
immigrants”, Switzerland has become a destination of vast immigration flows quite recently, in the last quarter of the XX century. Yet, Swiss economy was able to adjust quickly to a nearly 25% increase in population and labor force.

This study provides a theoretical rationale as well as empirical evidence for the existence of welfare-improving migration outcomes, at least for some labor market participants. I also discuss in details immigration policies in Switzerland. Although those are left behind the scope of the theoretical framework employed in this thesis, a reader should always keep in mind that the favorable migration outcomes are by large attributed to migration policies consistent with the current needs of the labor market.

The structure of the thesis is organized as follows. First, chapter one examines how emigration affects labor markets in migrant-sending countries. Following EU enlargement in 2004, Central and Eastern Europe experienced large waves of emigration. Given the magnitude and the speed of migration flows, I hypothesize that the migration waves had an impact on the distribution of wages, employment opportunities and the structure of production in migrant-sending countries. In order to identify groups of workers that lose or gain from emigration, I take advantage of detailed data provided by the Statistical Office of Lithuania. I use this data for estimating a labor market model with heterogeneous labor and imperfect rates of substitutions among different categories of workers. I consider five experience and three skill groups of workers.

The analysis is carried out in three steps. First, I estimate the wage setting curves in order to obtain the elasticity for each skill and experience group of workers. Second, the elasticity
of substitution between different categories of workers is estimated. Finally, I simulate wage
and employment effects of an exogenous labor supply shock caused by an emigration wave.
The simulations are based on the elasticities of the wage-setting curves for different sections
of the labor market and the elasticities of substitution between different groups of workers. I
find that the employment effect of migration is higher than the wage effect for all categories
of workers.

The second chapter contributes to the debate about economic determinants of interna-
tional migration flows and the consequential effects of migration on wages and unemployment
rates. The analysis is carried out within a two-country model that belongs to a general family
of search-and-matching models (e.g., Diamond, 1982 and Mortensen and Pissarides, 1994).

The main advantage of this approach is that it provides a clear basic structure of a labor
market while allowing for a manageable analysis of unemployment, wages and migration.
The central idea of this setting is that both firms (labor demand side) and workers (labor
supply side) have to spend resources before job creation and production can take place. A
larger number of unemployed workers searching for a job makes it relatively cheaper for a
firm to create a new vacancy. An increase in the number of vacancies, in turn, strengthens
the bargaining position of unemployed workers. Therefore, immigration does not necessarily
lead to a dramatic reduction of wages and increased unemployment, as in a Walrasian
paradigm. I show that under the search and matching framework a broad range of possible
migration effects can be generated. This is in line with the mixed evidence provided by the
empirical literature, which documents that the impact of migration on labor markets varies
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with time and location, and that it can be either positive or negative. In order to provide a theoretical rationale for the existence of diverse effects of migration on wages and unemployment, I develop two models with fully integrated migration decisions and non-linear costs of migration. In the first setting, migration costs represent pure costs of reallocation. As a result, migration affects only the unemployment rate, while wages remain unaffected. Additionally, this simple setting allows the determination of the value of the equilibrium migration rate, which depends on the structural characteristics of the labor markets in the two regions. This framework can potentially be extended to n regions. It can then be employed for obtaining (rough) estimates of the migrant distribution across different locations following, for instance, EU enlargement or the arrival of refugees.

In the second setting, I consider a case in which it is more costly for immigrants to stay in the host country than for workers who are born in the host country. The differentiated costs result in lower bargaining power of immigrants during the wage bargaining process and, thus, in lower wages for immigrants. I show that in this framework all labor market agents gain from migration. Firms pay lower wages on average and incur lower search costs due to the increased availability of workers. This in turn leads to a higher rate of vacancy creation. Immigrants are better off because they end up working in a structurally better region with higher wages and better prospects of finding a job than in their home country. The native workers benefit from the boost in labor demand as new jobs are created as well as from higher wages.

The third chapter presents an application of the theoretical framework of chapter two to
the migration between Switzerland and the EU. Migration effects on the Swiss labor market are derived. Being historically characterised by high immigration rates, Switzerland is now one of the countries most dependent on foreign workers of all European OECD countries. Today the share of foreigners in the total labor force of Switzerland constitutes 23.7%. First and second generation immigrants now make up more than a third of Switzerland's population over the age of 15. Starting from 2002, after enactment of a bilateral agreement of 2001 between Switzerland and the EU on the free movement of individuals, Switzerland's foreign resident population grew on average by 2.4% annually.

Given the magnitude of the inflow, it is not surprising that workers and some policy makers are concerned about potential negative labor market effects of immigration. Yet, most state officials, business representatives and employer organizations deny that the immigration of foreign employees displaces native workers or reduces their wage. They argue that the inflow of highly skilled employees to Switzerland occurs due to the lack of resident employees, and that the immigrants and the treaty hence do not only increase competitiveness and productivity in the country, but also prevent firms from outsourcing jobs, thus benefiting resident workers. Several empirical studies have examined labor market effects of the recent immigration wave to Switzerland, and most of them do not find any evidence of negative effects on wages of the native population. Moreover, examination of historical data series suggests that the largest net inflow of foreigners coincides with the highest growth of nominal wages and peaking number of newly opened vacancies. These observations go in line with the key assumptions and theoretical predictions of the models in chapter two.
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In order to obtain structural parameters characterizing the Swiss labor market, I first calibrate the standard one-country search model to generate the observed fluctuations in unemployment and job vacancies in Switzerland. I then use the parameters obtained from calibration for simulating the steady state versions of two two-country models for Switzerland and the EU. The results show, that the models can adequately predict observed percentage changes in unemployment rates in response to the immigration wave and resulting wage differences between immigrants and native workers.

Finally, a two-country search-and-matching model with migration and aggregate uncertainty is presented, as well as the tools developed for solving it numerically in MATLAB. The model aims at analyzing the response of decentralized labor markets to local shocks as well as at explaining migration patterns and outcomes related to cross-country differences in structural characteristics of labor markets in the presence of productivity shocks.
Chapter 2

Emigration and Imperfect Labor Markets: Evidence from Lithuania

2.1 Introduction

Of all forms of international flows, migration flows are among the most complex. Migration can no longer be attributed solely to the problem of underdevelopment, but should rather be recognized as an aspect of globalization (Newland, 2011). Migration persists even at very high levels of development, – such as within the European Union. Many developing – particularly middle-income – countries have become major immigrant destinations while still remaining sources of immigrants( Newland, 2011). These simple observations suggest intricate market interactions driven by migration.

Several studies have shown that, generally, a reduction of barriers to international migra-
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Migration is beneficial for the world as a whole (Klein and Ventura, 2009; Moses and Letnes, 2004). However, despite such an overall positive economic outcome, there are winners and losers in sending and receiving countries; and there are many economic, social, and institutional aspects of the impact of migration beyond the effect on countries’ GDP. While classifying the various types of migration impacts, it is necessary to distinguish between short and long-run perspectives, as well as between macro and micro level effects.

Given the significance of migration for the societies of sending and receiving countries, there has been a large volume of research on the impact of migration. Particular attention has been paid to labor market outcomes. An important issue that has emerged in the literature is the discrepancy between the analyses and prescriptions at the theoretical and empirical levels.

The stylized model of a competitive labor market has clear and unambiguous implications about how labor markets should adjust to a labor supply shift induced by migration. In a closed economy, a labor supply shock should inevitably change the price of labor, at least in the short-run. Despite the common-sense intuition underlying the model, empirical estimations yield puzzling and contradictory results. Utilizing a simulation-based approach, Borjas (2003, 2005) claims that migration has a substantial impact on labor markets in receiving and sending countries. At the same time, more data-driven econometric estimations that are not based on a theoretical model find considerably smaller or even negligible impact of migration on wages and employment (Card, 2005; Ottaviano and Peri, 2008).

According to Poot and Cochrane (2005), there are two potential explanations for this
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contradiction. First, there are substantial methodological differences across studies in how the estimations were performed. Moreover, the studies consider various geographical areas at different time intervals. Therefore, it is difficult to compare the results. Second, there are market forces at work that offset the potential pressure on wages and institutional factors that preserve markets from adjusting to the migration shocks as predicted by the neoclassical model.

Another stumbling block in migration studies is the direction of causality. In many real-world cases, it is difficult to tell whether migration has influenced wages and employment, or the change in economic conditions have induced migration, or both. To overcome this limitation of econometric analysis, there has been a search for truly exogenous migration shocks in labor markets, or so-called “natural experiments”\(^1\). Natural experiments are widely used to estimate the effects of migration policy changes in various locations (e.g., France (Hunt, 1992); Germany (Glitz, 2012), Israel (Friedberg, 2001)).

The aim of the present study is to analyze one such experiment. The study will take advantage of events that occurred in the Baltic countries over the past decade in order to quantify the economic impacts of migration-induced shocks of labor supply. Lithuania, together with Latvia and Estonia have recently experienced two massive waves of emigration. The first occurred in early 1990s followed the disintegration of the USSR, and was due to the repatriation of Russians from the newly independent countries. The second wave occurred

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\(^1\)In migration studies, a term “natural experiment” is defined as a large migration wave that occurred over the short period of time mainly due to political reasons. The term was first used in a study by Card, 1990 who analyzed the arrival of 125,000 Cuban immigrants in Miami between May and September 1980 to determine the labor market impacts of immigrant flows.
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after EU enlargement of 2004, when Ireland, Sweden, and the UK opened their boarders to workers from the Baltic states.

These political transformations abruptly changed economic opportunities of all workers for the Baltics countries. Due to the sudden changes in economic conditions, these two events can be considered natural experiments.

Given the magnitude and the speed of migration flows, I hypothesize that the migration waves had an impact on the distribution of wages, employment opportunities, and the structure of production in the source countries. In the present study I take advantage of high quality micro-data provided by Lithuanian Statistical Office in order to quantify the labor market impact of emigration.

The remainder of the chapter is organized as follows. The next section examines literature on economic effects of migration. Section 3 presents the theoretical model of migration impact. Section 4 provides the description of the data. The empirical estimation process and the final results are described in sections 5 and 6. Finally, section 7 draws the conclusions.
2.2 Economic effects of migration: a literature review

Theoretical aspects of labor market effects of immigration are usually described using a neo-classical model of labor supply and demand (Johnson, 1980). The economic impacts of international migration are essentially linked to the characteristics of the migrants and of the economies of migrant-sending and host countries. The effects of migration can vary with the skill level of migrants, the motives to migrate, and the capital structure of sending and receiving economies. It is also relevant whether and how quickly economies adjust to immigration through, for example, a change in capital, technology, or output variety (Dustmann, Glitz, and Frattini, 2008).

Therefore, most economic analyses of migration distinguish between the impacts of low and high-skilled migration, and between the long and short-run effects (Kerr and Kerr, 2011). In a simple short-run model, it is typically assumed that capital and technology are rigid, so that the primary effect of migration is labor supply shock. In a host country, immigrants lower the price of factors to which they are perfect substitutes and raise the price of factors to which they are complements. In a sending country, the exact opposite occurs.

In the long-run, capital and technology adapt to a new level of labor supply. For instance, a short-term change in the returns to capital is likely to change the long-run investment level. Investment in physical capital, in turn, will shift the demand for labor, thus bringing wages towards their pre-migration rates. Whether and how quickly these adjustments take place is an empirical question.
These conclusions of both one-sector and multisector theoretical models, however, have been criticised for excluding the possibility of dynamic and spillover effects that may arise from, for example, having a bigger and more diverse economy, a greater share of highly skilled and motivated people, and the presence of non-traded goods in a multisectoral economy (Borjas, 2009). In theory, such dynamic and spillover effects could be positive or negative, that is, they could raise or lower the productivity of the resident population, even in the long run. Economists are divided about the likely existence and direction of the net impacts arising from such effects (Hatton and Williamson, 2008).

This theoretical uncertainty has created a need for quantitative results and stimulated empirical research. The available literature on labor markets effects can be subdivided into two broad categories: simulation-based analyses (or the factor proportions approach) and so-called “area studies” (Okkerse, 2008).

The “area studies” estimate the effects of migration based on correlations between variations in wages or employment rates and variations in migration stocks or flows. Some methods exploit the geographical diversity in migration destinations (cross-sectional analyses); others deal with changes in migration patterns over time (time-series analyses). These studies usually do not rely on a theoretical background and produce more data-driven results (Hatton and Williamson, 2008).

Cross-sectional studies generally estimate the labor market impact of immigration by comparing conditions across localities in the country. These studies typically calculate the correlation between measures of immigrant penetration in local labor markets and measures
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of economic outcomes, such as wages (Card, 2001, 1990). The sign of this spatial correlation is then interpreted as indicating the direction in which immigrant supply shifts affect wages. A negative correlation suggests that immigrant-induced increases in labor supply lower wages. Although there is a lot of dispersion across findings, many studies find a near-zero spatial correlation (see meta-analysis of existing studies carried out by Poot and Cochrane (2005)). Cross-sectional studies potentially suffer from the endogeneity problem, which can be solved by using instrumental variables. However, it is difficult to find instruments that are highly correlated with the concentration of immigrants but uncorrelated with the wage or employment levels (Hatton and Williamson, 2008). Additionally, the studies that exploit the geographic clustering of immigrants and use differences across local labor markets to identify the impact of immigration are challenging because they omit possibly strong contemporaneous trends that tend to equalize economic conditions across cities and regions (Borjas, 2003).

One way to deal with the problems of econometric analysis is by looking at massive international migration flows, which have been mainly caused by reasons other than economic factors. Analyses of so-called “natural experiments” provide an opportunity for more robust estimations of migration effects. Natural experiments deal with enormous migration flows in limited periods of time compared with normal migration movements (Aydemir and Kirdar, 2013; Bauer, Flake, and Sinning, 2013; Silva et al., 2010). Their particular characteristics make natural experiments a valuable instrument for addressing the theoretical debates on migration effects.
In contrast to “area studies”, results from simulation-based analyses are theory-driven. An application of the partial equilibrium approach allows direct computing of the wage and employment effects arising from migration at a national level. The factor proportions approach distinguishes between migration effects on various groups of workers by employing a nested Constant Elasticity of Substitution (CES) production function with different skill categories of labor force. This idea goes back to Card (2001), who observed that the wages of high school and college graduates in the US, Canada, and the United Kingdom failed to move together for the last three decades of the 20th century. Using a model with imperfect substitution between similarly educated workers in different age groups, the authors argued that these shifts reflected changes in the relative supply of highly educated workers across age groups.

This approach has been widely adopted in the literature on migration. Obviously, migration is not evenly balanced across the groups of workers who have different educational backgrounds and work experience, and the magnitude of this supply imbalance changes over time. Borjas (2003) introduced a new approach for estimating the labor market impact of immigration by exploiting this variation in supply shifts across education-experience groups and by assuming that both migrants and local workers with different skill characteristics are not perfect substitutes. More recently Mishra (2007) used data on Mexican migration to the US from 1970 to 2000 and found that emigration of 1 percent of the workforce increased wages on average by 0.8 percent. Aydemir and Borjas (2007) applied a factor demand model to the same case and showed that the wage increases were higher for workers with a high-
school degree and some college education, and lower for college graduates and high-school dropouts.

In general, the simulation-based studies find a substantial impact of migration on wages. In a broad sense, these findings are consistent with the long-run effects of migration on wages predicted by the labor market theory. A recent study by Elsner (2013) contributes to the literature by introducing an age dimension to the analysis and by focusing on the short run effects of migration. The wage effects of emigration from Lithuania analyzed in this paper are significantly larger than those found in previous studies. Moreover, the findings reveal that younger workers staying in Lithuania gain as the result of emigration, while older ones lose.

It is necessary to mention that most of the previous studies utilizing factor proportions approach have assumed away wage rigidities and unemployment, which both can have an impact on wages at the same time as migration. There may be a number of reasons why migration causes these wage changes in the real world that go beyond a mere shift in labor supply and subsequent adjustments in labor demand. Firstly, there can be an underlying self-selection forces at work, when the majority of emigrants belong to unemployed part of the labor force prior to emigration. As the result of their departure, there will be only an effect on unemployment without any considerable subsequent adjustment of wage. Secondly, if the wage setting mechanisms in economy are strong, the emigration may increase the bargaining power of non-migrants. Because of the large number of young emigrants, the labor market for young workers became tighter, which means that the same number of firms competes for
fewer workers. For young stayers this means that they should be able to negotiate higher wages under the threat of emigration.
2.3 Theoretical framework

Wage-setting theory

To analyze the economic outcomes resulting from migration, I use the stylized theoretical model of the labor market that accounts for different rates of substitution between groups of workers with different characteristics, such as education and work experience. The model is based on a nested CES production function, in which each skill group of workers enters as a distinct labor input. This methodological approach closely follows Borjas (2003) and a recent study on Lithuania by Elsner (2013).

Moreover, this study will relax an assumption of perfectly competitive labor markets by employing a wage-setting framework, implying that wages adjust only partially to labor supply shocks. The wage-setting curve relies on the assumption that wages decline with the unemployment rate, although imperfectly. This assumption is widely supported empirically.

According to Blanchflower and Oswald (1995), there are at least four theoretical reasons to predict negative relationship between wages and unemployment. Firstly, the assumption of a wage-setting function can be derived from the contract models: a contract curve will generate a downward-sloping curve in w-u axis. Secondly, unemployment can serve as a motivator. In a booming labor market firms have to pay higher wages to ensure that workers, who know there are many other vacancies available, exert enough effort (Shirking model, Shapiro and Stiglitz, 1984). Thirdly, labor union and bargaining theories suggest that workers have relatively less bargaining power when unemployment levels are relatively
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high. Finally, the idea of persistent disequilibrium McCormick and Sheppard (1992) speaks in favour of the wage curve: if the labor market adjusts sluggishly, non-equilibrium states may routinely be observed, revealing negatively inclined set of points in wage-unemployment space.

For the purposes of the study it is not convenient to adapt a specific wage-setting model, mainly since it would require more detailed datasets and larger number of variables involved into the analysis, potentially blurring the migration effects. Instead, the elasticity of the wage curve can be thought of as the compound effect of wage-setting mechanisms and institutional factors.

Model of the migration impact

This section describes a wage-setting model in which labor market imperfections prevent market clearing. The model was originally developed by Levine (1999) for analyzing the welfare effects of East-West migration. More recently, the model has been employed in the analysis of wage and employment effects of immigration in Western Europe (Boeri and Brucker, 2005; Brucker and Jahn, 2011). This section presents a review of the model of Brucker and Jahn (2011)

Output is produced using different types of labor and capital. Let \( \tilde{N}_i \) denote the pre-migration labor force in each of the skill groups, where subscript \( l = 1, ..., n \) indicates the
type of labor. The total labor force after migration is given by

\[ N_l = \tilde{N}_l - \gamma_l M, \quad \sum |\gamma_l| = 1 \quad (2.1) \]

where M is a total emigration flow and \(0 < \gamma_l < 1\) is the share of workers of type \(l\).

Firms produce varieties of differentiated goods and follow the behaviour of monopolistic competitors. Production involves some fixed initial costs, but thereafter each firm produces output with constant returns to scale. Thus, production of a representative firm is given by

\[ Y_i = F(L^i, K^i), \quad (2.2) \]

where \(Y^i\) denotes a variety of output good, \(L^i\) is the vector of labor inputs, \(K^i\) is physical capital and the superscript \(i\) the firm index. Also, \(L_l \leq N_l\) must hold, since firms do not necessarily employ the entire labor force. The production function is assumed to be concave, increasing, twice continuously differentiable in all the inputs, and homogeneous of degree one.

Wages and labor demand are determined sequentially. Initially, wages remain fixed and the elasticity of wage with respect to the unemployment rate may differ for groups of workers with different education and experience. Later, after wages are negotiated, firms set prices and hire a certain number of workers, so that their profits are maximized (the marginal product of labor equals the wage rate). Assume that each firm faces a constant elasticity of demand \(\eta > 1\). Under the profit maximizing condition, the wage is given by

\[ w_l^i = \nu^{-1} P^i Y_{L^i}, \quad \forall l, \quad (2.3) \]
where the mark-up $\nu$ is $(1 - 1/\eta)^{-1}$, $P^i$ the product price of variety $i$ of the output good, and $Y^i_{L_l}$ the marginal product of labor.

Assuming that firms are identical, at the aggregate level, the following will hold $w^i_l = w_l$, $Y^i_{L_l} = Y_{L_l}$, and $P^i = P = 1$, where the price level is normalized to one. The real wage equals

$$w_l = \nu^{-1}Y_{L_l}, \quad \forall l \quad (2.4)$$

Equation (2.4) states what the demand for labor will be, given the real wage.

At the first step of the decision process, firms and employees negotiate wages, so that wages decline if the unemployment rate increases. Thus, the following aggregate wage-setting condition can be written:

$$w_l = f_l(u_l), \quad f_l' < 0, \quad \forall l \quad (2.5)$$

where $f_l$ is a function that captures the response of the wage to the unemployment rate $u_l = 1 - L_l/N_l$. The rationale underlying equation (2.5) is that a higher unemployment rate will result in workers having less bargaining power for demanding higher wages. This is consistent with a range of wage-setting models including collective bargaining and efficiency wage models.

Having obtained the wage-setting equation and the relation between the real wage and marginal product of labor, it is now possible to derive the employment response to a change in labor supply. The solution can be obtained by solving the system of equations which is determined by the wage-setting curves and the production function for each type of labor.
For each skill group, the system should satisfy the implicit function

$$\Omega_i(L, M) \equiv \nu^{-1} Y_{L_i}(L, K(N(M))) - f_i(u_i(L_i, N_i(M))) = 0, \quad \forall i, \quad (2.6)$$

From Equation (2.6) it can be seen that the capital stock may adjust to labor supply shocks, implying that $\partial K/\partial N \geq 0$. Differentiating this system implicitly with respect to a marginal labor supply shock caused by migration yields the following change in employment:

$$\frac{dL}{dM} = \left( \frac{\partial v^{-1} Y_L}{\partial L} - \frac{\partial f \partial u}{\partial u \partial L} \right)^{-1} \times \left( \frac{\partial f \partial u \partial N}{\partial u \partial N \partial M} - \frac{\partial v^{-1} Y_L \partial K \partial N}{\partial N \partial M} \right) \quad (2.7)$$

where $Y_L$ denotes a vector of the marginal products of labor in each cell of the labor market as stated in equation (2.4), $f$ the vector of wage-setting functions that determines the wage response to the unemployment rate as stated in equation (2.5), and $u$ the vector of unemployment rates.

After obtaining an equilibrium solution for the employment response, it is straightforward to derive the wage effects of migration:

$$\frac{dw}{dM} = \frac{\partial v^{-1} Y_L}{\partial L} \frac{dL}{dM} + \frac{\partial v^{-1} Y_L \partial K \partial N}{\partial N \partial M} \quad (2.8)$$

Equation (2.7) states that the employment response to migration increases along three parameters: the absolute value of the elasticity of the wage with respect to the unemployment rate; the adjustment of the capital stock to the labor supply shock; the elasticity between the marginal product of labor and the capital stock. At the same time the employment response to migration declines with the absolute value of the elasticity between the marginal product of labor and employment.
This wage-setting model represents the theoretical background for further empirical esti-
mations. In the empirical specification of the model labor will be distinguished by education
and experience. The wage-setting curves will be estimated separately for each type of labor,
while the labor demand functions for different types of labor are estimated using a nested
CES production function.
2.4 Description of the data

The empirical analysis of this study requires data collection for the period from 1998 to 2012 from two datasets: one for the estimation of the structural parameters that characterize the labor markets and one for the identification of the number of migrants per skill group.

The labor market data – i.e., annual figures on total labor force, unemployment rates, total population by age and education levels, – are taken from the Lithuanian Labor Force Surveys (LFSs). The data on real wages are obtained from the national Structure of Earnings Survey (SESs) and Household Budget Surveys (HBSs).

The Lithuanian LFS has been conducted since 1998, while the emigration wave started in 2004. The target population comprises all persons aged 15 years old and over, usually living in the households of the selected dwellings, including those who are temporarily abroad for a period of less than a year\(^2\). Apart from the data on average monthly number of employed, unemployment rates and total population, the LFS also provides figures on industry composition by different groups of workers employed.

The Lithuanian HBSs have been conducted since 1952 (Eurostat, 2004). The surveys contain detailed self-reported information on income from employment by age and the highest obtained educational qualification, income from unemployment and other personal characteristics such as number of children, marital status, the sector the respondents are employed in. To obtain the average real monthly wages, the variable *income from employment* is

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deflated by the harmonized consumer price index (HCPI, 2006=100) provided by the Eurostat Regional Statistics database. I restrict the sample to private sector workers only, since wage determination mechanism in public sector can differ from the conventional supply and demand interaction. Moreover, I consider only persons aged between 15 to 64, which are than aggregated in five 10-years categories. Depending on the highest obtained degree, each worker falls into one of three skill groups: low, medium and high. Abbreviation low indicates persons with pre-primary, primary and lower secondary education; persons with upper secondary and post-secondary non-tertiary education fall into the medium category. High denotes persons with first and second stage of tertiary education.

Unfortunately, Lithuanian Statistical Office only reports the data on emigrants by age profile, the distribution by skill group cannot be obtained directly from other national sources, as the latter do not provide detailed records about emigrants. The solution to the problem was proposed by Elsner (2013): the data needs to be compiled from datasets of the main migrant-receiving countries, namely Ireland and the United Kingdom.

The Irish Department of Social Protection provides yearly data on the number of Personal Public Service (PPS) numbers issued by nationality of applicants. Every worker who intends to be employed in Ireland has to apply for the PPS number. The dataset, therefore, captures all workers who legally emigrated from Lithuania with an intention to work in Ireland. The age and education distribution of the workers is obtained from the Census data provided by the Central Statistical Office of Ireland\(^3\).

\(^3\)Central Statistical Office of Ireland http://census.cso.ie/Census/
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The UK statistics on work permit data by the origin of recipients can be obtained from Accession Monitoring Reports (May 2004 to March 2009) issued by the UK Border Agency\textsuperscript{4}. The distribution of immigrations across education-age groups will be drawn from the Long-Term International Migration data estimated from the International Passenger Survey. The data is annual, covers the period from 1991 to 2010, and is provided by the UK office for National Statistics\textsuperscript{5}. 

\begin{itemize}
\item \textsuperscript{4}http:/www.ukba.homeoffice.gov.uk
\item \textsuperscript{5}http://www.ons.gov.uk/ons/index.html
\end{itemize}
2.5 Empirical estimation

The wage-setting equations

The first step of the analysis is to estimate the wage setting equations. Examining the raw data in Table 1, I expect the wage setting curves to vary across different skill groups of the labor market. Therefore, the elasticity between wages and the unemployment rate should be estimated separately by education and work experience groups. Specifically, for each of the group of workers the following equation is estimated:

\[ \ln w_{qjt} = \beta_{qj} \ln u_{qjt} + \eta' X_t + e_{qjt}, \] (2.9)

where \( \beta_{qj} \) is the elasticity between wage and unemployment rate, \( \eta \) is a vector of coefficients corresponding to control variables \( X_t \). A set of control variables includes the logs of the real GDP and variables that capture the development of prices and the impact of external shocks (HCPI, export and import performance indicators, oil price index). Finally, \( e_{qjt} \) is the error for each education-age group.

The econometric specification is similar to that used in Borjas (2003), but it differs from the usual approach by allowing the elasticity between wages and the unemployment rate to vary across education-experience groups.

Unobserved shocks may affect both wages and the unemployment rate. Moreover, unemployment may not only affect wage levels, but be itself a function of wages. In order to solve the potential endogeneity problem, equation (2.9) is estimated by 2SLS.
Table 2.1: Unemployment rates and wages by education & age, beginning and end of sample period

<table>
<thead>
<tr>
<th>Age</th>
<th>15-24</th>
<th>25-34</th>
<th>35-44</th>
<th>45-54</th>
<th>55-64</th>
<th>15-24</th>
<th>25-34</th>
<th>35-44</th>
<th>45-54</th>
<th>55-64</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>30.10</td>
<td>18.11</td>
<td>7.12</td>
<td>17.90</td>
<td>7.64</td>
<td>34.09</td>
<td>38.09</td>
<td>18.46</td>
<td>33.84</td>
<td>41.62</td>
</tr>
<tr>
<td>High</td>
<td>13.19</td>
<td>10.61</td>
<td>8.03</td>
<td>8.42</td>
<td>4.73</td>
<td>20.07</td>
<td>7.26</td>
<td>2.84</td>
<td>4.43</td>
<td>3.66</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>490</td>
<td>499</td>
<td>620</td>
<td>640</td>
<td>590</td>
</tr>
<tr>
<td>Low</td>
<td>618</td>
<td>630</td>
<td>782</td>
<td>807</td>
<td>745</td>
<td>966</td>
<td>984</td>
<td>1223</td>
<td>1261</td>
<td>1164</td>
</tr>
<tr>
<td>Med</td>
<td>837</td>
<td>854</td>
<td>1060</td>
<td>1094</td>
<td>1099</td>
<td>1309</td>
<td>1334</td>
<td>1657</td>
<td>1709</td>
<td>1577</td>
</tr>
</tbody>
</table>

Two types of instruments are considered. The first one, an industry composition index, is suggested by the regional wage curve literature (Blanchflower and Oswald, 2005; Brucker et al., 2014). It captures the change of unemployment due to the shift in sectoral structure driven by exogenous shocks. Presumably, some part of the employment growth in an education-age cell may be attributed to the concentration of workers in the considered cell in fast-growing industries. The industry composition index is calculated as

$$indcom_q = \sum_{i=1}^{n} g_{it} \frac{L_{qi,t-1}}{L_q} - g_t,$$

where $g_{it}$ is the employment growth in industry $i$ at time $t$, $L_{qi,t-1}$ is the employment of education group $q$ in industry $i$ at time $t - 1$, $L_q$ is total employment of the group, and $g_t$ the average employment growth over all education groups.

The second instrument is the export demand index. It captures shifts in labor demand driven by exogenous shocks in product markets. It is constructed as the GDP at constant prices of the seven largest trading partners in the OECD\textsuperscript{6} weighted by their share in Lithuan-

\textsuperscript{6}These countries are: Estonia, Germany, Netherlands, Poland, Russian Federation, Sweden, and the UK (http://stats.oecd.org)
nian exports. The export demand index can be considered as an exogenous variable, since Lithuanian economy is relatively smaller and the economic activity of the trading partners is not likely to be affected by the Lithuanian wage level.

Table 2 presents the regression results for each of the age/education group. All the coefficients have the hypothesized negative sign and are highly significant in most of the cells. Moreover, the test results say in favour of the chosen instruments. Hansen J statistics indicates that the instruments are appropriately uncorrelated with the disturbance term. The high value of the Cragg-Donald Wald F-statistics suggests that the instruments are not weak, which is supported by Shea’s partial $R^2$ (0.1118). Anderson’s LR test rejects it’s null hypothesis at 1% level, implying an adequate identification.

\footnote{Tests are reported for the pooled estimate of the wage curve.}
### Table 2.2: IV-estimates of the wage curve

<table>
<thead>
<tr>
<th>Ages</th>
<th>Coeff.</th>
<th>SE</th>
<th>$R^2$</th>
<th>Observ.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Age 15-24</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>-0.284***</td>
<td>0.033</td>
<td>0.75</td>
<td>15</td>
</tr>
<tr>
<td>Medium</td>
<td>-0.223***</td>
<td>0.043</td>
<td>0.57</td>
<td>15</td>
</tr>
<tr>
<td>High</td>
<td>-0.277***</td>
<td>0.027</td>
<td>0.87</td>
<td>15</td>
</tr>
<tr>
<td><strong>Age 25-34</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>-0.213***</td>
<td>0.041</td>
<td>0.48</td>
<td>15</td>
</tr>
<tr>
<td>Medium</td>
<td>-0.249***</td>
<td>0.057</td>
<td>0.74</td>
<td>15</td>
</tr>
<tr>
<td>High</td>
<td>-0.299***</td>
<td>0.095</td>
<td>0.51</td>
<td>15</td>
</tr>
<tr>
<td><strong>Age 35-44</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>-0.197***</td>
<td>0.027</td>
<td>0.67</td>
<td>15</td>
</tr>
<tr>
<td>Medium</td>
<td>-0.252***</td>
<td>0.055</td>
<td>0.77</td>
<td>15</td>
</tr>
<tr>
<td>High</td>
<td>-0.296**</td>
<td>0.115</td>
<td>0.63</td>
<td>15</td>
</tr>
<tr>
<td><strong>Age 45-54</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>-0.205**</td>
<td>0.588</td>
<td>0.61</td>
<td>15</td>
</tr>
<tr>
<td>Medium</td>
<td>-0.218***</td>
<td>0.029</td>
<td>0.58</td>
<td>15</td>
</tr>
<tr>
<td>High</td>
<td>-0.228***</td>
<td>0.049</td>
<td>0.47</td>
<td>15</td>
</tr>
<tr>
<td><strong>Age 55-64</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>-0.139***</td>
<td>0.016</td>
<td>0.77</td>
<td>15</td>
</tr>
<tr>
<td>Medium</td>
<td>-0.171***</td>
<td>0.018</td>
<td>0.82</td>
<td>15</td>
</tr>
<tr>
<td>High</td>
<td>-0.185***</td>
<td>0.021</td>
<td>0.76</td>
<td>15</td>
</tr>
<tr>
<td><strong>Pooled</strong></td>
<td>-0.246***</td>
<td>0.018</td>
<td>0.85</td>
<td>225</td>
</tr>
</tbody>
</table>

Hansen J-statistic (p-value): 0.8584
Cragg-Donald Wald F statistic: 57.31
Anderson canon. corr. LR statistic: 26.664

a) The dependent variable is the log Wage, independent is the log Unemployment.
b) The regressions include the log of real GDP and log of the HCPI as macro controls. IVs are the industry mix and export demand indexes.
c) The descriptive statistics are reported for the pooled estimates.
d) ***, **, * denote the 1%- , 5%- and 10%-significance levels, respectively.
The pooled estimate across all skill groups combined (Table 2) reveals the elasticity of the wage curve of 2.46. The result differs from the average estimates in the regional wage curve literature for the US and Western Europe (Bell, Nickell, and Quintini, 2002; Blanchflower and Oswald, 2005; Brucker et al., 2014), that usually find an elasticity of around 0.1. However, the high wage flexibility goes in line with the estimates of 0.416 for the Lithuanian wage curve (Martens and Pukeliene, 2007) and of 0.462 for the Latvian one (time periods are 1999-2006 and 1995-1996, respectively).

The regression results (Table 2) indicate that the elasticity of the absolute value of the wage curve is monotonically increasing with the skill level, with the single exception of the youngest group. This may reflect the lower union density and less effective collective wage agreements in the high-skilled group of the labor market. High wage flexibility in the segment of the workers aged 15-24 with low education level can be attributed to the absence of work experience resulting in higher wage sensitivity to the current economic conditions.

Remarkably, the wage flexibility changes very little across different age groups with the same education levels. This fact is supported by the study on the labor markets in Germany, Denmark, and the UK by (Brucker et al., 2014). However, the wage is considerably less responsive to unemployment level for the highest age group.
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The labor demand equations

The notation and analysis in this section closely follows Borjas (2003) and Elsner (2013). This structural approach to estimation of labor demand equation would make it possible to estimate not only the effect of a particular emigrant outflow on the wage of competing native workers, but also the cross effects on the wage of workers with other education-age characteristics. This approach imposes a certain structure on the aggregate production function that is represented by a three-level nested CES technology.

Assume that the aggregate production of a country is described by a standard Cobb-Douglas production function

\[ y_t = A_t L_t^\alpha K_t^{1-\alpha}, \tag{2.10} \]

where \( y_t \) is aggregate production, \( L_t \) is a labor input, \( K_t \) is a capital input and \( A_t \) is technology.

The total labor force is grouped into \( q=3 \) education groups and \( j=4 \) work experience categories. The workers across different education-experience cells are assumed to have different degree of substitutability. Grouping the labor force by education and experience gives:

\[
L_t = \left[ \sum_q \theta_{qt} L_{qt}^{(\delta-1)/\delta} \right]^{\delta/(\delta-1)}, \quad \sum_q \theta_{qt} = 1, \tag{2.11}
\]

\[
L_{qt} = \left[ \sum_j \theta_{qj} L_{qjt}^{(\rho-1)/\rho} \right]^{\rho/(\rho-1)}, \quad \sum_j \theta_{qj} = 1, \tag{2.12}
\]

where the aggregate \( L_t \) incorporates all workers who differ in both education and experience,
$L_{qt}$ all workers with education $q$, and $L_{qjt}$ all workers with education $q$ and experience $j$. The parameters $\theta_{q}$ and $\theta_{qj}$ capture the relative productivity of each skill group, while $\delta > 0$ is a parameter measuring the elasticity of substitution between labor of different education levels and $\rho > 0$ measures between the elasticity of substitution workers with similar education levels but different age. The productivity parameter $\theta_{qt}$ is allowed to vary across time since technological progress might affect the productivity of various types of labor in different ways. The relative productivity of workers across skill groups $\theta_{qj}$ is assumed to be constant over time.

From Equation (2.4) now it is possible to estimate the real wage rate as the marginal product of labor divided by the mark-up factor. Using Equations (2.11)-(2.12) and deriving equation (2.10) according to the chain rule, one can obtain the equation for the wage of a worker with skill $q$ and experience $j$:

$$\ln w_{qjt} = \ln \left( \nu^{-1} \alpha A_t \right) + (1 - \alpha) \ln K_t + \left( \alpha - 1 + \frac{1}{\delta} \right) \ln L_t$$

$$+ \left( \frac{1}{\delta} - \frac{1}{\rho} \right) \ln L_{qt} - \frac{1}{\rho} \ln L_{qjt} + \ln \theta_{qt} + \ln \theta_{qj},$$

(2.13)

where $\frac{1}{\rho}$ is the slope coefficient, while other parameters on the right-hand side of equation (2.13) vary along the dimensions, indicated by the subscripts, i.e. time, education and experience.

From equation (2.13) it is now possible to obtain the labor demand functions. This derivation follows the wage-setting framework in which labor demand is endogenously determined once wages are set. Employment is, consequently, the dependent variable, and the
wage is an independent variable.

Based on equation (2.13) the relative demand for workers with education $q$ and different age levels can be expressed as follows:

$$\hat{L}_{qjt} = D_t + D_{qt} + D_{qj} - \rho \ln w_{qjt} + \xi v_{qjt},$$  \hspace{1cm} (2.14)$$

where the time-specific fixed effect $D_t$ controls for the variability of the term $\rho \ln (\nu^{-1} \alpha A_t) + (1 - \alpha) \ln K_t + (\alpha - 1 + \frac{1}{\delta}) \ln L_t$. An interaction of time and education dummies $D_{qt}$ absorbs $(\frac{\theta}{\delta} - 1) \ln L_{qt} + \rho \ln \theta_{q}$. Finally, the education-age specific fixed effects $D_{qj}$ capture the variability of $\rho \ln \theta_{qj}$ and $\xi v_{qjt}$ is a disturbance term.

Because of the potential endogeneity issue, OLS can be inappropriate, and instrumental variables approach should be employed instead. Two instruments are used in the empirical analysis: the average number of dependent children under 18 years old and the log of unemployment benefit for each education-age group. Presumably, these instruments are correlated with the reservation wage without affecting labor demand.

**Table 2.3: 2SLS estimates of the elasticity of substitution across education-experience cells**

<table>
<thead>
<tr>
<th>$\hat{\rho}$</th>
<th>SE</th>
<th>p-value</th>
<th>$R^2$</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.671</td>
<td>0.709</td>
<td>0.046</td>
<td>0.949</td>
<td>225</td>
</tr>
</tbody>
</table>

Hansen J-statistic (p-value): 0.23
Anderson canon. corr. LR statistic: 58.425
Cragg-Donald Wald F statistic: 12.76

1 IVs are the average number of dependent children aged under 18 and the log of unemployment benefit.

From this regression both elasticity of substitution across experience groups $\hat{\rho}$ and pro-
ductivity parameters of experience groups of workers $\theta_{qj}$ can be obtained. The absolute value of the elasticity of substitution is 1.671, which is consistent with Elsner (2013) (1.58). The higher result may be due to the different identification strategy and bigger dataset. From the equation (2.14), 
\[
\hat{\theta}_{qj} = \frac{e^{D_{qj}/\hat{\rho}}}{\sum_{j=1}^{5} e^{D_{qj}/\hat{\rho}}}
\]

Knowing these two parameters, it is now possible to move up one level in the CES technology, and recover an additional unknown parameter $\delta$.

The equation identifying the relative demand for workers with different education levels is estimated analogously:

\[
\hat{L}_{qt} = D_t + D_q + \beta_q \tau_{qt} - \delta \ln w_{qt} + \varepsilon v_{qt},
\]  
(2.15)

where $D_t$ is the time-specific fixed-effect defined above and $D_t$ is the education dummy variable. The term $\beta_q \tau_{qt}$ is a control for the variation in $\delta \ln \theta_{qt}$, that different shifts in productivity across groups of labor with different education.

The labor composite $\hat{L}_{qt}$ can be computed as

\[
\hat{L}_{qt} = \left[ \sum_j \hat{\theta}_{qj} L_{qt}^{(\hat{\rho}-1)/\hat{\rho}} \right]^{\hat{\rho}/(\hat{\rho}-1)},
\]  
(2.16)

where the estimates of productivity parameters $\hat{\theta}_{qj}$ are calculated from the regression equation (2.14) as described above.

The estimated elasticity of substitution between education groups is smaller in absolute values that between age groups, suggesting that workers with different age are closer substitutes.
Table 2.4: 2SLS estimates of the elasticity of substitution across education groups

<table>
<thead>
<tr>
<th>( \hat{\delta} )</th>
<th>SE</th>
<th>p-value</th>
<th>( R^2 )</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.328</td>
<td>0.448</td>
<td>0.003</td>
<td>0.976</td>
<td>45</td>
</tr>
</tbody>
</table>

Hansen J-statistic (p-value): 0.38
Anderson canon. corr. LR statistic: 36.85
Cragg-Donald Wald F statistic: 16.22

1 IVs are the average number of dependent children aged under 18 and the log of unemployment benefit.

2.6 Simulation of the emigration impact

To simulate the wage effects of a labor supply shock due to emigration, the employment effects of migration have to be computed first. The general solution is stated in equation (2.7). The explicit solution can be derived from the nested structure of the production function. For a three dimensional case, the solution can be found in Bruecker et al. (2010).

In the case of the present paper only two dimensions are considered, so in total there are 3x5 = 15 types of labor. Using the notation from the nested production function (Bruecker et al, 2010), denote the vector \( \mathbf{x} = [x_{11}, x_{12}, ..., x_{15}, x_{21}, ..., x_{qj}, ..., x_{35}] \) where \( \mathbf{x} \in \{L, N, Y, L_u, f\} \).

The partial derivative of wages with respect to employment can be written as

\[
\frac{\partial \nu^{-1} Y_L}{\partial L} = \nu^{-1} \begin{bmatrix}
\frac{\partial Y_{L11}}{\partial L_{11}} & \cdots & \frac{\partial Y_{L1i}}{\partial L_{35}} \\
\vdots & \ddots & \vdots \\
\frac{\partial Y_{L35}}{\partial L_{11}} & \cdots & \frac{\partial Y_{L35}}{\partial L_{35}}
\end{bmatrix}
\]  

(2.17)
There are three types of partial derivatives in equation (2.17):

\[
\frac{\partial \nu^{-1} Y_{Lqj}}{\partial L_{qj}} = \frac{w_{qj}}{L_{qj}} \left[ s_{qj} \left\{ \frac{1}{\delta} - \frac{1}{s_q} \left( \frac{1}{\delta} - \frac{1}{\rho} \right) \right\} - \frac{1}{\rho} \right]
\]

for the diagonal elements of the matrix;

\[
\frac{\partial \nu^{-1} Y_{Lqj}}{\partial L_{qj'}} = \frac{w_{qj}}{L_{qj'}} \left[ s_{qj'} \left\{ \frac{1}{\delta} - \frac{1}{s_q} \left( \frac{1}{\delta} - \frac{1}{\rho} \right) \right\} \right]
\]

for the elements where both \(Y_L\) and \(L\) come from the groups of labor with different age, but same education levels;

\[
\frac{\partial \nu^{-1} Y_{Lqj}}{\partial L_{q'k'}} = \frac{w_{qj}}{L_{q'k'}} \left[ s_{q'k'} \frac{1}{\delta} \right]
\]

for the cells with different education and age. Variables \(s_{qj}\) and \(s_q\) denote the share of wages paid to workers of the respective cell in the total wage bill.

Using the wage setting equation:

\[
\frac{\partial f}{\partial u} \frac{\partial u}{\partial L} = \begin{bmatrix}
\frac{\partial \phi_{11}}{\partial u_{11}} & \frac{\partial \phi_{11}}{\partial L_{11}} & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots \\
0 & \cdots & \frac{\partial \phi_{35}}{\partial u_{35}} & \frac{\partial \phi_{35}}{\partial L_{35}}
\end{bmatrix}
\]

and

\[
\frac{\partial f}{\partial u} \frac{\delta N}{\partial u} \frac{\delta M}{\delta N} = \begin{bmatrix}
\frac{\partial \phi_{11}}{\partial u_{11}} & \frac{\partial \phi_{11}}{\partial N_{11}} & \frac{\partial \phi_{11}}{\partial M} \\
\frac{\partial \phi_{11}}{\partial u_{11}} & \frac{\partial \phi_{11}}{\partial N_{11}} & \frac{\partial \phi_{11}}{\partial M} \\
\vdots & \vdots & \vdots \\
\frac{\partial \phi_{35}}{\partial u_{35}} & \frac{\partial \phi_{35}}{\partial N_{35}} & \frac{\partial \phi_{35}}{\partial M}
\end{bmatrix}
\]

Then the wage equation (2.5) can be differentiated with respect to the employment changes in each cell of the labor market, yielding the wage response to emigration.
Prior to the simulations, the magnitude of the labor supply shock due to emigration $\Delta \frac{L_{qj}}{L_{qj}}$ has to be quantified. This fraction represents the share of emigrants with education $q$ and experience $j$ in total number of workers skills $qj$. While the number of workers in each skill group $L_{qj}$ can be directly taken from the Labor Force Surveys, the distribution of emigrants across different cells has to be computed from the data sources of migrant-receiving countries.

The simulations of the wage and employment effects of migration is based on the elasticities of the wage-setting curves in different cells of the labor market and the elasticities of substitution between the experience and education groups of workers. The parameters for the wage-setting curves are taken from the estimates of equation (2.9) and the parameters $\delta$ and $\rho$ from the estimates of equations (2.14)-(2.16). Because the long-run impact is simulated, the capital stock is assumed to adjust to an aggregate labor supply and the capital-output ratio remains fixed, as predicted by the Kaldor facts on economic growth (Bruecker et al., 2012).

Table 2.5: Calculated wage and employment effects of emigration of 1% of the labor force, by age and education

<table>
<thead>
<tr>
<th>Age</th>
<th>15-24</th>
<th>25-34</th>
<th>35-44</th>
<th>45-54</th>
<th>55-64</th>
<th>15-24</th>
<th>25-34</th>
<th>35-44</th>
<th>45-54</th>
<th>55-64</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>-0.03430</td>
<td>-0.07569</td>
<td>-0.05291</td>
<td>-0.01216</td>
<td>-0.03027</td>
<td>0.00012</td>
<td>0.02265</td>
<td>0.01566</td>
<td>0.00277</td>
<td>0.00560</td>
</tr>
<tr>
<td>Med</td>
<td>-0.08898</td>
<td>-0.07449</td>
<td>-0.07025</td>
<td>-0.04693</td>
<td>-0.05655</td>
<td>0.00025</td>
<td>0.01853</td>
<td>0.01770</td>
<td>0.01023</td>
<td>0.00965</td>
</tr>
<tr>
<td>High</td>
<td>-0.08608</td>
<td>-0.07796</td>
<td>-0.08948</td>
<td>-0.01540</td>
<td>-0.01442</td>
<td>0.00031</td>
<td>0.01657</td>
<td>0.01760</td>
<td>0.00315</td>
<td>0.00200</td>
</tr>
</tbody>
</table>

The first glance at the results suggests that the unemployment effect of migration is higher than the wage effect in all cells of the labor market. The most affected are the
workers in 25-34 and 35-44 age groups, where both considerable reduction of unemployment and increase in wages are present. For older groups, the wage effect is on average much smaller, possibly due to the facts that the wage is less flexible in this segment of the labor market, and that the vast majority of migrants is accounted for the age under 45 (figure 1 below).

However, the most surprising result is revealed for the youngest age group, where together with the sharp unemployment reduction, migration did not bring any significant change in wages. The smallest wage effect is detected for the youngest and less skilled workers – a puzzling outcome while only taking into consideration massive emigration level in this group (figure 1) and high wage elasticity estimated above.

Notwithstanding, it is also worth looking at the dynamics of unemployment for this group (figure 2 below).
The youngest age group experienced highest unemployment over the period considered, so most of the emigrants possibly belonged to unemployed category prior to their departure. Therefore, the youngest stayers benefited only in terms of reduced unemployment rather than increased wages. This explanation is also confirmed by the fact that after 2008 unemployment increased sharply in spite of enhanced emigration.

The results obtained in this section partly go in line with earlier study on Lithuanian labor market by Elsner (2013). Similarly, he found no effect on the wage distribution between high- and low-skilled workers and discovered smaller impact on the older groups. However, the magnitude of the wage effects in the present study differ dramatically from (Elsner, 2013). Certainly, these discrepancies should be treated with cautions. The wage effect estimates in Elsner’s study (from 0.5 to 9 percent for different experience groups) are performed for a short run; the immediate impact of the 2004 emigration wave was captured using four years of observations. Another potential difference may come from the labor market modeling.
2.7 Conclusions

This chapter investigated the effect of emigration on the labor market of a migrant-sending country. According to the Walrasian approach, emigration should decrease labor supply and lead to reduced unemployment and increased wages of the stayers. Taking advantage of the detailed data provided by Lithuanian Statistical Office, I test this hypothesis. Following the enlargement of the EU in 2004, Lithuania experienced a large wave of emigration. This change in migration policies is treated in the study as a natural experiment and exploited for comparing the key labor market indicators before and after 2004. Using the labor market model with heterogeneous labor and different substitutability among categories of workers, I identify groups of workers that lose or gain from emigration. The workers are segmented by education and age. The latter is used as a proxy for experience.

The results suggest, that emigration affected the unemployment rate more than the wages in all cells of the labor market. However, the effects are not homogeneous. The strongest impact is observed for the workers between 25-44 years old. For older groups, the wage effect is on average much lower, possibly due to the fact that the wage is less flexible in this segment of the labor market, but also since the vast number of emigrants belonged to the age under 45. For the youngest age group (under 25) emigration caused sharp unemployment reduction, without, however, any significant increase in wages. The youngest age group
experienced highest unemployment prior to emigration wave, so most likely the majority of
the emigrants in this group belonged to unemployed category.

This results can be useful for the policy makers of the countries that could face emigration
waves, once their workers are allowed to work abroad. Examples include EU candidate
countries as well as some other Eastern European countries, such as Belarus and Ukraine,
currently facing emigration towards the neighboring states (e.g., Poland and Russia), which
in turn are trying to compensate for the lack of domestic labor supply.

This study opens prospects for future research. With more data on migration from
Eastern Europe becoming available recently, it is important to check if the results hold
for other countries. In order to be convinced that the results are not limited, a similar
analysis should be carried out for more middle-income countries that recently joined the EU.
Additionally, return migration is another interesting subject that should be addressed. After
the crisis of 2008, employment prospects for migrants in the high-income EU countries have
reduced and many workers have returned to their countries of origin. The analysis of return
migration flows can help to determine to what degree migration is driven by employment
and wage differences.
Chapter 3

Labor Market Effects of Migration within Search-and-Matching Framework: a Theoretical Approach

3.1 Introduction

This chapter aspires to contribute to the debate regarding economic determinants of international migration flows and their consequential effects on the labor markets in terms of wages and unemployment rates. The analysis is carried out within a two-country model that belongs to a general family of search-and-matching models of the labor market (e.g., Diamond, 1982; Mortensen and Pissarides, 1994). The main advantage of this approach is that it provides a clear basic structure of a labor market that allows for a (manageable)
analysis of unemployment, wages, market tightness and migration with respect to essential labor market characteristics.

Unlike competitive paradigm, in this type of models workers do not compete with immigrants for a scarce number of jobs, which in turn boosts unemployment and reduces average wages. The search-and-matching approach allows for exploring the unemployment and wage effects that come from simultaneous adjustments of supply and demand sides of the labor market.

The central idea of this setting is that both firms and workers have to spend resources before job creation and production can take place. A larger amount of unemployed workers searching for a job makes it relatively cheaper for a firm to open a new vacancy, given that the probability of filling in this vacancy increases. Increased availability of new vacancies strengthens the bargaining position of unemployed workers. Therefore, immigration does not necessarily lead to a dramatic reduction of wages, as in Walrasian paradigm.

This argument is particularly significant for the analysis of impact of migration on the labor market of migrant-destination countries. A large number of empirical studies that depart from the Walrasian paradigm disagree regarding the direction and magnitude of migration effects on the host country. A particular point of this disagreement is the presence of wage effects. The only definite conclusion that can be drawn is that the impact of migration varies in time and location, and can be either positive or negative (Borjas, 1994).

The aim of this paper is to provide a theoretical rationale for the existence of diverse effects of migration on wages and unemployment within a framework, where migration de-
Decisions are fully integrated into the model. In order to tackle this problem, I introduce costs of migration, that non-linearly increase in share of immigrants. These costs may represent reallocation costs as well as quotas, labor and housing market restrictions and diminishing labor market value of some particular skills of the foreigners (e.g. language), which become excessive with increasing number of immigrants.

First, I present a version of the model, where the immigration costs only participate in immigration decision, thus representing reallocation costs. As the result, migration only affects the unemployment rate in the host region, while the wages remaining unaffected. This happens because firms react to a higher availability of potential employees (hence, lower search costs) by adjusting the vacancy rate. This simple setting, however, allows determination of the equilibrium migration rate given the structural characteristics of labor markets in two regions. This model can be potentially extended to $n$ regions and be useful for rough estimation of the migrants distribution across different locations following, for instance, the EU enlargement or arrival of refugees.

Next, I consider the model, where it is more costly for immigrants to stay in the host countries, then for workers who are born in it. The differentiated costs then result in lower bargaining power of workers during wage negotiations, and, consequently, in lower wages for immigrants. The outcome yields an increase in migration rate together with lower unemployment in a host region. This result comes from the fact that immigrations allows firms to pay lower wages on average and to incur lower search costs given an increased availability of unemployed workers. This in turn leads to a higher rate of vacancy creation and increased
labor market tightness. Immigrants are better-off, because they end up working in a “structurally better” region with higher wage and higher prospects of finding a job, then in their home country. The native workers benefit from the boost in labor demand due to newly created jobs as well as from the higher wages. With respect to this result, a reader should keep in mind that the model is limited to a partial equilibrium. The capital does not enter any of the equations of the presented setting explicitly. However, it is necessary to remember that the result holds only if there are sufficient funds available to support the boost in labor demand.

The rest of the paper is organized as follows. Section 2 discusses previous empirical and theoretical findings on the effects of migration. Section 3 revises the properties of the search-and-matching model for the autraky case. Section 4 presents two versions of a search-and-matching model for two labor markets linked by migration as well as the numerical examples.

3.2 Literature review

The effects of immigration have long been subject to extensive debates in empirical literature. A large number of empirical studies often offered contradictory results. While the studies within so called “nation approach” (Borjas, 2003; Borjas, Grogger, and Hanson, 2008) find an evidence of substantial wage effects of immigration on native workers, studies within “area approach” by Card (2009) and Ottaviano and Peri (2012) argue that the wage effects
are rather small and in some cases negligible. The estimates vary across countries, pointing out that the wage effects are related to the type of immigrants that a country receives. An additional question is whether a study allows for an adjustment of other inputs, specifically capital. This will naturally leads to a lower wage effect (Chiswick and Miller, 2015, p. 936).

One source of disagreement about the magnitude of the wage effect comes from the debates regarding the elasticity of substitution between native workers and immigrants with the same skills (Chiswick and Miller, 2015, p. 830). Obviously, with a lower rate of substitution between the immigrants and the natives and with skill-biased immigration, the native workers have bigger prospects for positive wage effects.

This issue is treated in a theoretical study by Chassamboulli and Palivos (2014), who employ search-and-matching framework that allows for skill heterogeneity and imperfect substitution between immigrants and native workers. The model is designed to analyze skill-biased immigration influx in the US. The authors also account for capital as an independent factor of production that serves as an additional channel of adjustment to immigration-induced changes in labor supply. The authors find, that the skill-biased immigration raised the overall net income of natives. Although the effect is unevenly distributed across skill groups.

Distinguishing labor by skill groups as well as taking capital into the consideration lays beyond the scope of the present study, since the model is intended to be later applied for the analysis of immigration into a small open economy with a perfect access to a world capital market and nearly uniform distribution of immigrants across skill groups (see chapter 4).
However, the results of the present model go in line with that of Chassamboulli and Palivos (2014), although at a higher level of aggregation.

A paper by Ortega (2000) is another study that is directly related to the matters discussed in this chapter. The author introduces higher search costs for a worker from country \( i \) while searching in country 2. His two-country model within a search-and-matching framework provides a rationale for the existence of positive effects of immigration on wages and employment rates of native workers. The presence of search costs for immigrants but not for the natives leads to his leads to wage heterogeneity. Firms in a migrant destination country can anticipate that immigrant workers with higher search costs will agree on receiving lower wages due to their lower bargaining position. Ortega (2000) shows that as a result of optimizing behaviors of firms and workers, there exist multiple equilibria that are Pareto-ranked. In one of such equilibria is welfare improving for everyone. Immigrants gain due to the fact that they work in a structurally better region, hence receiving higher wages than at home. Firms will pay lower wages on average, and due to the short-run scope of the model this does not affect the wages of the native workers. Finally, native workers gain from an increased number of available vacancies, thus, higher job finding probability.

The general result of Ortega (2000) is very similar to the one produced by a model in Section 4.8 below. However, Ortega’s setting does not allow for productivity differences between the regions or differences in unemployment benefits (values of outside option), since the two labor markets differ only in their separation rates. Another property of his model is a linear cost structure and a migration rate that is independent of the unemployment rate.
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in a migrant-sending country.
3.3 Review of the properties of the search-and-matching model for the autarky case

In this section I review main properties of a standard search and matching model with exogenous separation based on Pissarides (2000) and Miao (2014) in discrete time notation. The discussion below will be useful for the comparison with a two-country model presented later on in this chapter.

A basic search and matching model in discrete time

I assume there is a continuum of infinitely living identical workers, normalized to one. Each worker derives utility from consumption in each period $t$. The production technology is assumed to exhibit constant returns to scale. The only input is labor. The marginal productivity of labor is denoted by $p$. Each firm can employ maximum one worker. A firm starts operating by posting a vacancy at the cost $c$ and has to incur that cost until the vacancy is filled. When a vacancy is matched with a worker, a wage $w_t$ is negotiated and a firm generated profit $p - w_t$. Matches fall apart with an exogenous separation rate $s$.

The number of total matches in period $t$ is determined by a specific matching function which takes as arguments the total number of unemployed workers $u_t$ and vacancies $v_t$: $m_t = m(u_t, v_t)$. The matching function $m$ is assumed to be homogeneous of degree one, increasing in both arguments and concave. It is useful to define another variable, the market tightness, which is given by $\theta_t \equiv v_t/u_t$. Then, by the linear homogeneity property, the
probability that an open vacancy is filled (the job filling rate) can be expressed as:

\[ q(\theta_t) = \frac{m(u_t, v_t)}{v_t} = m\left(\frac{1}{\theta_t}, 1\right). \] (3.1)

Likewise, the probability for an unemployed worker to find a job (job finding rate) is given by

\[ f(\theta_t) = \frac{m(u_t, v_t)}{u_t} = m(1, \theta_t) = \theta_t q(\theta_t). \] (3.2)

From the properties of the matching function it follows that \( q'(\theta) < 0 \) and \( d \ln q(\theta)/d \ln \theta \in (-1, 0) \). Simply put, when the labor market is tighter, it becomes harder for a firm to fill in the vacancy and easier for a worker to find a job.

Next I define by \( N_t \) the aggregate employment at time \( t \). It is governed by the following dynamic equation:

\[ N_{t+1} = (1 - s_t) N_t + m(u_t, v_t), \] (3.3)

where \( s_t \) denotes job separation rate, and \( N_0 \) is given. Naturally, then the unemployment rate is defined as \( u_t = 1 - N_t \).

Another key assumption of the model is that the wage rate \( w_t \) is determined as an outcome of a Nash bargaining problem between a matched firm and worker. To state this problem, I must first elaborate the equations governing firm’s and worker’s decisions in this economy.
Let $U_t$ be the value of the worker while being unemployed and $W_t$ be the value of the worker while on the job. The asset value of unemployment state for the worker at time $t$ can be derived as a sum of the current period utility and the expected discounted value at time $t + 1$:

$$U_t = z + \delta[\theta_t q(\theta_t) W_{t+1} + (1 - \theta_t q(\theta_t)) U_{t+1}], \quad (3.4)$$

where $z$ can be broadly defined as the flow utility of leisure, $\delta = \frac{1}{1+r}$ is the time discounting rate and $\theta_t q(\theta_t)$ is the probability of obtaining a job at time $t$.

While on the job, a worker earns wage $w_t$. Also she faces an exogenous probability of loosing a job $s_t$ when a match dissolves. In this case, starting from the next period a workers becomes unemployed and the value drops to that of the unemployment state. Formally the value for an employed worker at time $t$ can be expressed as:

$$W_t = w_t + \delta[(1 - s) W_{t+1} + s U_{t+1}], \quad (3.5)$$

Now let $V_t$ be the value of an unfilled vacancy and $J_t$ be the value of the filled job. Then they should satisfy Equations 3.6 and 3.7:

$$V_t = -c + \delta[(q(\theta_t) J_{t+1} + (1 - q(\theta_t)) V_{t+1}], \quad (3.6)$$

where $c$ is the flow cost of the firm of keeping an unfilled vacancy and continuing to search.
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\[ J_t = p - w_t + \delta [sV_{t+1} + (1 - s)J_{t+1}] \]  (3.7)

I assume free entry to the market. Competition drives the value of the vacancy to zero \((V_t \equiv 0)\). Hence Equation 3.6 implies that

\[ J_{t+1} = \frac{c}{\delta q(\theta_t)}. \]  (3.8)

After substituting into (3.7):

\[ J_t = p - w_t + \frac{(1 - s)c}{q(\theta_t)}. \]  (3.9)

By combining (3.8) and (3.9) I obtain the dynamic equation governing behavior of labor market tightness \(\theta\):

\[ \frac{c}{\delta q(\theta_t)} = \left( p - w_{t+1} + \frac{(1 - s)c}{q(\theta_{t+1})} \right) \]  (3.10)

To complete the description of equilibrium, I must now get back to the determination of wages.

Wage determination

The Nash bargaining problem is given by

\[ \max_{w_t} (W_t - U_t)^\beta (J_t - V_t)^{(1-\beta)}, \]  (3.11)
subject to $W_t \geq U_t$ and $J_t \geq V_t$, where $\beta \in (0, 1)$ is the worker’s bargaining power.

Remembering that $V_t = 0$, the first-order condition is:

$$\beta J_t = (1 - \beta)(W_t - U_t).$$ (3.12)

Define the total surplus generated by a match as $S_t = J_t + W_t - U_t$ (*). Since from Equation 3.12 $J_t = \frac{1-\beta}{\beta}(W_t - U_t)$, it immediately follows that

$$\beta S_t = W_t - U_t.$$ (3.13)

Additionally, the free entry condition implies that the following equality holds:

$$(1 - \beta)S_t = J_t = \frac{c}{\delta q(\theta_{t-1})}.$$ (3.14)

After substituting Equations 3.7, 3.4 and 3.5 into (*), one can obtain:

$$S_t = p - z + \delta(1 - s)S_{t+1} - \delta \theta_t q(\theta_t)[W_{t+1} - U_{t+1}].$$ (3.15)

$$S_t = p - z + \delta(1 - s)[J_{t+1} + W_{t+1} - U_{t+1}] - \delta \theta_t q(\theta_t)[W_{t+1} - U_{t+1}]$$

$$= p - z + \delta(1 - s - \beta \theta_t q(\theta_t))S_{t+1}$$

$$= p - z + \frac{c(1 - s - \beta \theta_t q(\theta_t))}{(1 - \beta)q(\theta_t)},$$ (3.16)

where I used Equation 3.14 to derive the last equality.

Next use Equations 3.4 and 3.5 for rewriting Equation 3.13 as follows:
\[ \beta S_t = W_t - U_t \]
\[ = w_t - z + \delta(1 - s - \theta_t q(\theta_t))[W_{t+1} - U_{t+1}] \]  
\[ = p - z + \delta(1 - s - \theta_t q(\theta_t))\beta S_{t+1}. \]

Finally substituting 3.16 for \( S_t \) and 3.14 into the equation above and solving for \( w_t \) gives

\[ w_t = \beta(p + c\theta_t) + (1 - \beta)z. \]  

(3.18)

This completes the description of the partial equilibrium, which is fully determined by six variables \( N_t, u_t, v_t, \theta_t, w_t \) and \( q(\theta_t) \) such that they satisfy Equations (3.1) - (3.3), (3.10) and (3.18).

**Steady State**

Now let’s describe the steady state properties of the model by keeping all variables constant over time. Rewrite Equation (3.10) in the following form:

\[ \frac{c}{\delta q(\theta)}(1 - \delta(1 - s)) = (p - w). \]  

(3.19)

This gives us the first relationship between the wage and labor market tightness, the job creation curve (Figure 3.1). It is downwards sloping on the plane \((\theta, w)\) since \(q(\theta)\) is a decreasing function of \(\theta\). When wage increases, it becomes less profitable for firms to open vacancies. The job creation curve is an equivalent to the labor demand curve in Neoclassical economics.
The second relationship between $w$ and $\theta$ comes from Equation (3.18) for steady state:

$$w = \beta(p + c\theta) + (1 - \beta)z. \quad (3.20)$$

It yields an upwards sloping wage curve (Figure 3.1). When the labor market becomes tighter, wages increase since workers gain more bargaining power. It replaces the labor supply curve of Neoclassical labor market model.

In the intersection of the wage curve and job creation curve there is a unique steady state equilibrium $(\theta^*, w^*)$. By combining Equations (3.19) and (3.20) and solving for $\theta$, one can show that $\theta^*$ is indeed unique, since the RHS of the Equation (3.21) below is a strictly increasing and the LHS is a strictly decreasing function of $\theta$: 

![Figure 3.1: Wage and tightness in steady state equilibrium](image-url)
\[
(1 - \delta(1 - s)) \frac{c}{\delta q(\theta)} = -\beta c \theta + (1 - \beta)(p - z).
\] (3.21)

Figure 3.2: Vacancies and unemployment in steady state equilibrium

Next, with the \( \theta \) known, one can plot the job creation curve \( v = u \theta \) in \((u, v)\) coordinates (Figure 3.2 above). By considering the steady state version of employment dynamics (3.3): 
\[ s(1 - u) = m(u, v), \]
I obtain the relationship between unemployment and vacancy rates, a so called Beveridge curve. From the properties of the matching function it follows that the curve is downwards sloping and convex. On the intersection one can obtain the steady state equilibrium vacancy and employment rates.

Now comparative statics (for steady states) are straightforward. Consider an exogenous increase in productivity \( p \). This shifts both the job creation and the wage curves in Figure
3.1 upwards. Since $0 < \beta < 1$, the job creation curve shifts more and the system ends up in a new equilibrium with higher $w$ and $\theta$. It follows that the job creation curve in Figure 3.2 shifts counterclockwise, but the Beveridge curve remains unaffected. Thus, unemployment falls while vacancy rate increases.

An exogenous increase in worker's outside option $z$ shifts the wage curve in Figure 3.1 upwards, while the job creation curve remains in place. Thus, higher $w$ and lower $\theta$. Next, the job creation curve in Figure 3.2 shifts clockwise, increasing unemployment and lowering the vacancy rate. A similar result follows from an increase of worker’s bargaining power $\beta$. An increase of the separation rate $s$ drops the model into a new steady state equilibrium with lower $\theta$ and $w$, higher unemployment rate. However notably the greater separation rate also increases the number of vacancies.
Transitional dynamics

Substitute Equation (3.18) into (3.10) and rewrite as follows:

\[ \beta c \theta_{t+1} - \frac{(1 - s)c}{q(\theta_{t+1})} = (1 - \beta)(p - z) - \frac{c}{q(\theta_t)}. \]  

(3.22)

Equation (3.3) governing the dynamics of employment can be presented in terms of \( u \):

\[ u_{t+1} = (1 - s - q(\theta_t)\theta_t)u_t + s. \]  

(3.23)

I have now obtained a system of two non-linear equations in two unknown variables \((u_t, \theta_t)\). To analyze the properties of this system I should first linearize it around the steady state \((u^*, \theta^*)\):

\[
\begin{bmatrix}
\partial \theta_{t+1} \\
\partial u_{t+1}
\end{bmatrix} = \begin{bmatrix}
a_{11} & 0 \\
a_{21} & a_{22}
\end{bmatrix} \begin{bmatrix}
\partial \theta_t \\
\partial u_t
\end{bmatrix}
\]  

(3.24)

, where

\[
a_{11} = \frac{q'(\theta^*)/q^2(\theta^*)}{(1 - s)q'(\theta^*)/q^2(\theta^*) + \beta}
\]

\[
a_{21} = u^* q(\theta^*)[1 - \eta(\theta^*)]
\]

(3.25)

\[
a_{22} = 1 - s - q(\theta^*)\theta^*
\]

where \( \eta(\theta) = -\theta q'(\theta)/q(\theta) \) is the elasticity of the function \( q(\theta) \). Keeping in mind that \( q'(\theta) < 0 \), one can easily prove that \( \eta(\theta) \in (0,1) \) and therefore \( a_{21} > 0 \). Suppose that
1 - s - $\beta q(\theta^*)\theta^*/\eta(\theta^*) > 0$, then $a_{11} > 1$ and one can verify that $a_{22} \in (-1, 1)$. Thus, the steady state is a local saddle point. Note that $u_t$ is predetermined, while $\theta_t$ is not.

The phase diagram for the transitional dynamics is presented on the Figure 3.3. The locus $\Delta u_t = 0$ is downwards sloping and $\Delta \theta_t = 0$ is a horizontal line.

Consider a case when initial unemployment rate is $u_0$. The market tightness immediately jumps to the saddle path on the line $\Delta \theta_t = 0$. During the transition, $\theta_t$ is unchanged but $u_t$ decreases until it approaches steady state. Figure 3.4 shows the adjustment path of $v_t$. The vacancy rate initially jumps to the higher level and then steadily falls, keeping labor market tightness constant over time.

**Labor turnover**

Before proceeding with presentation of two-country model, let's first review how the outcomes of the model change due to the entrance and exit of new workers in and out of the labor
force. Denote the entrance rate by \( b \) and exit rate by \( \pi \). One can think of \( b \) as births or immigration. Assume all new entrants immediately replenish the pool of unemployed and start to look for a job. Exit from the labor force \( \pi \) may be thought of as retirement or emigration.

As before, there is an exogenous shock \( s \) that leads to job destruction and the total number of matches that occur in the labor market \( q(\theta)\theta uL \). Additionally there is now a flow of new entrants into unemployment \( bL \) and exits from unemployment \( \pi uL \). Thus, the evolution of the number of unemployed is given by:

\[
 u_{t+1}L_{t+1} - u_tL_t = s(1 - u_t)L_t + bL_t - \pi u_tL_t - q(\theta_t)\theta_t u_tL_t. \tag{3.26}
\]

For the steady state values of unemployment rate \( u \) and labor market tightness \( \theta \) it holds that:
\begin{equation}
\frac{u(L_{t+1} - L_t)}{L_t} = s(1-u)L_t + bL_t - \pi u L_t - q(\theta)\theta L_t. \tag{3.27}
\end{equation}

The rate of growth of the labor force \( \frac{L_{t+1} - L_t}{L_t} \) is given by the total entry less total exit: \( b - \pi \). The implicit assumption here is that the exit rates are the same both from employment and unemployment groups. Therefore, after dividing both sides by \( L_t \) and substituting \( b - \pi \) into 3.27, I obtain:

\begin{equation}
(s + b) - (s + b)u - q(\theta)\theta u = 0. \tag{3.28}
\end{equation}

The steady state level of unemployment from equation above is:

\begin{equation}
 u = \frac{s + b}{s + b + \theta q(\theta)}. \tag{3.29}
\end{equation}

Equation 3.29 is the new Beveridge curve. The entry rate \( b \) shifts it to the right, while the exit rate \( \pi \) has no effect on the locus. Thus, on average countries with higher entry rates have higher equilibrium unemployment then other countries. The exit rate does not affect the Beveridge curve because of the assumption that exit occurs at the same rate from unemployment and employment cohorts. If the rate of exit from unemployment is higher than the rate of exit from employment, the exit rate shifts the Beveridge curve downwards, implying lower equilibrium unemployment rate at given vacancy rate.

Consider no the determination of job creation and wage curve with exogenous exit rates \( \pi \). The value of vacant job remains unchanged, and as previously is given by:

\begin{equation}
V_t = -c + \delta[(q(\theta_t)J_{t+1} + (1-q(\theta_t))V_{t+1}], \tag{3.30}
\end{equation}
thus, from the zero-profit entry condition it follows that:

\[ J_{t+1} = \frac{c}{\delta q(\theta_t)}. \] (3.31)

The equation describing the value of filled job for a firm now is modified to reflect the fact that a job can be terminated for one of two reasons, the exogenous separation rate \( s \) and the exit (retirement) \( \pi \). Only the first one may move the value of the job to another positive value. The exit of a worker immediately yields zero returns with probability one. Hence, the value for \( J_t \):

\[ J_t = p - w_t + \delta(1 - \pi)[sV_{t+1} + (1 - s)J_{t+1}] \] (3.32)

Similarly, under the assumption that exit occurs both from employment and unemployment at the same rate, I can state the following equations for the worker’s:

\[ U_t = z + \delta(1 - \pi)[\theta_t q(\theta_t)W_{t+1} + (1 - \theta_t q(\theta_t))U_{t+1}] \] (3.33)

\[ W_t = w_t + \delta(1 - \pi)[(1 - s)W_{t+1} + sU_{t+1}] \] (3.34)

It easy to check that the job creation and wage equations are now describes by:

\[ \frac{c(1 - \delta(1 - \pi)(1 - s))}{\delta q(\theta)} = p - w, \] (3.35)
and

\[ w = (1 - \beta)z + \beta(p + c(1 - \pi)\theta). \quad (3.36) \]

In case of the job creation condition, the exit rate \( \pi \) is added to the rate of discount. The explanation is that higher exit rate increases the probability that the job will terminate sooner. Thus, the higher exit rate shifts the job creation curve downwards. At the same time, it shifts the wage curve downwards, moving the system to a new equilibrium with lowe wages. The effect on market tightness in this case is ambiguous and will depend on the values of other parameters.
3.4 Two country search-and-matching model with migration

Background

After having reviewed the main properties of the basic search-and-matching model, I now proceed with presenting a theoretical framework that will allow us to determine how two labor markets are linked through migration and what would be the effects on the equilibrium unemployment rate.

Environment

The economy consists of two regions indexed by $i = 1, 2$. In each region the market is decentralized and uncoordinated. Each labor market is represented by firms and workers, that dedicate time and resources to search for forming a match. The labor force in each region consists of $L_i$ individuals, out of which $E_i = u_i L_i$ are unemployed and $N_i = (1 - u_i) L_i$ employed, where $u_i$ is the unemployment rate. The possibility of on-the-job search is ruled out. Furthermore, I assume that in order to search for a job in a specific region, a worker has to live in it.

The total labor demand equals to the number of filled jobs plus the number of advertised vacancies. In order to establish a vacancy, a firm has to incur a search cost $c$ the vacancy is filled. Denote by $v_i$ the ratio between the number of vacancies and the total labor force,
then the mass of vacant firms that are searching for workers is $v_i L_i$. The matching process is governed by the constant return to scale matching function.

$$m_i L_i = m(u_i L_i, v_i L_i), \quad (3.37)$$

where $m_i L_i$ is the number of matches calculated as a function of $u_i$ and $v_i$. Denote by $\theta_i \equiv \frac{v_i}{u_i}$ the labor market tightness ratio and use the assumption of constant returns to scale matching function. Then the probability of a firm to fill a vacancy and the job-finding probability are, respectively:

$$q(\theta_i) = \frac{m(u_i L_i, v_i L_i)}{v_i L_i} = m\left(\frac{1}{\theta_i}, 1\right). \quad (3.38)$$

$$\theta_i q(\theta_i) = m(1, \theta_i). \quad (3.39)$$

**Migration**

Migration in this context acts as an alternative to searching for a job in the home region. Every period $t$ an unemployed worker in region $i$ compares the present discounted value of remaining unemployed at home ($U_{i,t+1}$) and the present discounted value of being unemployed after moving abroad ($U_{j,t+1}d(\mu_i)$). Given the fact that the value of unemployment state incorporates such labor market parameters as future probability of finding a job, future potential salary, the value of outside option and job separation rate (due to the recursive property of
the Bellman equations), the decision to immigrate is essentially based on a range of labor market characteristics of the two regions.

In a simplest version migration costs represent the costs of actual transferring and settling abroad. Later on in this chapter, I consider an extension of the model where immigrants will have higher costs while staying abroad.

The cost function $d \in (1, \infty)$ is assumed to be increasing in the rate of migration. With this respect, one can think of quotas on immigration for people of particular nationalities, restrictive housing market regulations (e.g. in Switzerland) and skills that are specific to immigrants from a certain region, such as language, which become excessive in a foreign labor market and do not contribute to job finding as much as when immigration is lower. This assumption is introduced for the modeling purposes and, thus, should be treated with caution while applying the model for empirical analysis. Recent literature is pointed out at the importance of diasporas for lowering the natural barriers to migration. As a result, the growing diaspora will catalyze more and more immigration (Collier, 2013). Therefore, for the analysis of migration in countries, where diasporas play important role (e.g., France, Germany, etc.), the assumption on the shape of the cost function must be relaxed and modified. One possible way is to consider a jump discontinuity in the costs of migration.

So, when making a decision in which region to search for a job, an unemployed worker discounts the value of unemployment state abroad by $\frac{1}{d(\mu_{it})} < 1$. Whenever $U_{i,t+1} < \frac{U_{j,t+1}}{d(\mu_{it})}$, a worker from region $i$ will emigrate, otherwise he/she will choose to stay and search for a job in her/his home region.
Bellman equations

Workers. Unemployed workers decide whether to search in their home region or to move and search abroad. The asset value of unemployment state at time $t$ is given by the current period utility plus the expected discounted value at time $t + 1$:

$$U_{i,t} = z_i + \delta \left[ \theta_{i,t}q(\theta_{i,t})W_{i,t+1} + (1 - \theta_{i,t}q(\theta_{i,t}))\max \left\{ U_{i,t+1}; \frac{U_{j,t+1}}{d(\mu_{it+1})} \right\} \right].$$

(3.40)

where $\delta = \frac{1}{1+r}$ is the time discounting rate, $z_i$ is the flow utility of leisure, $\theta_{i,t}q(\theta_{i,t})$ is the probability of obtaining a job in time $t$ and $i \neq j$.

Employed workers earn wage $w_{i,t}$ defined as a solution to a Nash bargaining problem. Active jobs in a region dissolve with exogenous rate $s_i$ and the affected workers enter the unemployment state starting from the next period. In this case, the value drops to the highest value of being unemployed in one of the two regions. Thus, the value for an employed worker at time $t$ is given by

$$W_{i,t} = w_{i,t} + \delta \left[ (1 - s_i)W_{i,t+1} + s_i\max \left\{ U_{i,t+1}; \frac{U_{j,t+1}}{d(\mu_{it+1})} \right\} \right].$$

(3.41)

Firms.

The flow cost of keeping an unfilled vacancy is $c$, therefore, the asset value of holding a vacant job is given by

$$V_{i,t} = -c + \delta[q(\theta_{i,t})J_{i,t+1} + (1 - q(\theta_{i,t}))V_{i,t+1}].$$

(3.42)

Because the free entry of firms is assumed, in equilibrium the value of vacancies will be driven to zero: $V_i = 0 \ \forall t$. Hence, equation 3.42 can be written
\[ J_{i,t+1} = \frac{c}{\delta q(\theta_{i,t})} \] (3.43)

The value of filled job equals to the present period profit plus the present discounted value of firm at time \( t + 1 \).

\[ J_{i,t} = p_{i,t} - w_{i,t} + \delta [s_i V_{i,t+1} + (1 - s_i) J_{i,t+1}] . \] (3.44)

Substituting equation 3.42 into 3.44 and imposing \( V_i = 0 \), I get:

\[ J_{i,t} = p_{i,t} - w_{i,t} + (1 - s_i) \frac{c}{q(\theta_{i,t})} . \] (3.45)

As in a standard Mortensen-Pissarides (MP) setting, the wage is modeled as the solution to the Nash bargaining problem, where firms and workers share the total surplus generated by the match according to:

\[ \beta (J_{i,t} - V_{i,t}) = (1 - \beta) \left( W_{i,t} - \max \left\{ U_{i,t}; \frac{U_{j,t}}{d(\mu_{rt})} \right\} \right) . \] (3.46)

The wage is renegotiated every period \( t \). Workers receive a constant fraction \( \beta \) of the total surplus that is expressed as a value that makes them indifferent between remaining on the job and becoming unemployed in either of the two regions.

An unemployed worker from region \( i \) will move to the location \( j \) if and only if \( \frac{U_j}{d(\mu_{rt})} > U_i \).

**Proposition 1:** At every time \( t \) migration can only be one directional. **Proof:** Allowing migration to both directions at the same time implies that the following two inequalities hold: \( \frac{U_2}{d_{i_1}} > U_1 \) and \( \frac{U_1}{d_{i_2}} > U_2 \).
Let \( U_2 = \alpha U_1, \alpha > 0 \)

Then

1) \( \frac{U_2}{d_1} > U_1 \Rightarrow \frac{\alpha U_1}{d_1} - U_1 > 0 \Rightarrow \left( \frac{\alpha}{d_1} - 1 \right) U_1 > 0 \Rightarrow \alpha > d_1 > 1 \) (since \( d_i > 1 \) by def.)

2) \( \frac{U_1}{d_2} > U_2 \Rightarrow \frac{U_1}{d_2} - \alpha U_1 > 0 \Rightarrow \left( \frac{1}{d_2} - \alpha \right) U_1 > 0 \Rightarrow \alpha < \frac{1}{d_2} < 1 \)

**Steady state**

The equilibrium in the labor markets of two regions in the steady state is governed by the system of ten equations (five for each region):

\[
U_i = z_i + \delta \left[ \theta_i q(\theta_i) W_i + (1 - \theta_i q(\theta_i)) \max \left\{ U_i, \frac{U_j}{d(\mu_i)} \right\} \right]. \tag{3.47}
\]

\[
W_i = w_i + \delta \left[ (1 - s_i) W_i + s_i \max \left\{ U_i, \frac{U_j}{d(\mu_i)} \right\} \right]. \tag{3.48}
\]

\[
J_i = \frac{c}{\delta q(\theta_i)}. \tag{3.49}
\]

\[
J_i = y_i - w_i + \delta (1 - s_i) J_i. \tag{3.50}
\]

\[
J_i = \frac{1 - \beta}{\beta} \left[ W_i - \max \left\{ U_i, \frac{U_j}{d(\mu_i)} \right\} \right]. \tag{3.51}
\]
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Wage determination

Once migration took place and maximum function can be decided, the wages are determined as usually in search-and-matching framework via Nash bargaining: \((1 - \beta)(W_i - U_i) = \beta J_i\).

After substituting equations (3.47), (3.48) and (3.50), it holds that:

\[
w_i = y_i - (1 - \beta)(1 - \delta(1 - s_i)) \frac{y_i - z_i}{1 - \delta(1 - s_i) + \beta \delta \theta q(\theta)}.
\] (3.52)

In a present simple setting wages do not depend on the rate of migration, since the productivities, separation rates and values of outside option of immigrants and native workers within a given region are assumed to be the same. Moreover, migration costs are assumed to be taken into consideration with respect to migration decision, representing only the costs of reallocation. Upon their arrival, migrants obtain the value of unemployment state that is common to all unemployed workers in that destination region. In a section 4.8 below I describe the result when this assumption is relaxed and immigrants keep bear the costs after moving abroad. As the result of higher search costs, ceteris paribus, immigrants will have a weaker bargaining position (lower ‘threat point’), which results in a lower equilibrium wage, than that of native workers.

Labor force dynamics

In order to close the model, I must describe the evolution of unemployment. Using our results from section 3.3, I proceed with describing the dynamics of the labor force and
unemployment rates. Let $L_1$ represent a total labor force of migrant sending destination and $L_2$ be the labor force of migrant receiving region.

Let $\mu$ represent the migration rate of unemployed. Then, for region one the exits from unemployment are due to emigration $\mu u_1 L_1$ and newly formed matches $q(\theta_1)\theta_1 u_1 L_1$. I also introduce a "birth" rate $b = \mu$, with which new workers enter the unemployment cohort. This assumption is done for two reasons. Firstly, to avoid the corner solution, where all the unemployed workers from region one emigrate. Secondly, this is done for tractability, since it allows to keep $L_1$ constant and on it’s basis determine $L_2$. Thus, the total unemployment in region one is given by $u_1 L_1$, so its evolution (for the steady state values of $u_1$ and $\theta_1$ is given by

$$u_1(L_{1,t+1} - L_{1,t}) = s_1(1 - u_1)L_{1,t} + bu_1 L_{1,t} - \mu u_1 L_{1,t} - q(\theta_1)\theta_1 u_1 L_{1,t}. \quad (3.53)$$

$$0 = s_1(1 - u_1) + bu_1 - \mu u_1 - q(\theta_1)\theta_1 u_1 - u_1 \frac{L_{1,t+1} - L_{1,t}}{L_{1,t}}. \quad (3.54)$$
The total change of the labor force, $L_1, t+1 - L_1, t$, is determined by the entry less exit from unemployment, $(bu_1 - \mu u_1)$. So substitution into 3.54 under the assumption that emigration and birth of unemployed compensate each other, leaves us with the steady-state level of unemployment unchanged.

$$u_1 = \frac{s_1}{s_1 + q(\theta_1)\theta_1}.$$  \hspace{1cm} (3.55)

Without the assumption above, emigration would shift the Beveridge curve to the left, implying lower equilibrium unemployment rate for a given vacancy rate.

For region two arrival of workers from abroad is equivalent to "birth" rate and growth of the total labor force. The total number of workers entering the labor force is $\mu u_1 L_1$. Keeping in mind that the rate of change in $L_2$ is given by $\mu u_1 \frac{L_1}{L_2}$, the steady-state level of unemployment can be derived as above:

$$u_2 = \frac{s_2 + \mu u_1 \frac{L_1}{L_2}}{s_2 + q(\theta_2)\theta_2 + \mu u_1 \frac{L_1}{L_2}}.$$  \hspace{1cm} (3.56)

Unemployment in a migrant destination region is positively related to that of migrant sending region. Moreover, it depends on the relative size of the labor force in two destinations.

**Solution of the system (3.47) -(3.51)**

It is trivial to verify that the system of equations (3.47) -(3.51) can be reduced to four functional equations in $(U_i; \theta_i)$ (see next page for details):
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\[ U_i = z_i + \frac{\beta c}{1 - \beta} \theta_i + \delta \max \left\{ U_i, \frac{U_i}{d(\mu_i)} \right\} \]  

(3.57)

\[ \frac{c}{1 - \beta} \frac{1 - \delta (1 - s_i)}{\delta q(\theta_i)} = y_i - (1 - \delta) \max \left\{ U_i, \frac{U_j}{d(\mu_i)} \right\} \]  

(3.58)
Derivation of Equations (3.57) and (3.58)

First, combining (3.49) and (3.51), and rearranging, I obtain:

\[ W_i = \beta \frac{c}{1 - \beta \delta q(\theta_i)} + \max \left\{ U_i, \frac{U_j}{d(\mu_i)} \right\} \]  \tag{3.59}

Next inserting (3.59) into (3.47) and using some algebraic manipulations, obtain (3.57):

\[ U_i = z_i + \delta \left[ \theta_i q(\theta_i) \beta \frac{c}{1 - \beta \delta q(\theta_i)} + \theta_i q(\theta_i) \max \left\{ U_i, \frac{U_j}{d(\mu_i)} \right\} \right] - \delta \theta_i q(\theta_i) \max \left\{ U_i, \frac{U_j}{d(\mu_i)} \right\} + \delta \max \left\{ U_i, \frac{U_j}{d(\mu_i)} \right\} \]  \tag{3.60}

In order to obtain equation (3.58), take (3.48) and rearrange as follows:

\[ [1 - \delta(1 - s_i)]W_i = w_i + \delta s_i \max \left\{ U_i, \frac{U_j}{d(\mu_i)} \right\} \]  \tag{3.61}

Combining (3.59) and (3.61) and rearranging, I get:

\[ [1 - \delta(1 - s_i)] \left( \beta \frac{c}{1 - \beta \delta q(\theta_i)} + \max \left\{ U_i, \frac{U_j}{d(\mu_i)} \right\} \right) = w_i + \delta s_i \max \left\{ U_i, \frac{U_j}{d(\mu_i)} \right\} \]  \tag{3.62}

Now, substitute expression (3.52) for \( w_i \):

\[ \frac{[1 - \delta(1 - s_i)] \beta c}{(1 - \beta) \delta q(\theta_i)} + (1 - \delta(1 - s_i)) \max \left\{ U_i, \frac{U_j}{d(\mu_i)} \right\} = y_i + (\delta(1 - s_i) - 1) \frac{c}{\delta q(\theta_i)} + \delta s_i \max \left\{ U_i, \frac{U_j}{d(\mu_i)} \right\} \]  \tag{3.63}
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\[ \frac{[1 - \delta(1 - s_i)]c}{(1 - \beta)\delta q(\theta_i)} \beta + \frac{[1 - \delta(1 - s_i)]c}{(1 - \beta)\delta q(\theta_i)} = y_i - (1 - \delta)\max \left\{ U_i, \frac{U_j}{d(\mu_i)} \right\}. \] (3.64)

After some rearrangement, I obtain equation (3.58).

**Multiple equilibria**

Three cases are possible:

1. No migration (autarky), when both \( U_1 > \frac{U_2}{d(\mu_1)} \) and \( U_2 > \frac{U_1}{d(\mu_1)} \) are true, so that there are no incentives for the unemployed in both countries to migrate. In this case we are back to a standard one-country search-and-matching model.

2. Full migration, when all the unemployed from one region leave. This corner solution is characterized by the following conditions: \( u_i = 0, u_j > 0 \) and \( U_i < \frac{U_j}{d(\mu_i)} \). This possibility is ruled out, since I have made an assumption implying that the stock of unemployed is constantly refilled.

3. Partial migration, when either of the two indifference conditions hold: \( U_i = \frac{U_j}{d(\mu_i)} \) or \( U_j = \frac{U_i}{d(\mu_j)}, i \neq j \). Both can be only true at the same time in the absence of migration costs. In this case \( u_{i,j} \geq 0 \), but all the incentives for migration are exhausted.

From now on the analysis will be focused on Case 3. The resulting migration flows and their effects on equilibrium can be observed by comparing results with the autarky case.
Analytical solution for Cobb-Douglas Matching function

For a specific form of the matching function, namely Cobb-Douglas with parameters \((0.5, 0.5)\), the model can be solved analytically. Then \(q(\theta) = \left(\frac{1}{\theta}\right)^{1/2}\) and \(f(\theta) = \theta^{1/2}\). This is a useful exercise that will provide an insight on the behavior of the outcome variables. The system (3.57) -(3.58) can be rewritten as:

\[
\begin{align*}
U_1 &= z_1 + \frac{\beta c}{1-\beta} \theta^1_1 + \delta \max \left\{ U_1; \frac{U_2}{d(\mu_1)} \right\}, \\
U_2 &= z_2 + \frac{\beta c}{1-\beta} \theta^1_2 + \delta \max \left\{ U_2; \frac{U_1}{d(\mu_2)} \right\}, \\
\frac{c}{1-\beta} \left[ \frac{1-\delta(1-s)}{\delta(\frac{1}{\theta^1_i})^{2}} \right] &= y_1 - (1 - \delta) \max \left\{ U_1; \frac{U_2}{d(\mu_1)} \right\}, \\
\frac{c}{1-\beta} \left[ \frac{1-\delta(1-s)}{\delta(\frac{1}{\theta^2_i})^{2}} \right] &= y_2 - (1 - \delta) \max \left\{ U_2; \frac{U_1}{d(\mu_2)} \right\}.
\end{align*}
\]

(3.65)

Without loss of generality, assume that migration goes from region 1 to 2. Therefore, as it is explained in Case 3 above, it should be true that \(U_1 = \frac{U_2}{d(\mu_1)}\). Since \(d > 1\forall \mu\), it follows that \(U_2 > \frac{U_1}{d(\mu_2)}\) and the maximum function can be decided.

This immediately implies that a system 3.65 can be split into two independent systems of equations for each region, from which I can obtain the unknowns \(U_i\) and \(\theta_i\).

\[
\theta_i = \left( \frac{-c(1 - \delta(1 - s)) + \sqrt{D}}{2\delta^2 c} \right)^2,
\]

(3.66)

where \(D = (c(1 - \delta(1 - s)))^2 + 4\delta^2 \beta c(1 - \beta)(y_i - z_i)\).

The value of unemployment state can be determined by substituting \(\theta_i\) into
The steady-state migration rate can be then obtained from the equilibrium condition

\[ U_1 = \frac{U_2}{d(\mu_1)} \].

The equilibrium unemployment rates are given by the equations (3.55) and (3.56), and the wages by (3.52).

A numerical example of one possible equilibrium with partial migration is presented in Table 3.1. The parameter values are from Ortega (2000) and Hagedorn and Manovskii (2008). Region 1 is a migrant-sending destination, region 2 is a migrant-receiving one. The relative size of the labor forces is 10:1, with region 2 being a smaller economy.

Table 3.1: Numerical example of equilibrium with partial migration

<table>
<thead>
<tr>
<th></th>
<th>Region 1</th>
<th></th>
<th>Region 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4.4</td>
<td>y</td>
<td>5.4</td>
</tr>
<tr>
<td>s</td>
<td>0.0097</td>
<td>s</td>
<td>0.0085</td>
</tr>
<tr>
<td>z</td>
<td>0.955</td>
<td>z</td>
<td>0.955</td>
</tr>
<tr>
<td>δ</td>
<td>0.99</td>
<td>δ</td>
<td>0.99</td>
</tr>
<tr>
<td>β</td>
<td>0.052</td>
<td>β</td>
<td>0.052</td>
</tr>
<tr>
<td>c</td>
<td>0.548</td>
<td>c</td>
<td>0.548</td>
</tr>
<tr>
<td>θ</td>
<td>0.2852</td>
<td>θ</td>
<td>0.4743</td>
</tr>
<tr>
<td>U</td>
<td>1.0502</td>
<td>U</td>
<td>1.0564</td>
</tr>
<tr>
<td>w</td>
<td>1.1423</td>
<td>w</td>
<td>1.1997</td>
</tr>
<tr>
<td>q(θ₁)</td>
<td>1.8724</td>
<td>q(θ₂)</td>
<td>1.4520</td>
</tr>
<tr>
<td>u</td>
<td>0.0178</td>
<td>u₂ autarky</td>
<td>0.0122</td>
</tr>
<tr>
<td></td>
<td></td>
<td>u₂ migration</td>
<td>0.0137</td>
</tr>
</tbody>
</table>

\[ \mu = 0.0059 \]

Region 1 is chosen to be the “structurally bad” region, with lower productivity and higher separation rate. For the present exercise the cost function \( d \) is chosen to be an exponential
CHAPTER 3. LABOR MARKET EFFECTS OF MIGRATION WITHIN SEARCH-AND-MATCHING FRAMEWORK: A THEORETICAL APPROACH

function of the migration rate. Table 1 presents the outcomes of the model without wage heterogeneity (denoted by M1) for two regions.

Because of our choice of assumptions about labor force dynamics in region 1, the outcome values in it are the same as for the autarky case. Region 2, however, after becoming a migrant receiving destination, faces higher equilibrium unemployment rate. The unemployment rate $u_2$ is decreasing when the differences in productivity and separation rates between two regions become larger (see Figures 3.6 and 3.7). At the same time the rate of migration increases (Figures 3.8 and 3.9). An increase in worker’s outside option $z_2$, keeping all other parameters fixed, leads to a higher equilibrium migration and unemployment. An increase of costs of vacancy opening $c$ for a firm in region two (worsened business environment) implies a lower equilibrium migration rate, making a region less attractive due to a lower potential job-finding probability.

![Figure 3.6: Dynamics of $u_2$ with the increase of difference in separation rate](image)

Figures 3.6 and 3.7 display the behavior of equilibrium unemployment rate in region 2.
with respect to changes in productivity and separation rate in Region 2, keeping all the parameters fixed in region 1.

Figure 3.7: Dynamics of $u_2$ with the increase of productivity difference

Figures 3.8 and 3.9 display the dynamics of migration rate with respect to exogenous changes in productivity and separation rate in region 2, keeping these parameters fixed in region 1.

Figure 3.8: Migration rate with the increase of difference in separation rate
Figure 3.9: Migration rate with the increase of productivity difference
3.5 Model extension: allowing for wage differentiation

In the present section I proceed with extending the two-country search-and-matching model in order to account for the wage gap between native and immigrant workers that is well documented by empirical studies (Aldashev, Gernandt, and Thomsen, 2012; Bartolucci, 2013; Miranda and Zhu, 2012).

Consider immigrant receiving country (region 2). Immigrants and native workers are now assumed to be heterogeneous with respect to their costs of staying in this region. Immigrants have to bear costs \( d(\mu) \) in every period after their arrival in a foreign region. Thus, values of while unemployed and working are discounted by \( d(\mu) > 1 \):

\[
U_I = \frac{z_2}{d(\mu)} + \frac{\delta}{d(\mu)} (\theta q(\theta) W_I + (1 - \theta q(\theta)) U_I), \tag{3.68}
\]

\[
W_I = w_I + \frac{\delta}{d(\mu)} (s_2 W_I + (1 - s_2) U_I), \tag{3.69}
\]

where \( w_I \) is a wage paid to an immigrant, \( \theta \) is a market tightness in the labor market of region 2.

When a firm meets a worker, it can distinguish whether she is an immigrant or a native. Given the wage bargaining process, explained below, a firm pays different wages and earns different profits depending on the type of worker. Thus, the value of the job filled by an immigrant is:
\[ J_I = y_2 - w_I + \delta(1 - s_2)J_I. \] (3.70)

After rearranging:

\[ J_I = \frac{y_2 - w_I}{1 - \delta(1 - s_2)}. \] (3.71)

As before, wages are the outcome of bilateral Nash bargaining between each firm and worker. The worker receives a value \( W_I \) when the agreement is reached and her lower threshold is \( U_I \). After a match occurs and production starts, a firm receives a value \( J_I \) and its lower threshold is \( V = 0 \). Wage \( w_I \) is then a solution to: \( (1 - \beta)(W_I - U_I) = \beta J_I \). After substituting into this expression equations (3.68) - (3.70) and solving for \( w_I \):

\[ w_I = y_2 - (1 - \beta)(1 - \delta(1 - s_2)) \frac{y_2 - \frac{\partial^2}{d(\mu)}}{1 - \delta(1 - \beta)(1 - s_2) - \frac{\delta}{d(\mu)}(1 - s_2 - \theta q(\theta))}. \] (3.72)

Native workers do not incur any costs of staying in their home region. So, their values while unemployed and on the job are given by expressions (3.73) and (3.74), respectively.

\[ U_N = z_2 + \delta (\theta q(\theta)W_N + (1 - \theta q(\theta))U_N), \] (3.73)

\[ W_N = w_N + \delta (s_2 W_N + (1 - s_2)U_N). \] (3.74)
The value of the job that is filled by a native worker with a wage \( w_N \) is given by

\[
J_N = y_2 - w_N + \delta (1 - s_2)J_N. \tag{3.75}
\]

\[
J_N = \frac{y_2 - w_N}{1 - \delta(1 - s_2)}. \tag{3.76}
\]

The wage \( w_N \) is a solution to a Nash bargaining problem between a native worker and a firm \((1 - \beta)(W_N - U_N) = \beta J_N\).

\[
w_N = y_2 - (1 - \beta)(1 - \delta(1 - s_2)) \frac{y_2 - z_2}{1 - \delta(1 - s_2) + \beta \delta \theta q(\theta)}. \tag{3.77}
\]

Then the expected wage in the economy is: \( w^e = \eta w_I + (1 - \eta)w_N \), where \( \eta = \frac{\mu U_1 L_1}{L_2 + \mu U_1 L_1} \) is the share of immigrants in total labor force. And the expected value of the job is given by:

\[
J^e = \eta J_I + (1 - \eta)J_N = \frac{y_2 - w^e}{1 - \delta(1 - s_2)}. \tag{3.78}
\]

From the zero-profit condition for vacancy posting I know that:

\[
J^e = \frac{c}{\delta q(\theta)}. \tag{3.79}
\]

After combining (3.78) and (3.79) and substituting expressions (3.72) and (3.77) for wages, I obtain the equilibrium tightness \( \theta \).

Migration rate is determined by the indifference condition \( U_1 = \frac{U_N}{d(\mu)} \). Knowing theta, I obtain \( U_N \) as a solution to (58)-(59) with a decided maximum function.
Equilibrium unemployment rate, as previously, is represented by

$$ u_2 = \frac{s_2 + \mu u_1 \frac{L_1}{L_2}}{s_2 + q(\theta_2)\theta_2 + \mu u_1 \frac{L_1}{L_2}}. $$

(3.80)

And, finally, I recover the vacancy rate as $v_2 = \theta u_2$.

Table 3.2 below presents a comparison of the outcomes of models without (M1) and with (M2) wage differentiation for Region 2, keeping all the exogenous parameters constant. An outcome of the model with wage differentiation for a migrant-destination region 2 is displayed in a third column (M2) of table. As previously, I use a Cobb-Douglas shape of the matching function.

The last column provides a comparison of the outcomes of the models without (M1) and with (M2) wage heterogeneity, given the same exogenous parameters for both regions. $\Delta$ is the absolute change in outcome variables (M1-M2).

<table>
<thead>
<tr>
<th></th>
<th>M1</th>
<th>M2</th>
<th>$\Delta = (M1 - M2) \times 10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.0058801117</td>
<td>0.0058801500</td>
<td>-0.0383</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>0.4742866394</td>
<td>0.4742878752</td>
<td>-1.2358</td>
</tr>
<tr>
<td>$u_2$</td>
<td>0.0136758029</td>
<td>0.0136757775</td>
<td>0.0255</td>
</tr>
<tr>
<td>$v_2$</td>
<td>0.0064862506</td>
<td>0.0064862554</td>
<td>-0.0048</td>
</tr>
<tr>
<td>$w^e$</td>
<td>1.1996552721</td>
<td>1.1996497998</td>
<td>5.4723</td>
</tr>
<tr>
<td>$w_I$</td>
<td>1.1943991902</td>
<td>1.1996497998</td>
<td>5.4723</td>
</tr>
<tr>
<td>$w_N$</td>
<td>1.1996553072</td>
<td>1.1996497998</td>
<td>5.4723</td>
</tr>
</tbody>
</table>

As we can see from these results, an introduction of differentiated wages into the model leads to an increased migration rate and at the same time to a lower unemployment. The
The reason behind this result is that immigrations allows firms to pay lower wages on average and to incur lower search costs given an increased availability of unemployed workers. This in turn leads to a higher rate of vacancy creation and increased labor market tightness. Immigrants are better-off, because they end up working in a “structurally better” region with higher wage and higher prospects of finding a job, then in their home region. The native workers benefit from the boost in labor demand due to newly created jobs as well as from the higher wages (compared to those in their home region).
Chapter 4


4.1 Introduction

This chapter deals with an application of the theoretical framework discussed in chapter two for the analysis of migration between Switzerland and the EU, and migration effects on Swiss labor market.

Being historically characterized by high immigration rates, Switzerland is now one of the most dependent countries on foreign workers of any European OECD country. Today foreigners’ total share in the labor force of Switzerland constitutes 23.7% (Figure 4.1 below).
First and second generation immigrants now make up more than a third of Switzerland’s population over the age of 15. Starting from 2002, after enactment of a bilateral agreement of 2001 between Switzerland and the EU on the free movement of individuals, Switzerland’s foreign resident population grew on average by 2.4% annually.

![Population structure by nationality](image)

**Figure 4.1: Population structure by nationality**

*Source: Data from Swiss Federal Statistical Office*

Currently, workers and some policy makers are concerned about the potential negative labor market effects of immigration. On the other hand, many business representatives and employer organizations deny that the immigration of foreign employees displaces native workers or reduces their wage. They argue that the inflow of highly skilled employees to Switzerland occurs due to the lack of labor supply, and that immigrants do not only increase competitiveness and productivity in the country, but also prevent firms from outsourcing jobs, thus benefiting resident workers. A number of empirical studies have examined labor
market effects of the recent immigration wave to Switzerland, and most of them do not find any evidence of negative effects on wages of the native population. Moreover, examination of historical data series suggests that the largest net inflow of foreigners coincides with the decline in wage share and the peaking number of newly opened vacancies. These observations go in line with the key assumptions and theoretical predictions of the models in chapter two, making Swiss labor market a perfect candidate for testing the performance of this theoretical framework.

In order to obtain structural parameters characterizing the Swiss labor market, I first calibrated the standard one-country search model to generate the observed fluctuations in unemployment and job vacancies in Switzerland. Then, I use obtained calibrated parameters for simulating the steady state versions of two-country models for Switzerland and the EU. The results reveal that the models can adequately predict observed percentage changes in unemployment rates in response to immigration waves and captures the wage differences between immigrants and native workers.

Finally, in this chapter, a two-country search-and-matching model with migration and aggregate uncertainty is presented, as well as the tools developed in order to solve it numerically in a Matlab environment. The model aims at analyzing the response of decentralized labor markets to local shocks as well as at explaining migration patterns and outcomes related to cross-country differences in structural characteristics of labor markets in the presence of exposure to productivity shocks.

The remainder of the chapter is organized as follows. The next section deals with cal-
CHAPTER 4. TWO-COUNTRY SEARCH-AND-MATCHING MODEL AND IMMIGRATION TO SWITZERLAND: A CALIBRATION APPROACH

ibration and simulation of a search and matching model to match the labor market data for Switzerland in order to obtain the key exogenous parameters. Section 3, presents the outcomes of a simulation of a two-country model with migration between Switzerland and the EU. Finally, Section 4 presents an extended two-country model with migration flows that allows the model to capture fluctuations over the business cycle, together with a numerical solution to it.
4.2 Background on labor market characteristics and immigration in Switzerland

Today foreigners’ share of total labor force in Switzerland constitutes 23.7%\(^1\), which makes Switzerland one of the most dependent on foreign workers of any European OECD country (Becker, Liebig, and Sousa-Poza, 2008).

Between 2002 and 2011, Swiss foreign resident population grew on average by 2.4% percent annually, with a large share of foreign population immigrating due to work reasons (Basten and Siegenthaler, 2013). In 2008, the total number of immigrants reached a peak of +161,629 persons, with the net immigration of +103,363 persons.

![Figure 4.2: Dynamics of migration flows in Switzerland](image)

*Figure 4.2: Dynamics of migration flows in Switzerland

*Source:* Data from Swiss Federal Statistical office

\(^1\)source: all the numbers are own calculations based on data from Swiss Federal Statistical Office (SFSO)
Between the second quarter of 2002 and the second quarter of 2015 the foreign labor force in Switzerland grew by 363,425 persons (+41.3%), which in percentage change is substantially more than an increase in the labor force of Swiss citizens (341,516 persons or +10.7%, which by a large degree is attributable to naturalization). This is a vast inflow in relation to the total labor force of 4.084 million workers in 2002. This raised some reasonable doubts about the potential negative outcomes of such migration inflows. The evolution of nominal and real wages over the same period can be observed from (Figures 4.3 and 4.4 below). Between the years 2004 and 2008 corresponding to an increase in net immigration, the real wages remained unchanged.

Figure 4.5 presents how wage development compares to GDP. One can notice that between 2004 and 2008 the wage share has fallen. As we will see later in this chapter, the theoretical model of chapter two will also predict a decrease in expected (i.e., average for immigrants and natives) wages.
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Figure 4.3: Evolution of nominal wages (basis 1939=100)

Source: Data from Swiss Federal Statistical Office

Figure 4.4: Evolution of real wages (basis 1939=100)

Source: Data from Swiss Federal Statistical Office
Immigration went hand in hand with creation of new jobs, thus the unemployment rate in Switzerland remained relatively low (see Figures 4.6 and 4.7). Between 2007 and 2009, when net total immigration reached its peak, the total number of newly vacancies also significantly increased. The vacancy rate (calculated as the total number of opened vacancies divided by total labor force) has slightly dropped in 2007-2008 due to a vast increase of the labor force. However, in 2009 it has returned to it’s long run value.
CHAPTER 4. TWO-COUNTRY SEARCH-AND-MATCHING MODEL AND IMMIGRATION TO SWITZERLAND: A CALIBRATION APPROACH

Figure 4.6: Number of opened vacancies

Source: Data from Swiss Federal Statistical Office

Figure 4.7: Yearly values of unemployment and vacancy rates

Source: Own calculations based on data from Swiss Federal Statistical Office

These observations make Switzerland a very particular and useful example of immigration policies that can be taken into account by EU countries facing large immigration inflows at the present time.
The beginning of large "immigration waves" to Switzerland started in the end of 1980s with the introduction of an agreement on free movement of people within the European Economic Area (EEA). Before all immigration was subject to quotas, which imposed an annual upper limit on the number of foreigners allowed to enter the country for the first time. Most of all stay permits were issued for the seasonal workers. However, since the late 1980s, a large portion of immigrants were no longer subject to control. A downturn in the early 1990s lead to an increased unemployment which strongly affected foreign workers. This was partly due to qualification mismatches, as the result of the shift in labor demand towards more qualified workers (Becker, Liebig, and Sousa-Poza, 2008).

This led to an introduction of a three-circular concept into immigration policy. The first circle represented immigrants from the EEA. In case labor demand was not met, the second circle (citizens of the USA, Canada, New Zealand and Australia) were allowed to enter the labor market. Finally, the third circle consisted of the workers from the rest of the world. The main outcome was that recruitment of low-skilled workers from Yugoslavia, which at that time represented 15% of the newcomers, became impossible (Becker, Liebig, and Sousa-Poza, 2008). Additionally, seasonal work permits were restricted to people from the EU/EFTA region and thus recruitment policy gradually shifted towards allowing only highly qualified foreigners; this particularly regarding permanent immigration.

In 1998 the 3-circles model was replaced by the 2-circles, which only distinguished between EU/EFTA citizens and the rest of the world. However, the feature outcome of the 1991 policy remained: foreigners could only be recruited in exceptional cases and if they met
A new immigration wave to Switzerland started in 2002, after enactment of a bilateral agreement of 2001 between Switzerland and the EU, regarding the free movement of individuals (a Free Movement of Persons Treaty (FMP)). According to this agreement, EU nationals have the same living and working rights as Swiss citizens, except for voting rights, which means a complete liberation of immigration from the EU area. The legislation however, still restricts the entry of non-EU immigrants to skilled personnel, considered as those required by the economy and able to integrate into Swiss society. As the result, in 2014, the total number foreigners reached 1,937,447 persons with a total population of 8,139,631 persons. Additionally, 270,000 cross-boarder commuters hold a job in Switzerland (Basten and Siegenthaler, 2013).

Given the magnitude of the inflow, it is predictable that workers and some politicians are concerned about the potential negative labor market effects. Yet, most business representatives and employer organizations deny that the immigration of foreign employees displaces native workers, or reduces their wage. They argue that the inflow of highly skilled employees to Switzerland occurs because of the lack of qualified resident employees and that the immigrants increase competitiveness and productivity in the country, as well as prevent firms from outsourcing jobs; thus benefiting resident workers. Since 2002, the average formal qualifications of immigrants have exceeded those of the resident workforce. Immigrants make up a substantial part of the workforce in the science, chemical, pharmaceutical and biotechnological industries in which 45% of workers are foreigners (Basten and Siegenthaler,
The split in public opinion becomes most evident while analyzing the results of the Swiss Immigration Referendum (Italian: "Iniziativa popolare contro immigrazione di massa" February, 2014). The initiative aims at limiting immigration through quotas, as legislation had before the bilateral agreements between Switzerland and the EU. Metropolitan centers (such as Zurich, with about 40% of foreigners in total population) have rejected the initiative with 58.5% of votes against. At the same time, rural cantons and isolated cities voted with 57.6% and 51.3% voting for.

After the beginning of the recent immigration wave in 2002, debates on the effect of immigration in Switzerland reached their peak. A number of empirical studies have examined the labor market effects of the new immigration wave to Switzerland in general, and the effects of the introduction of the FMP in particular. The results from economic studies suggest that immigration in Switzerland was at least not welfare deteriorating, with most of these studies do not find any evidence that wages or employment opportunities of native employees are negatively affected by the recent inflow of foreign workers.

For instance, Basten and Siegenthaler (2013) argue that their results from a quasi-experimental approach reveal that immigration has in fact reduced unemployment and increased employment of residents in the last decade. The authors conclude that immigration to Switzerland helps to avoid job outsourcing due to skill shortages and increases the competitiveness of the economy. As the result, immigration does not harm wages while improving employment prospects of the native workers.
Beerli and Peri (2015) exploit a difference-in-difference approach and estimate the effects on native labor market outcomes such as wages and employment, for the inflow of foreign workers after opening the border to EU immigrants. The authors find no significant impact on average native wages and employment, nor on wages and employment in different skill groups of native workers.
4.3 Application of search-and-matching model for the analysis of Swiss labor market

In the present section I apply search and matching framework in order to explain some stylized facts about the labor market in Switzerland. First, I estimate a standard search and matching model, calibrated to match Swiss data in order to obtain structural parameters of the labor market. Next, I proceed with calibration and simulation of two-country models of chapter 2.

Search and matching model with aggregate uncertainty: set up and estimation

Consider a stochastic discrete time version of the Diamond-Mortensen-Pissarides (DMP) model, discussed in chapter 2. The model can be formulated as follows: The value of filled job and the value of unfilled vacancy for a firm, respectively are:

\[ J(p) = p - w(p) + \delta(1 - s)EJ(p'), \quad (4.1) \]
\[ 0 = -c(p) + \delta q(\theta(p))EJ(p'), \quad (4.2) \]

where it has been anticipated that in equilibrium the value of a vacancy will be driven to zero, \( V(p) = 0 \), because of free entry.

The values of staying unemployed and while on the job for a worker, respectively, are
given by:

\[ U(p) = z + \delta [f(\theta(p))EW(p') + [1 - f(\theta(p))]EU(p')] , \quad (4.3) \]

\[ W(p) = w(p) + \delta [(1 - s)EW(p') + sEU(p')] . \quad (4.4) \]

The outcome of Nash bargaining:

\[ \beta J(p) = (1 - \beta) [W(p) - U(p)] . \quad (4.5) \]

where the exogenous productivity state \( \log(p) \) evolves according to:

\[ \log(p)' = \rho \log(p) + \epsilon' , \quad \epsilon \sim \mathcal{N}(0, \sigma^2) . \quad (4.6) \]

\( E \) denotes the expectational operator with respect to \( \log(p)' \) conditional on \( \log(p) \). Equations (4.1)-(5) build a system of five functional equations in the five unknown and time-invariant functions \( J(\cdot) \), \( U(\cdot) \), \( W(\cdot) \), \( \theta(\cdot) \) and \( w(\cdot) \) defined over the state \( \log(p) \). The unemployment rate \( u \) is another state variable of the dynamic system but evolves independently according to

\[ u' = 1 - [(1 - s)(1 - u) + m(u, v(u,p))] \]

\[ = 1 - [(1 - s)(1 - u) + m(1, \theta(p))u] . \]

Whenever notation \( p \) is used instead of a \( \log(p) \), it should be read as \( \exp(\log(p)) \). Once I obtain the function \( \theta(\cdot) \), for given states \( u \) and \( p \) , I can easily recover \( v(u,p) \) and \( u' \). The wage rate is given by

\[ w(p) = \beta p + (1 - \beta)z + c(p)\beta \theta(p) , \quad (4.7) \]
and that the system of equations in (4.1)-(5) can be reduced to a single rational expectations
functional equation in only one unknown function $\theta(\cdot)$:

$$\frac{c(p)}{\delta q(\theta(p))} = E \left[ (1 - \beta)(p' - z) - c(p')\beta(\theta(p')) + \frac{(1 - s)c(p')}{q(\theta(p'))} \right]. \quad (4.8)$$

**Solution of the model**

Rewrite Equation (4.2) as

$$\frac{c(p)}{\delta q(\theta(p))} = EJ(p'), \quad (4.9)$$

and substitute in Equation (5) to obtain

$$\frac{c(p)}{\delta q(\theta(p))} = \frac{1 - \beta}{\beta} E[W(p') - U(p')]. \quad (4.10)$$

From (4.4) and (4.3) I know that

$$W(p) - U(p) = w(p) - z + \delta[(1 - s) - f(\theta(p)))]E[W(p') - U(p')].$$

Iterating one period ahead and taking expectations, obtain

$$E[W(p') - U(p')] =
E \left[ w(p') - z + \delta[(1 - s) - f(\theta(p'))]E'[W(p'') - U(p'')] \right],$$

Equation (4.10) can then be stated as

$$\frac{c(p)}{\delta q(\theta(p))} = \frac{1 - \beta}{\beta} E \left[ w(p') - z + \delta[(1 - s) - f(\theta(p'))]E'[W(p'') - U(p'')] \right].$$
CHAPTER 4. TWO-COUNTRY SEARCH-AND-MATCHING MODEL AND IMMIGRATION TO SWITZERLAND: A CALIBRATION APPROACH

Using the forward version of (4.10), produces:

\[
\frac{c(p)}{\delta q(\theta(p))} = \frac{1 - \beta}{\beta} E \left[ w(p') - z + \delta [(1 - s) - f(\theta(p'))] \frac{\beta}{1 - \beta} \frac{c(p')}{\delta q(\theta(p'))} \right] = E J(p'). \tag{4.11}
\]

Finally, using Equation (4.7) and \( f(\theta(p)) = q(\theta(p)) \theta(p) \), and rearranging terms, obtain

\[
\frac{c(p)}{\delta q(\theta(p))} = E \left[ (1 - \beta)(p' - z) - c(p') \theta(p') + \frac{(1 - s)c(p')}{q(\theta(p'))} \right].
\]

To show that Equation (4.7) indeed holds, note that (4.11) implies that

\[
J(p) = \frac{1 - \beta}{\beta} [w(p) - z] + [(1 - s) - f(\theta(p))] \frac{c(p)}{q(\theta(p))},
\]

while the combination of Equation (4.1) and (4.2) yields

\[
J(p) = p - w(p) + (1 - s) \frac{c(p)}{q(\theta(p))}.
\]

The combination of the two indeed gives

\[
w(p) = \beta p + (1 - \beta)z + c(p) \beta \theta(p).
\]

Reformulation for the numerical solution

Following notation of Miranda and Fackler (2002) Chapters 8.6 and 9.9, \( f(s, x, Eh(s', x')) \), and \( g(s, x, \epsilon') \) I express the rational expectations equilibrium in Equations (4.6)-(4.8) in the form

\[
0 = f(s, x, Eh(s', x'))
\]

\[s' = g(s, x, \epsilon').\]
The state is captured by \( s \),
\[
\text{s} = \log p.
\]

The response variable is denoted by \( x \),
\[
\text{x} = \theta.
\]

\( h(s, x) \) captures the expectational variable,
\[
\text{h} = (1 - \beta)(p - z) - c(p)\beta\theta + \frac{(1 - s)c(p)}{q(\theta)}.\]

The equilibrium function \( f(s, x, Eh(s', x')) \) then reads,
\[
\text{f} = Eh(s', x') - c(p)\frac{\delta q(\theta)}{\delta q(\theta)}.
\]

Finally, the state transition \( g(s, x, \epsilon) \) reads
\[
\text{g} = \rho \log(p) + \epsilon'.
\]

**Calibration and simulations**

Our approach to calibration follows that of Hagedorn and Manovskii (2008). Shimer (2005) argues that the standard calibration of the model fails to capture the high volatilities of unemployment and vacancies, which is not consistent with the U.S. data. Hagedorn and Manovskii (2008) proposed an alternative reasoning for determination the value of non-market activity, that results in an adequate behavior of the model.
The model period is chosen to be one month, which corresponds to the frequency of the data on unemployment and vacancies. All the values obtained from the data with quarterly or yearly frequencies are appropriately disaggregated.

Labor productivity \( p \) is measured in the data as quarterly seasonally adjusted real GDP per person. In the model the evolution of productivity is represented by AR(1) process:

\[
\log p_{t+1} = \rho \log p_t + \epsilon_{t+1}, \quad \epsilon \sim N(0, \sigma^2).
\] (4.12)

In the quarterly data I find an autocorrelation 0.9786 and sigma 0.0001743 (Figure 4.8), which after adjustment to monthly frequency results in the value of \( \rho = 0.9988 \) and \( \sigma^2 = 0.0031 \). The mean of the productivity is normalized to one.

---

\( ^2 \)Real GDP data (1980q1-2014q4) is constructed by State Secretariat for Economic Affairs SECO http://www.seco.admin.ch/themen/00374/00456/04878/index.html?lang=en. I divide it by quarterly population figures to obtain per capita values.
The values for job finding rates and job separation rates for Switzerland are taken from Hobijn and Sahin (2009), who used GMM to estimate the steady-state job finding and separation rates for a broad sample of countries. The average monthly separation rate for Switzerland is 1.19% and monthly job finding rate is 13.35%. I also set $\delta = 0.99^{1/12}$ at a monthly frequency.

The functional form of the matching function should ensure that the job finding probability is between zero and one. Following Hagedorn and Manovskii (2008), I use the following matching function: $m(u, v) = \frac{uv}{(u+v)^{1/l}}$. The value of the matching function parameter $l$ is then chosen to match the average job finding rate from the data ($l=0.1335$).

In order to obtain the value for the flow capital cost incurred by firms while keeping vacancies open, I again use the results from Hagedorn and Manovskii (2008). They recognize the presence of capital in the model, which, however, does not change any of the equations and only results in re-interpretation of the productivity process.

They consider the aggregate capital stock $K$. The number of active firms equals to $v+1-u$, out of which $1-u$ are operating and $v$ are searching for a worker. Then, the amount of operating capital is $K \frac{1-u}{v+1-u}$ and the amount of idle capital amounts to $K \frac{v}{v+1-u}$.

Next, the authors define the aggregate constant returns to scale production function as $F(K \frac{1-u}{v+1-u}, A(1-u))$, where $A$ is labor-augmenting productivity. Also, define $k = \frac{K}{A(v+1-u)}$ and $f(k) = F(k, 1)$. Finally, denote by $k^*$ the value of the $k$ that satisfies the equilibrium condition $f'(k) = \frac{1}{\delta} - 1 + d$, where $d$ is the capital depreciation rate.

It is possible to show that the flow capital cost of posting a vacancy equals $(\frac{1}{\delta} - 1 + d)kA =$
\( F_K \frac{K}{v+1-u} \), where \( F_K \) denotes the derivative of \( F \) with respect to its first argument (see Hagedorn and Manovskii (2008) for derivations). Decomposing:

\[
\frac{F_K K}{v + 1 - u} = \frac{F_K}{F} \frac{1 - u}{v + 1 - u} \frac{F}{1 - u}.
\] (4.13)

I now compute the steady state values for each factor using Swiss data. According to the estimates of Swiss National Science Foundation, the capital income share \( F_K \) remained constant over the past 30 years and amounts to a value of 0.33\(^3\)

I also use Short-Term Labour Market Statistics by OECD\(^4\) to obtain an average of monthly seasonally adjusted values of \( u \) and \( v \) for the years 1975 to 2014. Since \( u = 0.02234 \) and \( v = 0.01106 \), the value of the factor \( \frac{1 - u}{v + 1 - u} \) amounts to 0.9988.

Since labor productivity is normalized to one \( (F_L = 1) \), it holds that

\[
\frac{1 - u}{F} = \frac{F_L A (1 - u)}{F} = \frac{1 - F_K K}{v + 1 - u} = 1 - 0.33 \frac{1 - u}{v + 1 - u} = 0.6704.
\] (4.14)

Substituting all the factor values into equation 4.13, I obtain the steady state capital flow cost of posting a vacancy \( c = 0.4916 \).

Two more parameters have to be identified: the value of worker’s outside option \( z \), and worker’s bargaining power \( \beta \). I know that the value of the outside option \( z \) should be close to that of the productivity level \( p \), and the value of \( \beta \) has to be relatively small (see Hagedorn and Manovskii (2008) for discussion). I also take into account that the choice of \( \beta \) and

\(^3\)http://www.snf.ch/en/researchinFocus/newsroom/Pages/news-140318-mm-labour-share-in-switzerland-remains-constant.aspx

determine the elasticity of wages with respect to productivity. Therefore, I target the elasticity value of 0.9 obtained for Switzerland by Duque, Ramos, and Surinach (2006) using panel data from 1970 to 2004 for OECD countries. I explore different specifications of the model. I pick the one that gives the most consistent with the data results with respect to target values and cyclical properties of unemployment and vacancy rates.

Table 4.1 below contains the resulting calibrated parameter values.

Table 4.1: Calibrated parameters and target values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>value of outside option</td>
<td>0.979</td>
</tr>
<tr>
<td>$\beta$</td>
<td>worker’s bargaining power</td>
<td>0.05</td>
</tr>
<tr>
<td>$l$</td>
<td>matching parameter</td>
<td>0.4216</td>
</tr>
<tr>
<td>$c$</td>
<td>cost of vacancy when $p=1$</td>
<td>0.49</td>
</tr>
<tr>
<td>$\delta$</td>
<td>rate of discount</td>
<td>0.99^{1/12}</td>
</tr>
<tr>
<td>$s$</td>
<td>separation rate</td>
<td>0.0119</td>
</tr>
<tr>
<td>$\rho$</td>
<td>persistence of productivity process</td>
<td>0.9988</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>variance of innovation in productivity process</td>
<td>0.0031</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Target</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average job finding rate</td>
<td>0.1335</td>
<td>0.1335</td>
</tr>
<tr>
<td>Average market tightness</td>
<td>0.502</td>
<td>0.509</td>
</tr>
</tbody>
</table>
Tables 4.2 and 4.3 below present summary statistics for the variables obtained from the data and outcomes from the calibrated model.

Table 4.2: Summary statistics, monthly Swiss data, 1975:2014 (OECD database)

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$v/u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>.435</td>
<td>.121</td>
<td>.501</td>
</tr>
<tr>
<td>Quarterly autocorrelation</td>
<td>.960</td>
<td>.955</td>
<td>.972</td>
</tr>
<tr>
<td>Correlation Matrix</td>
<td>1</td>
<td>-.445</td>
<td>-.976</td>
</tr>
<tr>
<td></td>
<td>$v$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>.629</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$v/u$</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.3: Results obtained from the calibrated model

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$v/u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>.407</td>
<td>.142</td>
<td>.629</td>
</tr>
<tr>
<td>Quarterly autocorrelation</td>
<td>.858</td>
<td>.554</td>
<td>.641</td>
</tr>
<tr>
<td>Correlation Matrix</td>
<td>1</td>
<td>-.417</td>
<td>-.613</td>
</tr>
<tr>
<td></td>
<td>$v$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>.973</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$v/u$</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>
Two-country search and matching models with migration and

labor market outcomes in Switzerland

Having obtained the fitted values of outside option $z$ and bargaining power $\beta$, I can now proceed with the calibration of the models of chapter 2 (section 4) for Switzerland and EU-28 region.

I allow the two regions differ in productivity levels and job separation rates. In the data I find that the average yearly productivity per person in Switzerland is 1.167 times higher than the value for EU-28 region. I keep this ratio while calibrating the model. The value of productivity in EU-28 region is normalized to match the target of average unemployment rate of 10 percent. The value of job separation rate for EU region I obtain from Hobijn and Sahin (2007) by taking the average for European countries. I set the ratio of labor force size in Switzerland to the EU-28 equal to 1:55 (according to the data from Eurostat).

The left side of Table 4 below presents the outcome of calibrated simulation for the model without differentiated wages, while the right hand side - the outcome of the model with wage differentiation.

As expected, introduction of differentiated search costs and wages leads to a lower unemployment effect, which is partly absorbed by the expected wage reduction, and higher equilibrium migration rate.

Next, I compare obtained above results with the data from Switzerland. The case of

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5Eurostat Table: Labour productivity per person employed (EU28=100), years 2003-2014 (code: tesem160)
Model 2 with wage differentiation is more interesting, since, according to the data, there is an observed wage gap between the wages of Swiss and foreign workers.

The large recent inflow of foreign labor force to Switzerland from the EU has occurred after FMP treaty (starting from 2002). Hence, I treat the values of unemployment and wages in 2001 as steady-state autarky values and compare them to those in 2006 (post-migration steady-state values) in order to determine the impact of immigration wave on wages and unemployment rate. The year 2006 is chosen for several reasons. First, the time period should not include the economic crises of 2008. Second, due to the short run scope of the two-county model, I need to consider data over a short time period prior to and after 2002. I choose a time span of 5 years.
I use the data by Swiss Federal Statistical office on average monthly unemployment rates and on CPI adjusted median nominal wages for the years of interest, and calculate percentage changes over the period as well as the percentage differences in wages of immigrants and native workers.

Table 4.5: Model outcomes compared to Swiss data

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
<th>change in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>u rate</td>
<td>0.04309</td>
<td>0.07368</td>
<td>71.01</td>
</tr>
<tr>
<td>w_e</td>
<td>1.06954</td>
<td>1.06953</td>
<td>-0.0012</td>
</tr>
<tr>
<td>w_{native}</td>
<td>1.0695</td>
<td>0.13090</td>
<td>56.11</td>
</tr>
<tr>
<td>w_{immigr}</td>
<td>1.0681</td>
<td>4861</td>
<td>16.11</td>
</tr>
</tbody>
</table>

The results reveal that the model adequately predicts an increase in unemployment rate as well as the existing wage gap between native and immigrant workers. However, it fails to explain an increase in the expected wages, observed in data. This suggests that there are some unaccounted factors. If we remind Figure 4.5 above, we can conclude that the evolution of the adjusted wage share is more consistent with the model outcome on lowered expected wages. Indeed, according to the data from European commission, the adjusted wage share decreased from 72.35 % in 2001 to 68.48 % in 2006. Therefore, consideration of the aggregate economic development provides a more meaningful interpretation of the simulated results.
4.4 Extension: two-country search-and-matching model with migration and aggregate uncertainty

In this section I extend the two country search-and-matching framework with migration, presented in Chapter 2, by introducing aggregate uncertainty. I also develop a tool for solving it numerically in Matlab environment (Appendix B).

The advantage of this exercise is twofold. First, this allows to account for the cyclical properties of unemployment and vacancy rates. Second, depending on the value that a state variable (productivity) takes in each period and in each country, a resulting migration rate is calculated. Thus, on average migration in both directions becomes possible within this setting, allowing to capture the net migration flows between two countries.

Thereby, the model aims at analyzing the response of decentralized labor markets to local shocks as well as explaining migration patterns and outcomes related to cross-country differences in structural characteristics of labor markets in the presence of exposure to productivity shocks.

Environment

Assumptions:

- I consider a discrete time continuous state model.

- There is one match surplus realization (productivity) associated with all successful
matches in a region at any time.

- The wage is set via a Nash bargaining mechanism.

- The opportunity of migration to a labor market abroad is governed by a Poisson process.

The economy consists of two regions indexed by $i = 1, 2$. In each region the labor market is decentralized and uncoordinated. Each labor market is represented by firms and workers, that dedicate time and resources to search for forming a match. The labor force in each country consists of $L$ individuals, out of which $uL$ are unemployed and $(1 - u)L$ employed, where $u$ is the unemployment rate. Furthermore, the total labor demand is equal to the number of filled jobs plus the number of advertised vacancies. Denote by $v$ the ratio between the number of vacancies and the total labor force, then the mass of vacant firms that are searching for workers is $vL$. The matching process is defined by the following

$$mL = m(uL, vL), \quad (4.15)$$

where $mL$ is the number of matches calculated as a function of $u$ and $v$. Denote by $\theta \equiv \frac{v}{u}$ the labor market tightness ratio and use the assumption of constant returns to scale matching function. Then the probability of a firm to fill a vacancy and the job-finding probability are, respectively:

$$q(\theta) = \frac{m(uL, vL)}{vL} = m(\frac{1}{\theta}, 1) \quad (4.16)$$
\[ \theta q(\theta) = m(1, \theta) \]  

New matches generate a surplus \( p_i \) that evolves according to

\[ \log(p_i)' = \rho \log(p_i) + \epsilon', \]

where \( \epsilon \) is assumed to be i.i.d. with mean zero, standard deviation \( \sigma \), and skewness \( \gamma_i \) that is region-specific. The negatively skewed productivity shocks are necessary for our modeling purposes for two reasons. First, the introduction of skewness gives us a possibility to model one region as relatively adverse one (i.e. being exposed to negative shocks more frequently). Second, as is shown in Ruge-Murcia (2012), skewness plays an important role for adequate data representation because in reality large negative realizations of productivity shocks tend to happen more frequently than large positive ones. By contrast, the symmetric Normal distribution implies that negative and positive shocks of the same magnitude occur with the same frequency. Consequently, argues Ruge-Murcia (2012), leaving the skewness out leads to a misspecification: in order to account for the large negative shocks present in the data the Normal distribution tends to amplify the actual standard deviation.

**Migration**

Migration in this context acts as an alternative to searching for a job in the home region. In every period the unemployed workers in location \( i \) receive an opportunity to migrate with Poisson probability \( \pi^m \). After receiving this opportunity, an unemployed worker in
region \(i\) compares the present discounted value of remaining unemployed at home (\(U_i\)) and the expected discounted value of being unemployed after moving abroad (\(\frac{U_j}{d_i}\)). Migration is costly, therefore the value of unemployment state abroad is discounted by \(\frac{1}{d_i} < 1\). Whenever \(U_i < \frac{U_j}{d_i}\), a worker from region \(i\) will emigrate, otherwise he will choose to stay and search for a job her/his home region.

**Bellman equations**

For notation parsimony from now on the subscripts \(i, j\) suppressed unless necessary to distinguish between regions. However, I must keep in mind that each of the following Bellman equations holds for each of the two regions.

**Workers.** Unemployed workers decide between searching in their home region or abroad, once they have received an opportunity to migrate. The asset value of unemployment state is:

\[
U(p) = z + \delta [\theta q(\theta) E W(p') + (1 - \theta q(\theta)) E U(p')] + \pi^\mu \max \left[ \tilde{U} - U(p); 0 \right],
\]

where \(\delta = \frac{1}{1+r}\) is the time discounting rate, \(z\) is the flow utility of leisure, \(\theta q(\theta)\) is the probability of obtaining a job, and \(\tilde{U} = \frac{U_j(p_j)}{d_i}\) for a worker from \(i\) is the value of being unemployed and searching for a job after emigrating to \(j\).

Employed workers earn a flow of wage \(w(p)\), defined as a solution to a Nash bargaining problem. Active jobs in a region dissolve with exogenous rate \(s\), in this case the workers
enter the unemployment state. Thus, the value of employment is given by

\[ W(p) = w(p) + \delta[(1 - s)E W(p') + sE U(p')] \] (4.20)

**Firms.** The behavior of firms is essentially the same as in a standard stochastic dynamic model (e.g. Hagedorn and Manovskii, 2008; Shimer, 2005). The flow cost of keeping an unfilled vacancy is \( c \), therefore the asset value of holding a vacant job is given by

\[ V(p) = -c(p) + \delta[q(\theta)E J(p') + (1 - q(\theta))E V(p')] \] (4.21)

And the value of filled job equals to

\[ J(p) = p - w(p) + \delta[sE V(p') + (1 - s)E J(p')] \] (4.22)

The labor market dynamics in region \( i \) is then governed by:

\[ u'_i = u_i + s_i(1 - u_i) + \pi^\mu I_{u_j > 0}u_j - \pi^\mu I_{\bar{u}_j > 0}u_j - \theta_i q(\theta_i)u_i, \] (4.23)

where \( I \) is an indicator function equal to 1 whenever the condition in brackets is fulfilled, and is 0 otherwise.

As in a standard Mortensen-Pissarides (MP) setting, the wage is modeled as the solution to the Nash bargaining problem:

\[ \beta(J(p) - V(p)) = (1 - \beta)(W(p) - U(p)) \] (4.24)

Although a number of studies starting with Shimer (2005) claim that the wage setting mechanism needs to be altered, since this standard setting failed to account for the cyclical
properties of the key variables. However, these empirical studies do not provide a theoretical justification for any other approach to wage determination. Moreover, Hagedorn and Manovskii (2008) suggested that the problem lies not in the wage setting mechanism, but rather in the way the model is typically calibrated. Using an alternative calibration strategy of the MP model, the model with a Nash bargaining assumption was shown to be consistent with the cyclical volatility of unemployment and vacancies for the US data.

**Equilibrium and out-of-steady-state dynamics**

Equations (4.19)-(4.22) build a system in time-invariant functions $J()$, $U()$, $W()$, $V()$, $\theta()$ and $w()$ defined over the state $\log(p)$. The following equilibrium conditions impose the steady state of the model:

1. $u' = u$: steady-state unemployment

2. $V(p') = V(p) = 0$: free entry for new vacancies

3. $\frac{\beta}{1-\beta}J(p) = W(p) - U(p)$: Nash bargaining in equilibrium

Under the equilibrium conditions (1)-(3) it is possible to show that the system can be boiled down to only two rational expectation functional equations in $\theta()$ and $U()$.

$$U(p) = z + \delta E U(p') \beta \gamma(p) \theta(p) + \pi^\mu \max[\tilde{U} - U(p); 0]$$ (4.25)
CHAPTER 4. TWO-COUNTRY SEARCH-AND-MATCHING MODEL AND IMMIGRATION TO SWITZERLAND: A CALIBRATION APPROACH

The equilibrium tightness ratio given asset values:

$$\frac{c(p)}{\delta q(\theta(p))} = E \left[ (1 - \beta)(p' - z - \pi^\mu max[\tilde{U}(p') - U(p'); 0]) - c(p')\beta \theta(p') + \frac{(1 - s)c(p')}{q(\theta(p'))} \right]$$

(4.26)

The wage in terms of $\theta$ and the asset values can then be recovered as follows:

$$w(p) = \beta p - (1 - \beta)(z + \pi^\mu max[\tilde{U} - U(p); 0]) + c(p)\beta \theta(p)$$

(4.27)

Tightness, wages, migration direction and the value functions in the model are all functions of only the match surplus realizations across regions, therefore they all react instantaneously to the shocks on $p$. Unemployment, in contrast, does not adjust instantaneously. Imposing an equilibrium condition (1) on equation (4.23), I obtain the steady-state unemployment:

$$u_i = \frac{s_i + \pi^\mu I_{[\tilde{U}_j - U_i > 0]} u_j}{s_i + \theta_i q(\theta_i) + \pi^\mu I_{[\tilde{U}_i - U_j > 0]}}$$

(4.28)

In response to a productivity shock, unemployment rates first jump to a new level, and then converge back to the steady state.

**Unemployment value**

Define by the expression $\tilde{U}_i = z_i + \delta[\theta_i q(\theta_i)EW_i(p'_i) - (1 - \theta_i q(\theta_i))EU_i(p'_i)]$ the net present discounted value of of being unemployed and searching for a job only in country $i$.

$$U_i(p_i) = z_i + \delta[\theta_i q(\theta_i)EW_i(p'_i) - (1 - \theta_i q(\theta_i))EU_i(p'_i)] + \pi^\mu max[\tilde{U}_j \frac{d_i}{d_j} - \tilde{U}_i; 0]$$

(4.29)

- case 1: $\frac{U_i(p_i)}{d_i} - U_i(p_i) \leq 0$, then

$$U_i(p_i) = z_i + \delta[\theta_i q(\theta_i)EW_i(p'_i) - (1 - \theta_i q(\theta_i))EU_i(p'_i)]$$

(4.30)
• case 2: $\frac{U_j(p_j)}{d_i} - U_i(p_i) > 0$, then

$$U_i(p_i) = z_i + \delta[\theta_i q(\theta_i)EW_i(p'_i) - (1 - \theta_i q(\theta_i))EU_i(p'_i)] + \pi^\mu[\bar{U}_j - \bar{U}_i]$$

(4.31)

Technical notes: derivation of equations (11)-(13)

From Equilibrium condition (2) and Equation (4.21) I obtain:

$$EJ(p') = \frac{c(p)}{\delta q(\theta)}$$

(4.32)

Iterating Equilibrium condition (3) one period ahead and plugging into Equation (4.29):

$$E[W(p') - U(p')] = \frac{\beta}{1 - \beta} \frac{c(p)}{\delta q(\theta(p))}$$

(4.33)

After substituting (4.30) into Equation (4.19) and rearranging, I obtain Equation (4.25).

In order to obtain Equation (4.26), I subtract (4.20) from (4.19) and iterate one period ahead. This results in:

$$\frac{c(p)}{\delta q(\theta)} = \frac{1 - \beta}{\beta} E[w(p') - z - \pi^\mu max[\bar{U}(p') - U(p'); 0]] +$$

$$\delta[(1 - s) - \theta(p')q(\theta(p'))] \frac{\beta}{1 - \beta} \frac{c(p')}{\delta q(\theta(p'))}$$

(4.34)

Finally, using a one period ahead version of (4.27) and rearranging, I get Equation (4.26).

To show that Equation (27) indeed holds, note that (4.34) implies that

$$J(p) = \frac{1 - \beta}{\beta} \left[w(p) - z - \pi^\mu max[\bar{U}(p) - U(p); 0] + [(1 - s) - \theta(p)q(\theta(p))] \frac{c(p)}{q(\theta(p))}\right]$$

(4.35)

On the other hand, the combination (4.21) and (4.22) yields

$$J(p) = p - w(p) + (1 - s) \frac{c(p)}{q(\theta(p))}$$

(4.36)

The combination of (4.35) and (4.36) results in Equation (4.27).
Chapter 5

Conclusions and Discussion of the

Results of Chapters 3 and 4

Migration is an inherent feature of the global labor market of the XXI century. Regional agreements, liberalization of migration policies and increasing accessibility of long distance travels are contributing to escalation of international migration flows. Growing concerns among policy makers in migrant-sending and receiving countries demand detailed and comprehensive studying of possible migration outcomes from various perspectives. Quoting William Easterly, “Bad ideas on migration and development block understanding that migration IS development”. A deeper understanding of migration effects can help the societies to deal with migration in the most efficient and beneficial way.

A recent theoretical literature on migration attempts to provide a rationale for the existence of diverse migration outcomes documented by numerous empirical studies. The main
outcome of the third chapter of this study is a theoretical foundation for the existence of positive effects of migration on the welfare of native workers and migrants in terms of employment opportunities and wages.

The two-country models of migration in the third chapter are developed under the search-and-matching framework, which is widely used for the analysis of the business cycle properties of unemployment and vacancy rates. However, the application of search models for the analysis of migration has been rather limited. I show that the search-and-matching framework can be used to generate positive (and diverse) effects of immigration. I argue that search frictions combined with additional frictions associated with reallocation of labor, can be useful for explaining the unemployment and wage effects of migration, as well as the wage gaps between immigrants and native workers. This result is not attainable with traditional Harris-Todaro (1970) models of migration.

Additionally, the results suggest that the search-and-matching framework allows for determination of the magnitude of migration flows depending on the structural parameters of two labor markets linked by migration. Moreover, the calibration exercise of chapter 4 shows that the search-and-matching models with migration are consistent with the data from Switzerland.

Due to its tractability, search-and-matching approach to modeling migration opens many doors to fruitful applications and further research. First, the framework can be potentially extended to n regions and used for estimation of the migrant distribution across different locations depending on the labor market characteristics of the host countries. Second, af-
ter introducing workers’ heterogeneity, it can be employed for the analysis of the optimal distribution of migrants across different locations depending on the sectoral structure of an economy and current labor market needs.
Bibliography


