CLEARING MECHANISM IN REAL AND FINANCIAL MARKETS

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A mia Mamma, mio Babbo, mia Sorella

Al dottor Ugo De Giorgi che contro la statistica mi ha tenuto in vita

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General Abstract

The present thesis is a collection of contributions concerning the application of the clearing principle in real and financial markets. In particular, two issues are deepened. The first concerns the effect of bilateral clearing of interbank deposits in the reduction of systemic risk. The second focuses on the endogenous formation process of clearing houses that manage payments. The methodology applied in the three contributions is the network analysis combined with agent-based models and computer simulations. The work is structured in four sections. The first defines and describes the more popular clearing agreements from a legal and economic point of view and provides a presentation of the three collected papers. The second section investigates the effect of bilateral netting on financial stability during a liquidity crisis. The third section analyzes how bilateral netting affects the resilience of an interbank credit network in the event of a shock affecting the assets of an intermediary. The fourth section analyzes the endogenous formation of clearing houses in both corporate barter circuits and interbank payment networks.

Long Abstract

This thesis is a collection of contributions developed during the Phd program at the University of Trento under the supervision of Prof. Edoardo Gaffeo. The red thread that binds these contributions is the compensation mechanism and the application of the network analysis to credit network investigation, respectively from the theoretial and methodological point of view.

The work is structured in 4 sections. The first introductory section deals with a presentation of bilateral and multilateral netting agreements. In particular, an overview of the regulatory environment and the economic rationality of these agreements is provided first. The introduction continues developing a presentation of the network analysis as a methodology applied to the investigation of credit networks in which the prerequisites for reading the three documents are found. Lastly, we introduce the three contributions collected with a general overview of the issues explored in each of them.

The first contribution examines the impact of the On Balance Sheet (OnBS) bilateral netting on the systemic risk in contexts of liquidity crisis. In particular, we compare the performance of a mandatory enforcement of several types of OnBS netting agreements to the one of the gross standard settlement for different metrics mapping the financial stability. So placed, the identification of the optimal policy proposal becomes a multi-objective maximization problem. To this end, we have identified the M.O.O.R.A (multi-objective optimization on the basis of ratio analysis) method as a tool to discriminate the various treatments. Finally, we find that a mandatory policy to bilaterally net mutual interbank exposures between the shocked and the unshocked banks is the the optimal policy option for a wide array of parameters and topologies tested. In addition to enriching the crisis management literature, this contribution is part of the research stream that investigates the development of financial crises and systemic resilience through the application of static networks. Specifically within the literature that uses fixed point algorithms à la Eisenberg and Noe (2001) and Lee (2013).

The second contribution investigates the impact of a mandatory enforcement of the OnBS bilateral netting on the financial stability for crisis situations generated by shock that originates on the external assets of a bank intermediary. To this end, we compare contagion dynamics under OnBS netting and a standard gross settlement through the lenses of a mean-field model and computer simulations. We found that OnBS netting tends to reduce the number of defaults, preserves the aggregate amount of bank capital and takes its toll on retail deposits. Hence, our analysis provides support for a policy mix that combines bilateral netting with a national insurance scheme for retail deposits. This contribution is included in the literature that investigates the default cascades through the application of the static credit

networks. In particular, within the class of models that uses the Furfine's algorithm (2003).

The third contribution deals with the endogenous formation of clearing houses in real and financial environments. More precisely, the formation process of the clearing houses is investigated as an act of coordination among agents by means of an agent based model and computer simulations. In particular, the conditions for the formation of clearing houses within real and financial payment networks are investigated. The first case concerns the creation of corporate barter circuits, the second investigates the development of clearing houses that managed interbank payments in the Free Banking America. We find out that the clearing houses activation is critically affected by the presence of transaction costs, spatial frictions, business cycles and credit network topologies. Given the heterogeneity of the cases presented, our work can be inserted into different strands of literature such as those dealing with credit networks, clearing houses, trade credit, institutional and monetary economics.

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INTRODUCTION

1. PAYMENT SYSTEM AND SETTLEMENT MODES

The payment system is one of the foremost institutions characterizing a modern economy and it can be defined as a set of rules, contracts and procedures that allow the transfer of monetary wealth. A retrospective not only enables us to look at the current functioning of payment systems in advanced economies, but also makes it possible to clearly identify the two principles that underlie settlement procedures: the liquidity principle and the clearing principle. The liquidity principle pertains to the idea that transactions amongst agents adhering to a system are carried out on a gross basis and through the transfer of payment means. From a technical perspective, the principle finds a direct equivalent in the settlement procedure commonly known as Real Time Gross Settlement (RTGS). In addition to the gross settlement, the main feature of RTGS systems is timing, since payments are made continuously. Typical examples of such settlement modes are the Swiss interbank payment system SIC or the European Target2. The main advantage of such settlement mode is the immediate extinction of obligations arisen amongst members of the payment system; the main disadvantage is the weight of the procedure in terms of the costs for acquiring the necessary resources. The clearing principle is based, instead, on the idea that the agents participating in a payments system hold simultaneously credit and debit obligations toward the system. Therefore, an effective settlement method of the aforementioned obligations would consist in the offsetting and the expenditure of payment means only for their net value. Contrary to Real Time's payment systems, netting procedures take place on a fixed date for all obligations arising between agents over a set period of time. Within the payment system, the clearing principle is reflected in the Deferred Netting System (DNS). DNS have two categories, bilateral (BNS) and multilateral (MNS). In the former category, bilateral netting is applied at a specific date exclusively for reciprocal obligations between two agents; in the latter, netting is carried out for all the obligations that a single agent holds towards the system. Usually, netting procedures take place in a clearing house where members of the system have an account. Therefore, the netting procedure turns out to be a complex process, which can be made of three phases: the exchange of proofs of purchase of obligations to be settled; the tally of net balances; the transfer of the net balances to the members' accounts. The main advantages of clearing systems are to be found in the decrease of liquidity costs and of the operational risk during the execution of the payments. The main disadvantage is to be found in the credit risk that, inevitably, emerges whenever payments are not regulated continuously; such risk pertains to the possible default of one or more members of the clearing house. An example of payment systems making use of clearing is the Italian Bi-comp for retail payments. Note that from the difference between the two principles emerges a tradeoff: on the one hand, the liquidity principle grants security against the credit risk; on the other hand, the costs necessary to settle each transaction can restrain the efficiency of the system. Historically, regulators have been particularly careful not to tip the scales in favor of none of the systems, weighing the costs and the advantages of both. It is not uncommon to identify both settlement modes within the same payment system. For example, in regard to wholesale payments, in Italy TARGET2 (T2) is particularly relevant. T2 allows the regulation in the central bank currency of individual transactions denominated in Euro using an RTGS system. However, TARGET2 is flanked by EURO1, which allows the execution of transactions in MNS, with an end-of-day settlement through TARGET2.

Often, due to their analogy, it is easy to confuse the settlement regime in place within the payment system with the settlement procedures established by companies, States or individuals on real or financial markets. The former regards the manner in which central banks, private banks, operators who provide payment services (etc.) transfer monetary wealth on their own account or on behalf of their clients. The latter concerns the settlement procedure for contracts stipulated outside of the payment system for the ordinary occurrences of economic life. For this second instance as well, the principles stated for the payment system are valid. Other settlement procedures for obligations will flank the payment system. For example, to the RTGS and DNS procedures described above, a type of settlement known as Gross Settlement (GS) is typically added, allowing counterparties to opt for a gross payment (as for RTGSs) but not continuous (as for DNSs). In order to clarify the difference between payment system and settlement agreements, we discuss the case of two companies holding two reciprocal credit and debit obligations and whose settlement is implemented through a netting agreement. At the contract expiring date, the net debtor issues his bank the order to execute the payment towards the bank of the net creditor's account. The two banks could be members of a system that operates under the RTGS regime: in such case, they companies would settle their exposures using the principle of clearing, whereby the transfer of wealth between them would be handled by the banks through a system that operates in accordance with the principle of liquidity. The theoretical contributions gathered in the present thesis revolve around the theme of payment agreements in real and financial markets. The first and second contributions discussed here will focus on the impact that bilateral netting agreements between banks can have on systemic risk. The third will investigate the dynamics of multilateral clearing within a real and financial payments networks. The following section presents an overview of the types of agreements that will be discussed in this thesis.

2. BILATERAL NETTING

Bilateral netting is a tool commonly used in real and financial markets. With general reference to the Italian system, the clearing principle is made explicit in the article *no.* 1241 of the civil code, which states: *"Quando due persone sono obbligate l'una verso l'altra, i due debiti si estinguono per le quantità corrispondenti, secondo le norme degli articoli che seguono"*[1] (1242-1252), providing normative framework. Equally important is article *no.* 1252, which states: *"Per volontà delle parti può aver luogo compensazione anche se non ricorrono le condizioni previste dagli articoli precedenti. Le parti possono anche stabilire preventivamente le condizioni di tale compensazione."*[2] The following contributions will consider the three most common forms of bilateral netting, namely payment netting, novation netting and close-out netting.

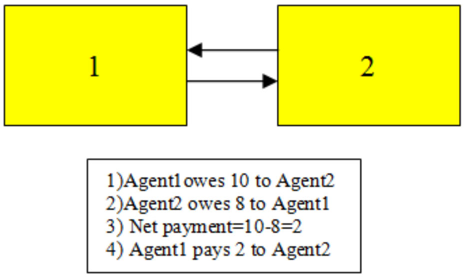
2.1. Settlement netting

Settlement netting or payment netting is a type of agreement in which two parties decide to clear the liquidity fluxes to be transferred at a set date. This type of agreement does not have an impact on the nominal value of obligations in place between the two contracting parties, since its object is limited to the quantity of the pre-established payment means to transfer. The purpose of this type of

[1] "When two persons are obliged to one another, the two debts are extinguished for the corresponding quantities, according to the norms of the following articles"
[2] "By the will of the parties involved, compensation may take place even if the conditions set out in the previous articles do not occur. The parties may also establish the conditions for such compensation in advance".

agreement is to reduce the liquidity risk and the operational risk, in addition to alleviate the costs of obtaining financial liquid resources. The decrease of the liquidity risk is caused by the clearing procedure itself, which reduces the absolute amount of liquidity to be transferred, so that the probability that one of the two parties involved does not possess the means of payment necessary to close the transaction is lowered. Furthermore, netting allows both parties to lower the costs of obtaining payment means. Such costs can be either direct, if the contracting party lacks means of payment, or indirect, if one considers the opportunity cost of an alternative usage of such means of payment. In regard to the decrease of the operational risk or to the possibility of suffering losses deriving from the inadequacy or failure of procedures, technical systems, or unforeseen events, whenever the exposures amongst agents are reduced, the potential impact of the manifestation of such risk is reduced as well. Usually the payment netting is defined in a clause of a more general agreement (Master Agreement), although autonomous payment netting agreements are not rare (Stand Alone Payment Netting Agreement); however, sometimes they might be required by law. Figure 2.1 shows an example of payment netting.

Figure 2.1. Settlement netting



2.2. Novation netting

The term "novation netting" refers to a type of contract or clause which allows to extinct mutual obligations between two parties, creating a new obligation that absorbs the multiplicity of fluxes of the previous obligations in a single net flux. Contrary to settlement netting, this type of clause has a significant impact on the financial and legal statements of the contracting parties, redefining obligations and claims amongst parties. After the novation netting there will be only one creditor, the net creditor counterparty, and only one debtor, the net debtor counterparty. The value of previous obligations, which is necessary to determine the value of the "renewed contract", is established through standard procedures or defined by the counterparties. Within the Italian context, the legal basis for the novation principle is to be found in article *no.* 1230 of the civil code; as for the clearing principle, instead, it is to be found to the aforementioned articles *no.* 1241 and 1252. The economic purpose of novation netting agreements is primarily to reduce the liquidity risk of the counterparty, to reduce management costs of multiple contracts, to mitigate the operational risk at the deadline, and to reduce credit risk. Management costs of various contracts and of risk evaluation of such contracts decrease, and so do potential policies aimed at hedging single operations. The liquidity risk decreases because, on the one hand, the lower the net exposition, the lower the probability that the net debtor requests a payment extension; and because, on the other hand, it allows the net creditor to extinguish his exposition without the need to recuperate liquid resources. Operational risk decreases in the same way as described for payment netting. The credit risk emerges from the bankruptcy of the counterparty; as a consequence of netting procedure, credit risk decreases for the exposure of the new contract is lower than that of the initial contracts.

Figure 2.2. a) Before Novation Netting

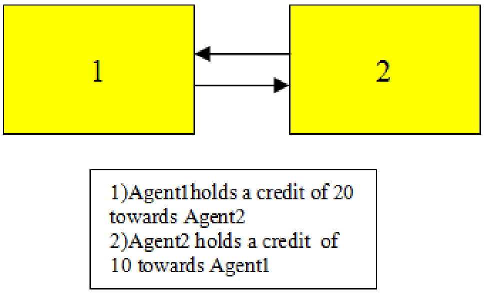
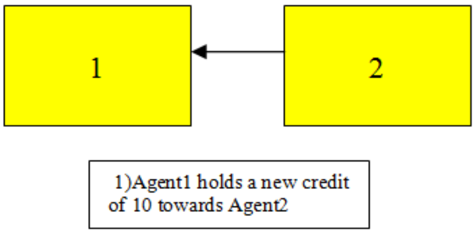


Figure 2.2. b) After Novation Netting



2.3. Closing out netting

Closing out netting clauses are generally part of the Master Agreements and have ample circulation amongst banks and financial intermediaries for operations concerned with derivatives or for interbank deposits. Usually, Master Agreements are a type of agreement reached amongst parties which define the terms regulating one or more transactions as well as the services characterizing the object of the contract. Within the financial community, bank and sector associations tend to create and present some typified schemes of Master Agreements in order to standardize operators' behaviour and therefore lower the uncertainty and the bargaining costs of the operations. Amongst the most commonly used schemes are the ones regulating derivatives transactions (see the ISDA Master Agreement) and the ones on deposit netting (see IDNA Master Agreement sponsored by BBA). The closing out clause allows for the clearing of the bilateral expositions of two agents upon the occurrence of a contingency, typically the default of the counterparty. The purpose of such agreements is to reduce credit and liquidity risk. These types of clauses are situated within the financial collateral agreements. The European legislation defines such types of clauses in the 2002/47/EC directive (amended by the 2009/49/EC), which has been in force in Italy since the implementation of the Legislative Decree *no.* 170 of May 21, 2004. In this, article 1.f) states:

"'close-out netting provision' means a provision of a financial collateral arrangement, or of an arrangement of which a financial collateral arrangement forms part, or, in the absence of any such provision, any statutory rule by which, on the occurrence of an enforcement event, whether through the operation of netting or set-off or otherwise:

(i) the obligations of the parties are accelerated so as to be immediately due and expressed as an obligation to pay an amount representing their estimated current value, or are terminated and replaced by an obligation to pay such an amount; and/or

(ii) an account is taken of what is due from each party to the other in respect of such obligations, and a net sum equal to the balance of the account is payable by the party from whom the larger amount is due to the other party;"

Netting clauses and agreements are mentioned by the Basel Accords II[3] and III concerning possible "discounts" on capital reserves in countries where agreements are legally enforceable. For what concerns European legislature, the Regulation 573/2013 on the prudential capital requirements for credit institutions and the Banking Recovery and Resolution Directive 2014/59 / Eu consider legitimate the use of netting agreements. The applicability of the close-out netting is the main issue of this type of clause, due to the heterogeneity of national legal systems. Therefore, the contracting parties must obtain solid legal opinions in order to avoid the risk of not being able to use the guarantee at the time of the counterparty's failure (Regulation *no.*573/2013 article 194 point 1). This opinion has been strengthened and articulated by the EBA:

"On the issue of whether an opinion must be specific to the relevant transaction covered and to the technique employed by the institution or whether it can be a generic one, it depends mainly on the nature of the two. If an institution engages in the same type of transaction, with counterparties located in the same jurisdiction and uses the same credit risk mitigation technique, then it can rely on the same opinion. For example, if an institution uses a master netting agreement for which a generic opinion exists, it can use that opinion as long as the latter clearly indicates that the agreement is legally effective and enforceable in all the jurisdictions relevant to the transactions covered by that agreement"[4].

With regard to the Banking Recovery and Resolution Directive 2014/59 / EU, the applicability of the close-out netting clauses is guaranteed by Article 77, stating:

"Member States shall ensure that there is appropriate protection for title transfer financial collateral arrangements and set-off and netting arrangements so as to prevent the transfer of some, but not all, of the rights and liabilities that are protected under a title transfer financial collateral arrangement, a set-off arrangement or a netting arrangement between the institution under resolution and another person and the modification or termination of rights and liabilities that are protected under such a title transfer financial collateral arrangement, a set-off arrangement or a netting arrangement through the use of ancillary powers".

A further economic motivation to use close-out-netting clauses, in addition to the ones in common with the aforementioned payment and novation netting, is the possibility to eliminate "cherry picking" behaviors in case of the counterparty's default. Take into consideration, for example, the case in which two banks are under a swap contract. When the default occurs, the bank in crisis could internalize the inflows while neglecting its own outflow obligations towards the counterparty. Furthermore, the decrease of the liquidity risk assumes greater importance in the event of the counterparty's default, in comparison to the instances in which netting agreements are closed in periods of regular counterparties' activity. This is because often times the default party does not hold enough liquid resources, so that, even if the creditor covered by financial insurance had access to

[3] Paragraph *no.* 188 Basel II

[4] https://www.eba.europa.eu/single-rule-book-qa/-/qna/view/publicId/2013_23

these resources before other categories of creditors, such resources could yet prove to be insufficient. The clearing procedure reduces the probability of this occurrence.

Figure 2.3. a) Before Closing out netting

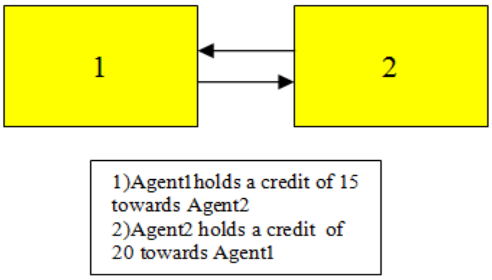
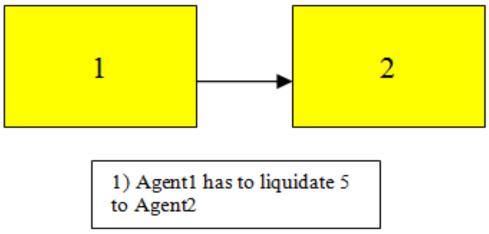


Figure 2.3. b) After Closing out netting



3. MULTILATERAL NETTING

Multilateral netting is a practice enabling the application of the clearing principle in the event of debit and credit relations amongst three or more individuals. Usually, in the Italian system, clearing agreements are of private nature and are based on contractual limitations (contracts of confirmation or adhesion; for the case of corporate barter circuits sometimes the institution of *permuta* is considered). Given the higher complexity of the procedure, multilateral netting is usually performed by a specialized operator, the clearing house. The clearing house is the authority which agents address to offset their debits and credits. The clearing procedure is made of three phases. In the first phase, an exchange of information takes place between the parties involved and the clearing house; in the second phase, the tally of the net balances is carried out; in the third phase, the payment is finalized. In some cases, mainly in the clearing houses of payment systems, adherence to a clearing house does not extinguish the obligations relating to the contracts that are to be offset until the conclusion of the procedure. In other cases, as it happens in the clearing houses for some kind of derivatives, the clearing house is interposed within a bilateral relationship and gives rise to two new ones between itself and the respective counterparties, extinguishing the previous relationships.[5] From an economic perspective, it can be seen that the adherence to the clearing house and the relations amongst the agents are grouped together in a single balance on the account of each participant. By design, the sum of all the actions of the agents is zero. The advantage for the agents is similar to the advantages

[5] For an in-depth analysis of the legal principles that can delimit the field the clearing houses see: Olivieri, G. (2002), Compensazione e circolazione della moneta nei sistemi di pagamento, Milano, Giuffrè; Santoro, V. (2007), Il diritto dei sistemi di pagamento, Milano, Giuffrè.

described for bilateral netting – *i.e.* lowering of the operational risk and of the liquidity risk. The clearing house provides the offsetting service under the payment of commission allowing for the sustainment of the management costs. The clearing house's management entails a credit and liquidity risk. By interposing itself between the agents, the clearing house needs to be able to accommodate creditors and to collect debtors' payments. The credit risk manifests itself whenever a debtor defaults and is not able to extinguish his obligation. In order to mitigate the credit risk, the clearing houses usually request collaterals, apply the margins system and are equipped with capital. In regard to the liquidity risk, in addition to the margins system, the supervisor of the clearing house is equipped with financial resources to face possible liquidity shortages.

Figure 3.1. a) Before multilateral netting

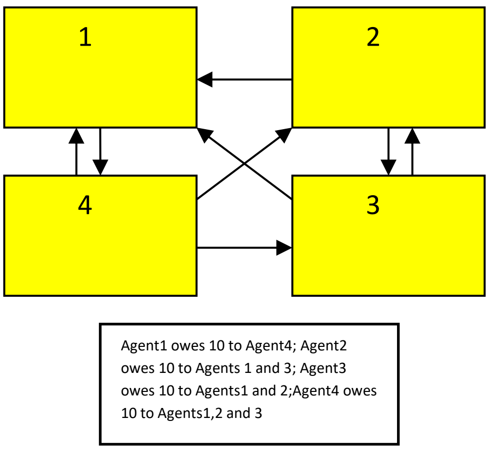
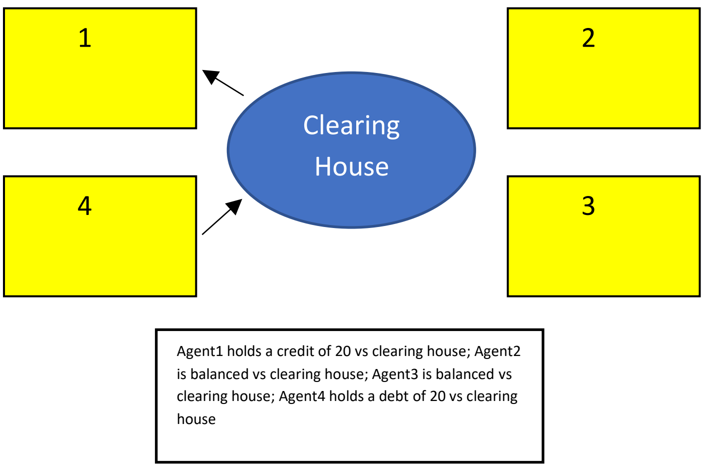


Figure 3.1. b) After multilateral netting



4. METHODOLOGY

The methodology used in the three contributions presented in this thesis is network analysis. After a first development in the sciences such as physics, biology, epidemiology and medicine, this methodology has spread as a useful tool for the analysis of social dynamics and it is nowadays commonly used in sociology, anthropology, psychology and economics. In economics it is widely employed in the study of trade and financial dynamics.

The main feature of network analysis is to consider the economic environment as a set of relationships represented by a graph composed of nodes and links. The former represent agents, companies, banks or, more generally, the actors of an economic system, the latter the economic relationships that exist between them. In this section we will explain and briefly show the types of graphs that will be presented in this thesis and how it is possible to construct and interpret them. The purpose of this introduction to network analysis is to provide the reader with the prerequisites for addressing the reading of the contributions collected in this thesis. Within each contribution the reader can find the descritpion and the building procedure for the all the topologies presented, for a detailed overview of the network theory and its application in the social sciences we suggest taking Jackson (2010) as a reference point.

4.1. The graphs

A graph is the representation of a network and it is composed of nodes and links. The simplest way to define a graph of N nodes is through the explication of the adjacency matrix. Figure 4.1 shows the adjacency matrix X of dimension N^2 of a graph composed by N nodes.

Figure 4.1. Adjacency Matrix.

$$\boldsymbol{X} = \begin{bmatrix} 0 & \cdots & x_{1j} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & \cdots & 0 & \cdots & x_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{Nj} & \cdots & 0 \end{bmatrix}$$

Each term $x_{i,j}$ ($\forall i, j \in N$) of the matrix can assume values 0 and 1 which indicate the presence or not of a relationship between the subject *i* and the subject *j*, respectively 1 in the former and 0 in the latter case. An adjacency matrix can be *directed* or *undirected*, in the first case the direction of the relationship between two nodes is highlighted while in the second not. For this reason, the two adjacency matrices are defined differently.

For the directed graph, the relation between the agents *i* and *j* is identified by the terms $x_{i,j}$ and $x_{j,i}$. When the terms $x_{i,j}$ and $x_{j,i}$ are respectively 1 and 0 such relation is directed from *i* to *j*, viceversa in the case in which $x_{i,j}$ is 0 and $x_{j,i}$ is 1. When both elements of the matrix are 1, a bilateral relationship is recorded.

As far as *undirected* graphs are concerned, the adjacency matrix only indicates the presence of a relation between two nodes without identifying the direction. In this case the adjacency matrix is symmetric, so when the term $x_{i,j}$ is equal to 1 the term $x_{j,i}$ is 1 too. The principal diagonal is composed of zeros for both adjacency matrices considered, that because it would be non sense to report the relationship that an agent has with himself. Figure 4.2 shows an example of directed and undirected adjacency matrices.

Figure 4.2. Matrices A and B respectively identify an undirected and a directed network

$$A)\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \qquad B)\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

The presented adjacency matrices are commonly referred to as *unweigheted* given that they signal the relationships between two nodes but do not indicate their "scale". One way to represent the difference between the relationships that exist in a network is to attribute weights to links. Figure 4.3 shows two weighted adjacency matrices, respectively for *directed* and *undirected* cases. Looking at the matrices shown, it easy to notice that the terms are no longer dichotomous variables but instead they are positive real values.

Figure 4.3. Matrices C and D respectively identify a directed and an undirected weighted network

$$C)\begin{bmatrix} 0 & 10 & 0 & 2 \\ 0 & 0 & 4 & 1 \\ 0 & 1 & 0 & 0 \\ 2 & 7 & 0 & 0 \end{bmatrix} \qquad D)\begin{bmatrix} 0 & 2 & 0 & 4 \\ 2 & 0 & 10 & 1 \\ 0 & 10 & 0 & 0 \\ 4 & 1 & 0 & 0 \end{bmatrix}$$

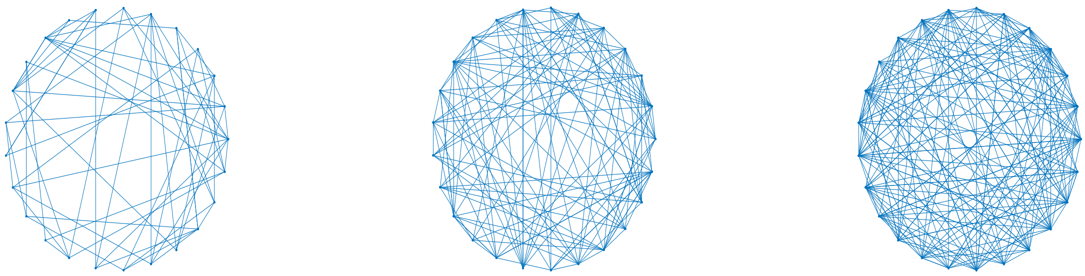
The construction of adjacency matrices, weighted and unweighted, can occur through the retrieval of empirical data or can be generated randomly. In this work we consider random credit networks generated by stochastic processes. In addition to being random, the networks that will be presented have the characteristic of being static. In network theory, the word static indicates the fact that all connections and nodes of a network are generated at a given time. The static term is opposed to the dynamic one, it indicates processes where the number of nodes and connections varies over time according to the behaviors of the network members or by exogenous imposition. In the continuation of this section we present some features of the best known random networks.

4.2. Random networks

The topology of random networks is dictated by stochastic processes that define the level of connection of the nodes. In this section we present three of the most known topologies. The networks

that are presented in each contributions can be traced back to these three types (albeit in some cases with some minor variation).
The first topology we take in consideration is the Erdos Renyi. The main feature of this type of random graph is that each node has the same attachment probability of the others. An Erdos Renyi network can be identified by only two variables, the number of nodes *N* and the attachment probability *p*. This type of network generates uniform topologies as the expected value of the number of links afferent each node is constant. Increasing the value of *p* increases the number of connections afferent to each node. The Figure 4.4 shows three types of Erdos Renyi networks for three different connection levels.

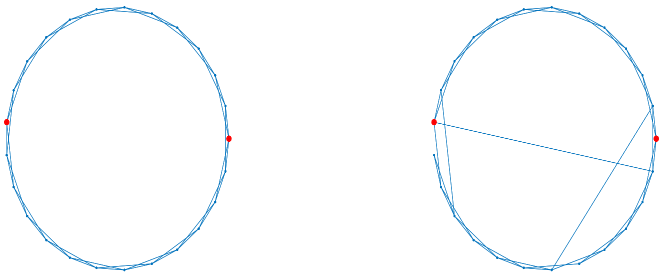
Figure 4.4. Erdos Renyi random networks. The graphs A, B and C are generated respectively by (N=25; p=0.1),(N=25; p=0.2) and (N=25;p=0.3)



a) b) c)

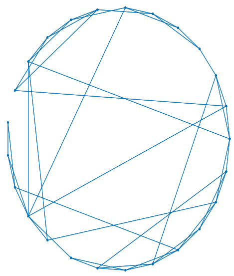
The second popular topology is the Small World one. The principal feature of this topology is that the nodes belonging to this network can be reached by others by means of a small number of intermediate steps. The process of generating this topology is based on the operation called re-wiring. Starting from a circular structure, the re-wiring consists initially in deleting some links within the network and then reassign them randomly. This procedure was developed by Watts and Strograz, in Figure 4.5 we represent the functioning of this algorithm by means of a simple example. Considering the topologies *a* and *b*, the differences between them are due to the re-wiring of three links. Due to the re-wiring effect, the shortest distance between the two nodes marked in red is different. Precisely, it is equal to 6 steps for the graph *a* and of two steps for graph *b*. Assuming that the probability of re-wiring of each link is β (0 <β <1), the graph tends to resemble a random homogeneous graph as the parameter β approches one. Figure 4.6 shows a Watts and Strogratz random network for a β=0.3.

Figure 4.5. The graph *a* is a lattice ring with 4 connections per node. The graph *b* is the same as *a* except for the re-wiring of 3 connections.



a) b)

Figure 4.6. Watts and Strogatz random network generated by (N=25; β=0.3)

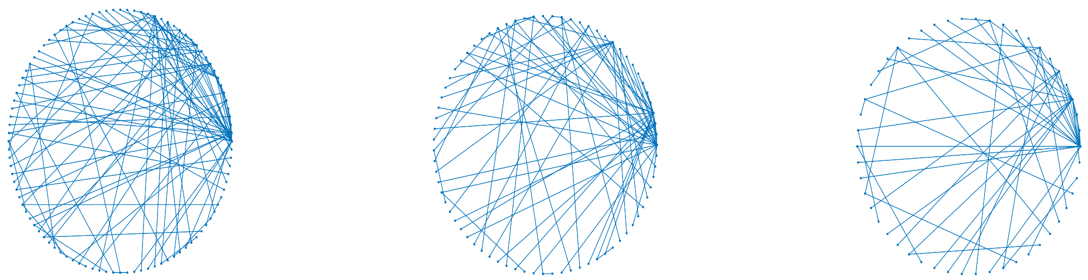


The third type of random networks is that generated by means of power law distributions. The main feature of these types of networks is that of having a low number of nodes possessing a high number of connections and the remaining majority of nodes characterized by a lower one. One way to generate this type of topology is by using the Barabasi and Albert's algortihm. This procedure consists of several steps. The first is to identify a set of nodes. To this set are added one at a time additional nodes characterized by connection probability proportional to the number of links held by the nodes existing in the system. From the analytical point of view, the probability that a new node is connected to an already existing i-th node can be expressed by the formula (1).

$$P_{new} = \text{Number of links of the node } i \text{ / Number of total nodes of the system} \qquad (1)$$

The Figure 4.7 shows three networks of different sizes generated by the same initial set of nodes and connections using the Barabasi and Albert's method. It easy to note that there are some nodes with a high number of connections alongside a larger number of nodes characterized by a smaller number of links and visually identifiable by the lower concentration of blue color near the node.

Figure 4.7. Barabasi and Albert random networks composed of a)100 b)75 c)50 nodes



a) b) c)

One of the ways to describe and analyze networks is through the use of appropriate indicators. In general, indicators can provide information on individual nodes or on the network as a whole. For example, the index *degree* provides us with information on the number of links of a node. This index, for direct networks, can be decomposed into the two indexes In and Out-Degree, which respectively indicate the connections comming in and out of a node. If we wanted to know the average connection level per node in the network, we could use the *average degree index* calculated as the sum of the degree of each node and divided by the total number of nodes. We present the Table 4.1 that introduces some of the most famous indexes to the reader. The indexes used in the contributions are defined and explained in detail in the respective contributions.

Table 4.1. Indexes and definitions

Indexes and definitions	**Description**
Degree of a node	The number of connetions a node has to other nodes.
Degree Distribution	The probability distribution of the degrees over the network.
Distance	The shortest path in terms of number of links between two nodes.
Diameter	Largest distance between two nodes.
Average Path Lenght	Mean of the distances between nodes.
Betweenness Centrality	Measure of the ability of a node to connect other nodes.
Closeness Centrality	Measure of how easy a node can be in contact with other nodes.
Triplet (closed/open)	When three nodes are connected each other by three (closed) or by two (open) undirected ties.
Clustering	Concentration index obtained by dividing the number of closed triplets by the number of all triplets (open+closed).
Reciprocity	How many nodes in a directed graph are linked to each other by reciprocal linkages.

4.3. Focus on random credit network

In this work we consider credit networks. Which means that the links among the nodes of a network represent debit and credit relationships. The adjacency matrix of a network illustrates the outstanding obligations. By convention, we hypothesize that the credit relationships of the creditors with respect to the debtors are represented in the lines of the adjacency matrix. So, reading the matrix from left to right allows to identify the credit positions $x_{i,j}$ of the i-th creditor towards the j-th debtor. Since the credit of agent *i* towards agent *j* can also be defined as the debt of agent *j* towards agent *i*, the j-th column is associated with the debts of the j-th agent. To identify the value of network obligations and build the weighted adjacency matrix, we proceed by steps. The first step is to choose whether to determine exogenously the total value of system credits or, alternatively, to leave this value as the determinant of an endogenous procedure. In the first case the total credit value of the system is set at the beggining of the process and the identification of the value of the individual credits follows a path from the general to the particular. More precisely, a criterion of distribution of the fixed value has to be identified. To make the concept clearer we provide a couple of simple examples. If the purpose of the network builder is to identify homogeneous credits, after having fixed the total amount of credit value the easiest way is to divide the total credit value by the total number of links. On the other hand, assuming that the objective is that to assign the same credit value stock to each node, it can be reach by divide the total credit value by the number of nodes. Finally, the value of each single loan has to be set. The endogenous procedure usually follows the reverse path fixing the value of each credits and then computing the systemic amount. Commonly, for these type of networks the value of individual credits is assign by resorting to probabilistic distribution. The probabilistic method for assigning the value of individual credits consists in the extraction of the value from a predetermined probability distribution. This technique is usually empoyed for medium-large networks. This is due to the fact that in order to have adherence between the desired distribution and the realization of the stochastic process it is necessary to reach a "critical mass" in terms of the number of nodes and connections.

The application of network analysis to investigate credit dynamics implies considerable advantages. First of all, it allows to present the agents of an economic system in terms of budgets and, starting from financial statements, to show the relationships that arise among the network participants. Secondly, network analysis makes it possible to contextualize a given credit phenomenon within the economic system and to highlight its relevance. This relevance may not be the result of the phenomenon itself, but rather of the location within the financial network where it happended. Thirdly, network analysis provides the tools for a direct investigation of the network as a whole that can be very useful for regulators and policy makers to get a general understanding of economic dynamics. Regarding the application of static network analysis to credit networks, it can be useful from different points of view. In the presence of empirical data, network analysis allows to identify the network topology and to give an accurate description. In non-empirical investigation, static analysis allows the creation of artificial credit networks with topologies close to real ones on which to test policy proposals, analyze contagion dynamics and assess the level of network resilience to a shock. The static analysis is therefore particularly indicated if one has to analyze a sudden phenomenon or an event that does not extend over time. The weakness of this methodology is that it does not allow the analysis of repeated phenomena, of strategic interaction and of the modification of the network topology in response to economic phenomena. In contrast to static analysis there is the dynamic one that allows to analyze the evolution of a network over time, this evolution can be the result of stochastic variables or the product of the interaction between agents. Interesting for the economic analysis is the application of network analysis to strategic behavior, given that such behavior depends structurally on the position that an economic actor assumes within the system and on the interaction with other actors. The strength of these models is that by setting appropriately the behavioral rules of the agents it is possible to give a realistic representation of different economic phenomena as well as to provide possible explanations on the economic forces that have contributed to develope a certain topology. Moreover, this methodology is able to represent complex and repeated dynamics over time among heterogeneous agents, increasing in this way the level of realism of the models based on this approach. The main limitation consists in the difficulty of identifying the behavioral rules of the agents. Indeed, agent behaviours can be considerably heterogeneous and change according to undefined patterns.

5. PAPERS OVERVIEW

Once the clearing agreements are presented, the contexts in which the effects of such agreements are tested will be briefly shown. In regard to bilateral netting contracts, we will observe their effects on the reduction of systemic risk due to crises that develop in the interbank market. For what concerns the multilateral netting system, we will develop an agent based model that analyzes the endogenous formation of the clearing houses in real and financial markets.

5.1. First Contribution

In the first contribution, we will attempt to analyse the effect of interbank deposit netting during the process of contagion of a financial crisis. In particular, using network analysis as the chosen methodology to study interbank dynamics, we will investigate whether a mandatory enforcement of bilateral netting agreements by the supervisory authorities can have relevant effects on the systemic risk. Then, we will compare the netting performance to the standard gross settlement mode. The focus of this first contribution is on crises triggered by bank runs as in Lee (2013)[6]. Lee algorithm fits into the class of models that analyzes a liquidity crisis through network analysis like Gai and Kapadia (2010) and Gai, Haldane and Kapadia (2011). As stated in Hurd (2015), the peculiarity of Lee's algorithm is that of having adapted the Eisenberg and Noe (2001) algorithm to a liquidity crisis. In this way the model presented in this paper is part of a broad line of literature that uses the methodology developed by Eisenbeg and Noe (2001). We will define the systemic risk using four metrics apt to evaluate the severity of a financial crisis and we test their dynamics for networks characterized by different topologies, loans heterogeneity, and shock of different magnitudes. Specifically, we will take into consideration the number of banks going through a period of illiquidity; the entity of such illiquidity; the degree of liquidity of the system at the end of a contagion process and the depth of the interbank market. Having set the metrics, an evaluation of the trade-offs that the policymaker faces when different welfare objectives return contrasting results. Once defined in this way, in fact, the

[6] Bank run and liquidity crisis are phenomena for which there is a huge strand of literature. Within this is contribution the reader will find a literature review dealing with network and liquidity crisis models, for a broader perspective and without pretension of exhaustiveness we suggest Eichengreen (2002), Reinhart and Rogoff (2009), Kindleberger (1978), De Soto (2006), Rick (2016), Allen and Gale (2007).

main goal of financial stability can be immediately translated into a multiobjective decision-making problem. In order to better convey the study, a basic example shall be provided.

Figure 5.1. The balance sheets

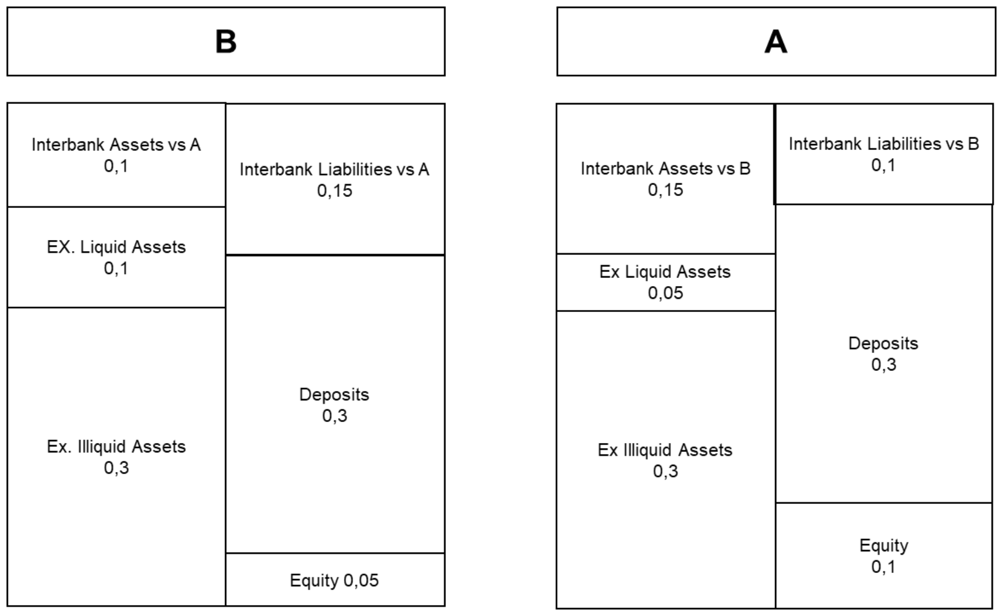


Figure 5.2. The shock and the beginning of the contagion

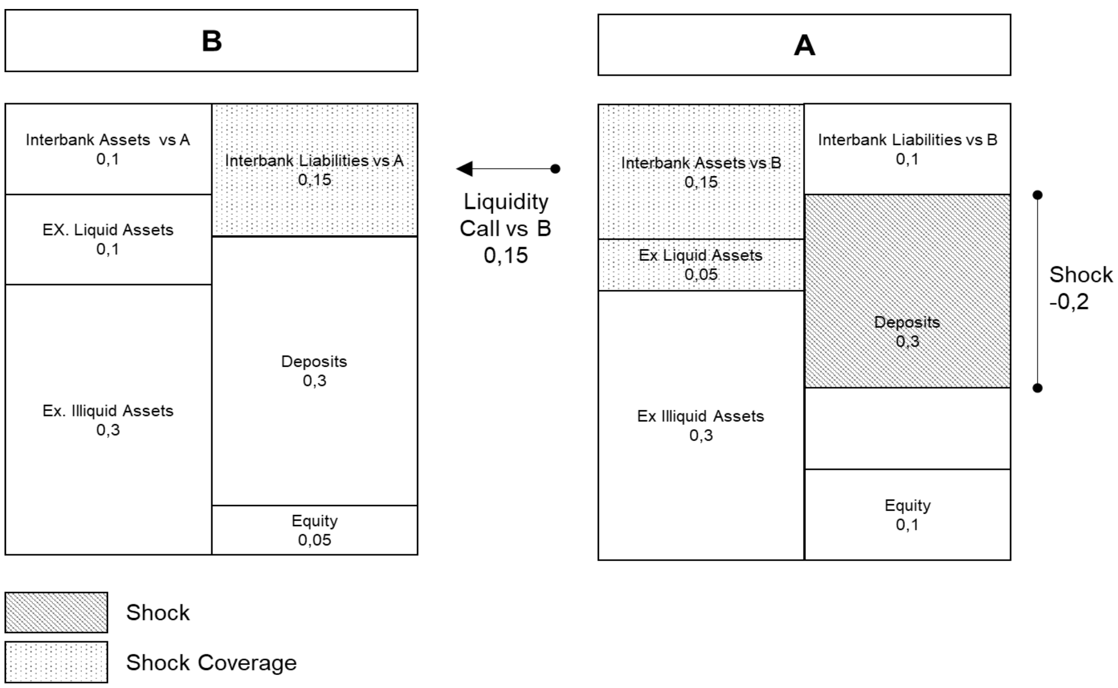
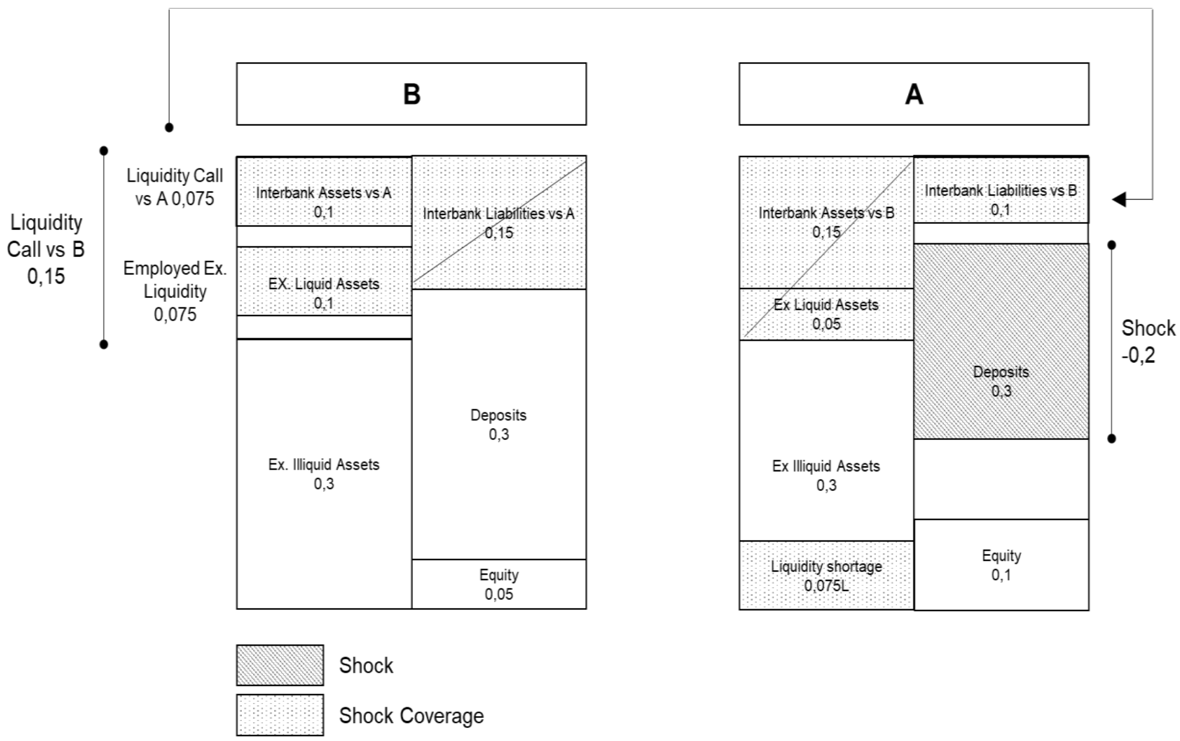


Figure 5.3. Contagion dynamic



Consider a simple financial system formed by two banks A and B, as shown in Figure 5.1 The accounting reports of the banks are constituted on the asset side by interbank credits, external assets, and external liquidity reserves. On the liability side, accounting reports are constituted by equity, external deposits, and interbank debts. Bank A is hit by a shock that may represent a bank run that will drain its liquidity reserves and force it to recall its interbank credits to obtain the necessary liquidity. External assets are hypothesized as hardly liquidable, therefore whenever a bank has exhausted its external liquid reserves and called interbank loans back, it must obtain liquidity from the Central Bank. We will define this case liquidity shortage. In the case depicted in the Figure 5.2, Bank A is hit by a shock of 0.20 and will face the need for liquidity using its liquid reserves for 0.05 and calling back the interbank loans to bank B for 0.15. In this case the shock is equal to the sum of the interbank and external liquid assets; if it was lower, the shocked bank would employ interbank and external liquid assets proportionally. Bank B will internalize A's request for 0,15 and will front it using 0.075 of liquid reserves and calling back 0.075 of interbank loans to A (following the aforementioned proportionality). Bank A, at this stage, will be forced to ask the central bank for a credit line in order to repay Bank B which, in turn, will use the resulting liquidity to extinguish the payments still due to A (Figure 5.3). Summing up, for the gross settlement case, Bank A marks a liquidity shortage of 0.075 and sees its liquid and interbank resources dried up; on the opposite end, Bank B does not need to resort to the central bank, although it uses the major part of its liquid resources. Considering the four metrics employed, one bank needs the Central Bank aid, the total liquidity shortage amount is 0,075, the external liquidity after contagion is 0,025 and the interbank markets consists of Bank B credit towards A for 0,025. How would the enforcement of the bilateral deposit netting by the supervisory authorities have changed the dynamic? If, upon the occurrence of a bank run, authorities applied bilateral netting to their own interbank deposits, the scenario would change, as the Figure 4.4 shows. In this context, given the decrease of the interbank assets' volume due to the clearing, Bank A is able to call back a lower absolute value of receivables having to mark a liquidity shortage of 0.1 (Figure 5.5). Bank B internalizes a request for 0.05 to be covered with 0.05 of liquidity (Figure 5.5). Notice how the crisis dynamic is substantially different from the former. If we consider the metrics applied, we observe that in this second case: the number of banks asking for help to the Central Bank is one; the value of the liquidity shortage is higher in the net case; Bank B is able to preserve more liquidity with respect to the gross settlement case; during the netting process and the contagion process all the interbank availabilities are employed.

Figure 5.4. Netting procedure

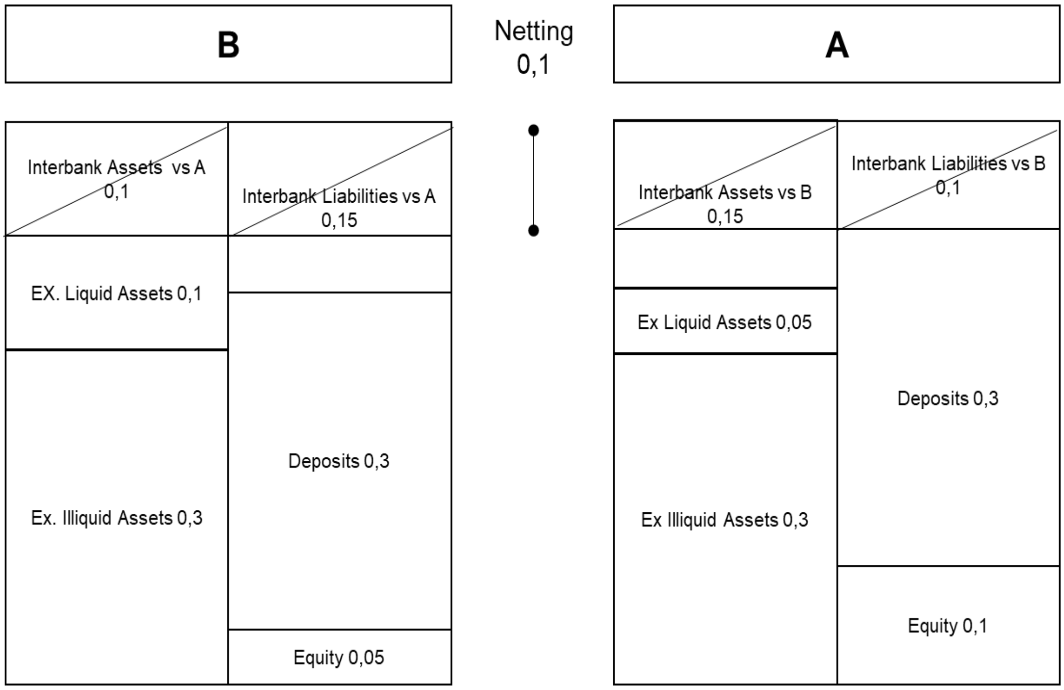
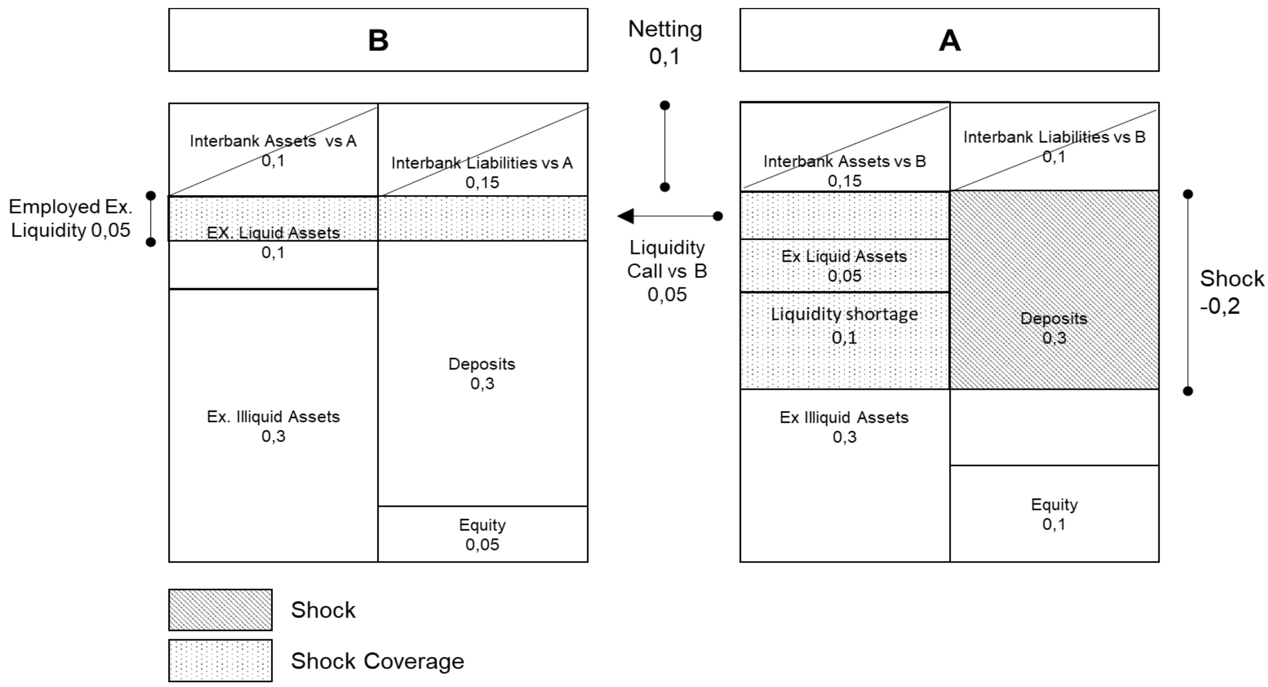


Figure 5.5. The shock and the contagion process



Evidently, a mandatory enforcement of bilateral netting is not neutral towards the development of the financial market contagion, as well as towards the metrics taken into consideration. In the first contribution these dynamics are analyzed for complex financial networks. This paper adds to the extant literature which employs network analysis for the study of financial crises. It makes an original contribution to the field in two ways. Firstly, to the best of our knowledge, this is the first work to address the close-out netting issue in the context of liquidity crises. Secondly, the paper elaborates a wide-ranging and multidimensional approach to systemic risk, highlighting the trade-off which regulators might be confronted by during a liquidity financial crisis.

5.2. Second Contribution

The second contribution analyses the effect of bilateral netting agreements as a crisis management tool utilized by supervising authorities and governments. Differently to what was done for the first contribution, we will study: the ability of bilateral netting to alleviate financial crises triggered by a shock hitting a bank's external assets; the health condition of the system after the contagion process for what concerns bank capitalization; the "risk shifting" effects innate to the netting procedure. Three metrics are adopted to compare the benchmark gross settlement performance with the net case performance: default number; capitalization level of the system at the end of the contagion process; impact on external deposits to the bank system. A simple example shall be provided in order to grant a qualitative understanding of the issue addressed by the second contribution. To this purpose, we take into consideration a fictitious financial system, made of three banks – bank A, B and C- linked by interbank credits and debits. The balance sheet of each bank is made of the asset from interbank credits and external assets, the liability from an equity share, external deposits, and interbank debts. As the Figure 5.6 shows, Bank A is tied to bank B and C, which, in turn, are tied to each other.

Figure 5.6. Balance sheets

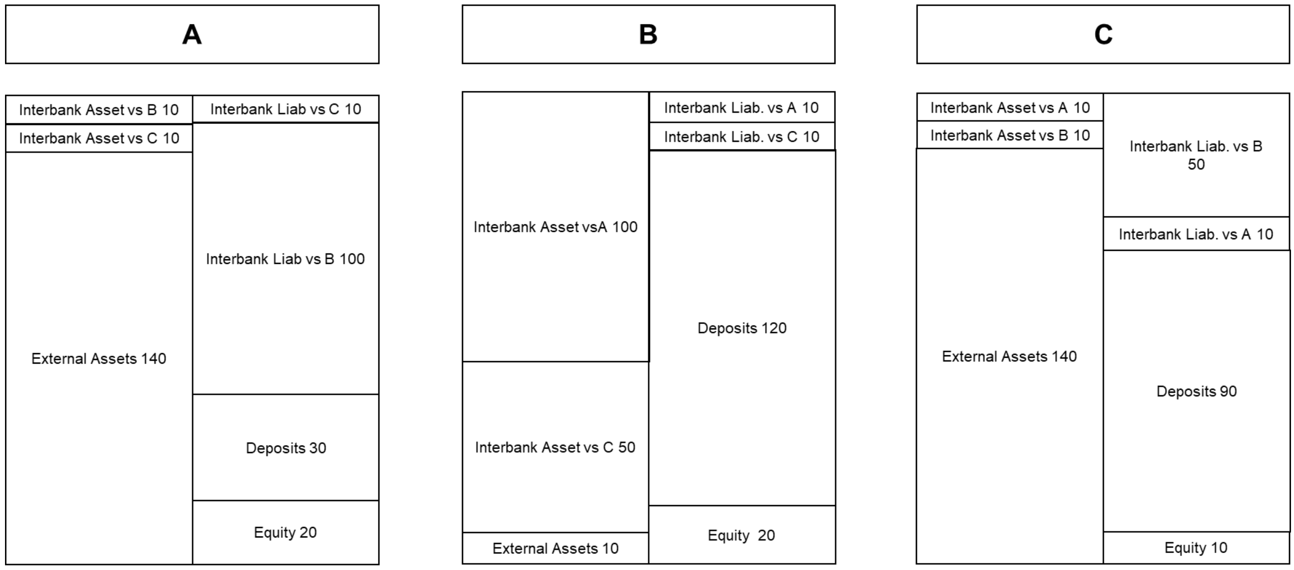


Figure 5.7. Asset shock on Bank A

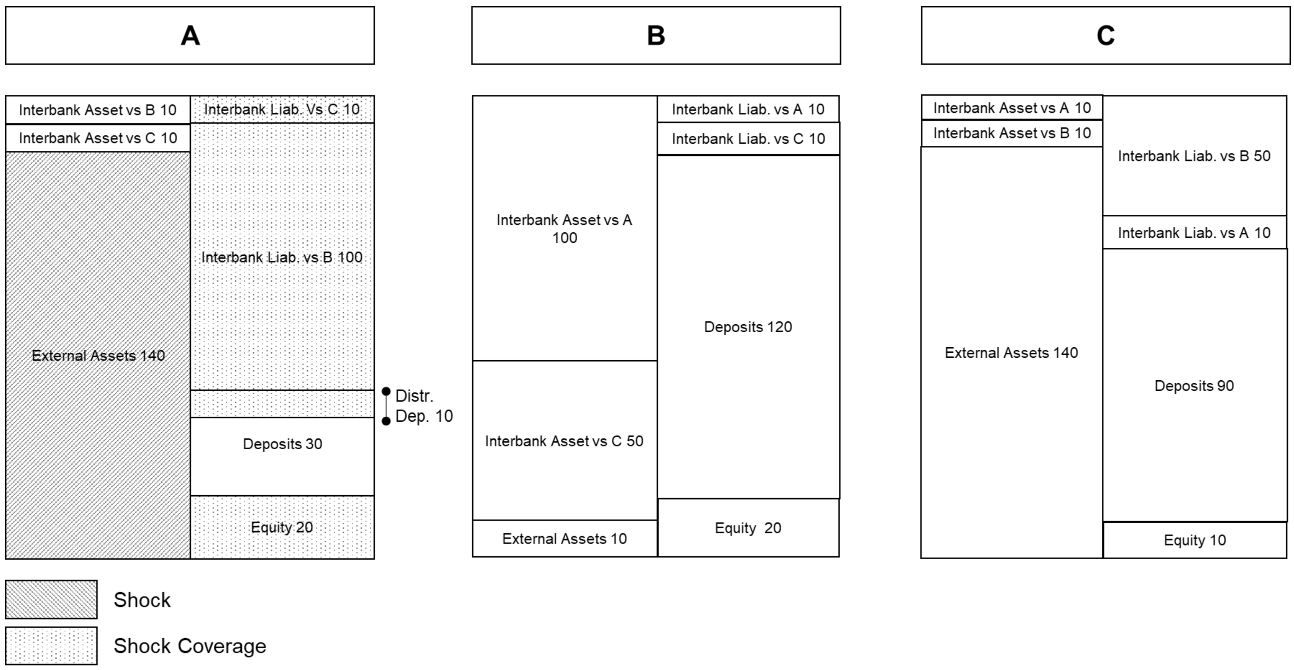
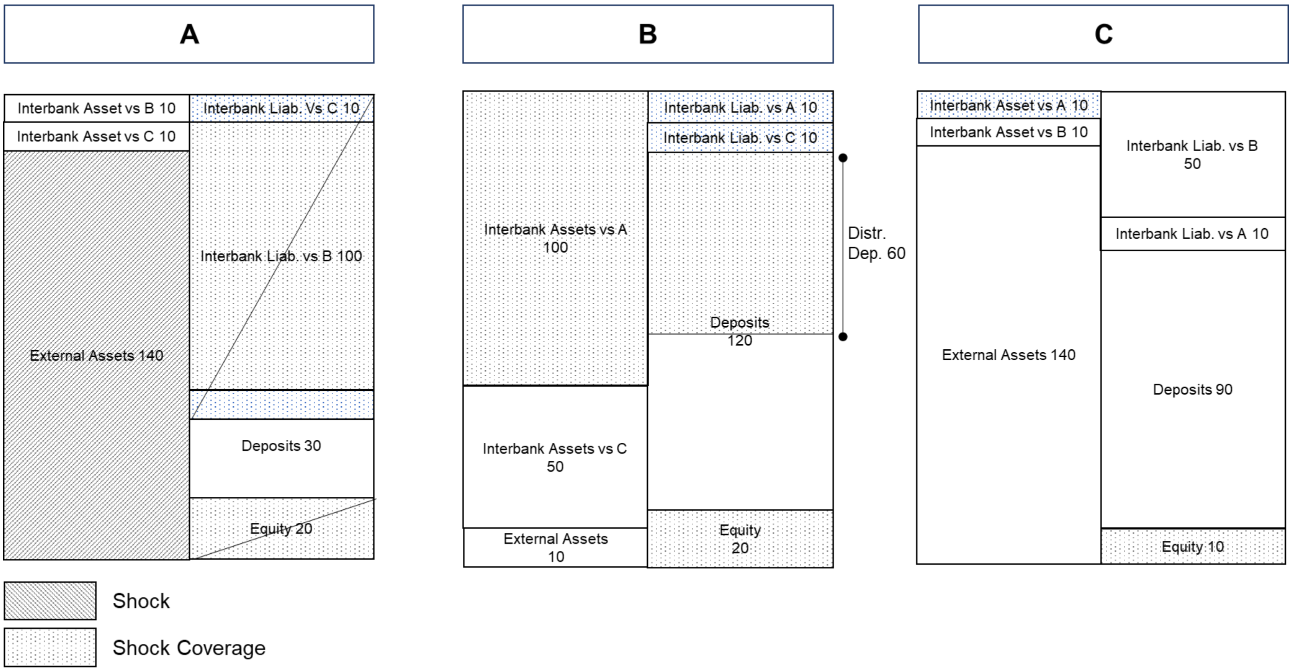


Figure 5.8. Beginning of the contagion



The example shows a shock of hitting all the external assets of Bank A, that is,140. The shock wipes out all the external of the Bank A triggering a default dynamic exactly comparable to the one developed in Nier et al. (2007). Since the shock is higher in value than the amount of the bank's equity, Bank A defaults. The fraction of the shock, which is not absorbed by the capital, will be transmitted to Banks B and C, while a minor part is absorbed by the depositors of bank A (Figure 5.7). This is due to the lower seniority of interbank deposits in comparison to external deposits. At this stage, Bank B and C import losses on interbank credits towards A (Figure 5.8). Given that Bank B and C are not sufficiently capitalized, they will default. Furthermore, B will export a loss towards the other banks of the system. Notice how, in this basic example, the shock is able to cause a domino effect bringing all three banks to default and draining the capital of the entire system. What would have happened if a close-out netting clause among banks was in place? Following the rise of the shock, the implementation of the netting would have isolated each other banks A and C (Figure 5.9). As in the previous instance, Bank A would have gone into default; however, a higher fraction of the loss would have been laid off on the external deposits of Bank A. Notice how the shock distribution amongst the participants differs from the previous case. Bank B would import losses, causing the default of the bank and a higher portion of loss is covered by Bank B deposits (Figure 5.10). Bank C is completely isolated on the asset side thanks to the netting procedure, avoiding the contagion process. In the simple example proposed above, the implementation of the netting clause avoids the defaults of the intermediary C and manages to preserve the capital better than in the standard case. By contrast, those who hold deposits in Bank A and B, would be subjected to losses, which would otherwise be laid off on interbank debts. In this contribution we present an investigation of this issue for different topologies, shock magnitudes, bank sizes, and loan heterogeneities. The paper's originality lies in the attempt to provide a theoretical framework for understanding the subject of bilateral netting in the context of an asset shock. The paper adds to the extant literature which employs network analysis for the study of default cascades. As is the case with the former contribution, also this one shows how policy guidelines in a time of crisis change according to the network topology, to the kind of shock, and to the kind of intermediary which is hit; furthermore, it emphasizes how the regulator's choices, in most cases, are driven by the importance that he attaches to the specific objectives to be met.

Figure 5.9. Netting procedure and shock on Bank A

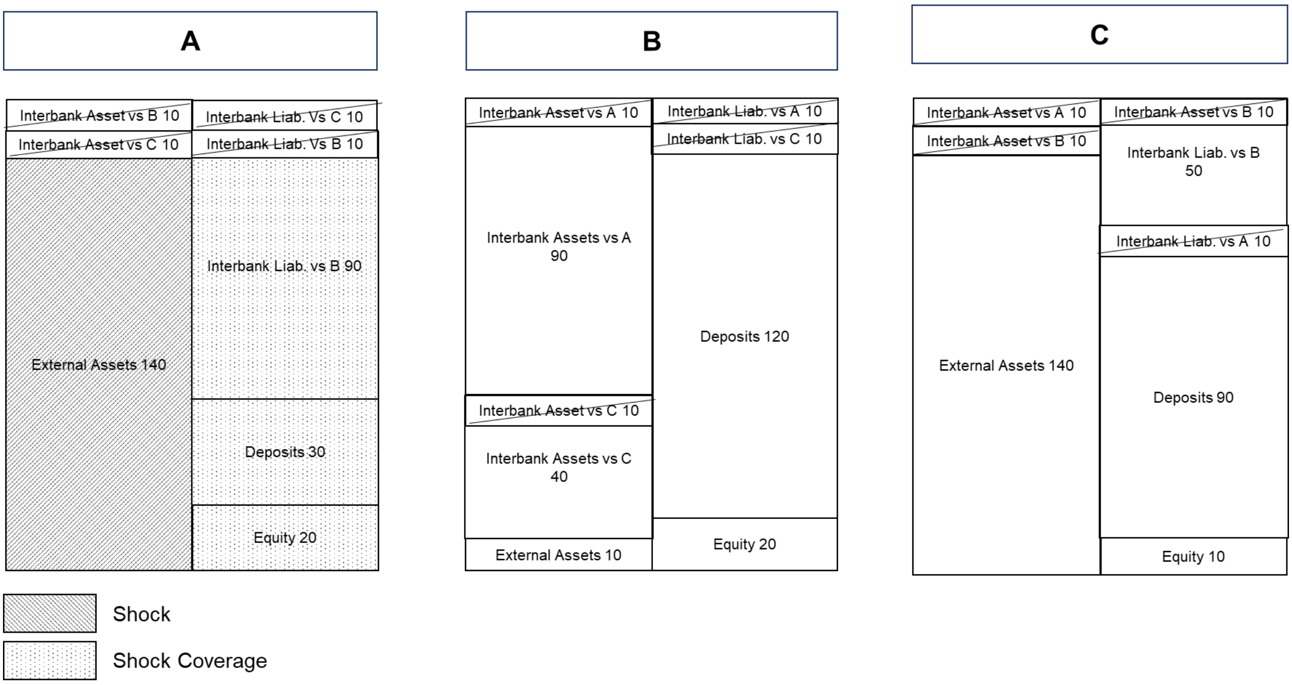
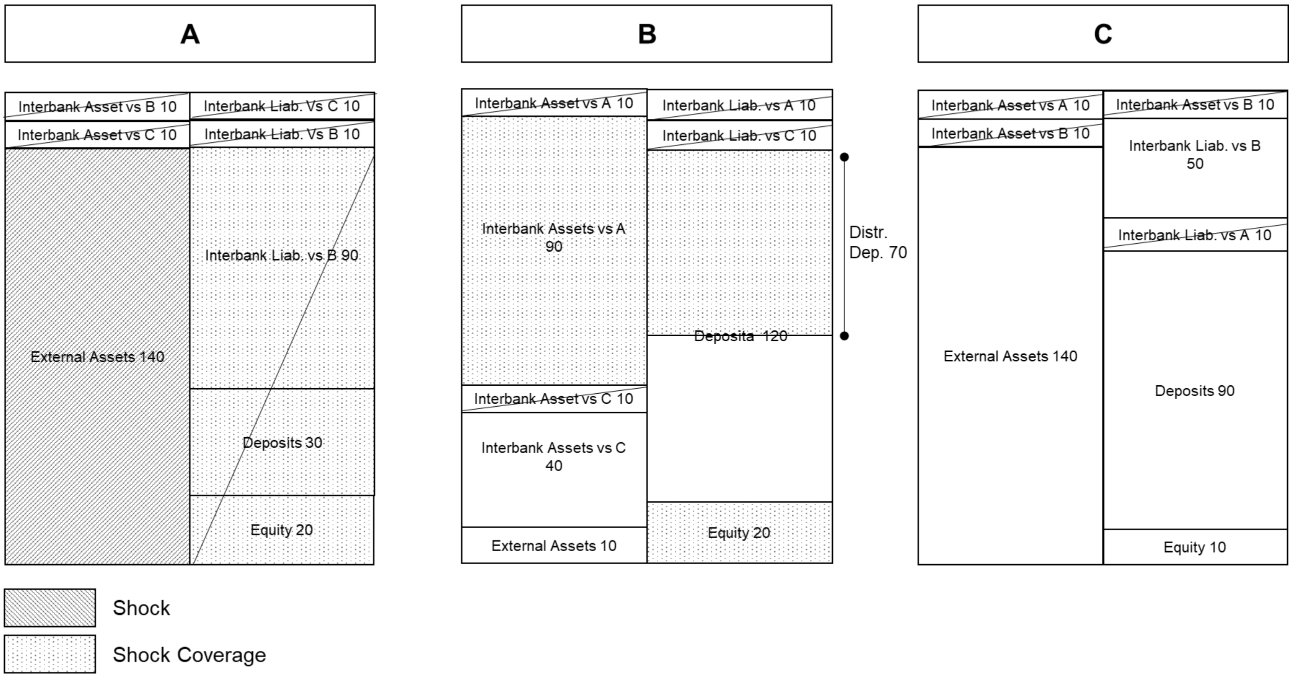


Figure 5.10. Contagion process



4.3. Third Contribution

The third paper focuses on the endogenous formation of clearing houses within payments networks. To this purpose, we develop an agent-based model elucidating what dynamics lead the agents to regulate their transactions through gross settlement, bilateral netting or multilateral netting when these systems are in competition with each other. In this contribution as well, we recognized network analysis as the best-suited methodology to expose the dynamics which informs the contexts under scrutiny. In particular, the contribution attempts to observe the use of multilateral clearing by some company circuits. Notwithstanding, the model can also be employed to provide an explanation of the birth of the first clearing houses, which handled interbank payments in United States in the times of free banking as well as, more generally, in contexts where multiple actors engage in credit and debit relations with each other. The reasons for the use of clearing mechanism are multiple. Typically, the reduction of both liquidity and operational risk in ordinary settlement procedures. A motivation of non secondary importance concerns the fact that clearing allows to add a funding channel to the traditional ones. Reducing the net exposures among counterparties this procedure prevents them for having to collect liquidity on the market or for having to draw on their liquidity stocks.

For real markets, drawing on the experience of clearing houses for commercial credits such as Wir and Sardex, we investigate whether, within a payment network amongst companies, a space for multilateral netting can exist. To this purpose, we made use of the aforementioned agent-based model in which agents are able to decide whether to regulate their transactions in a gross system or in a clearing house. The essential variables for the choice have been: interest rates, credit risk, expectations on interest rates trend, and management costs of the clearing house. Such variables will be applied to various business cycle scenarios, tested for various network topologies of payments and for a spatial friction limiting the coordination amongst agents through Monte Carlo simulations. The results of simulations are indicative of how, upon the presence of coordination frictions, managing the development of clearing houses is challenging, except for specific phases of the business cycle. Thus, the model confirms the uniqueness of the aforementioned experiences in real markets. Subsequently, in order to investigate whether there is a potential incentive to applicate to multilateral clearing, an algorithm is developed that allows to identify the order of magnitude of credits that can be settled within a clearing house given a network of payments. From the outcome of this procedure, notice that for large clearing houses there is a great potential with results which, in the most favorable cases, reach almost all the receivables of the system settled in clearing houses. From the findings of the present study, it appears clear that, despite the existence within the system of a high number of multilateral netting possibilities, which might be preferred by agents to gross settlement, the presence of frictions limiting the coordination amongst agents causes the possibilities of multilateral netting to remain the only options. Nonetheless, considering the combined effect of the recent legislation of payment system (PSD2) and the growth of large platforms for the exchange and sale of goods amongst companies (for example the Amazon B2B sector), it is not difficult to foresee an increase in the use of multilateral compensation for two reasons. The first is that platforms that allow the meeting of demand and supply of goods and services ease the coordination amongst agents. Indeed, the platform manager could easily become manager of a clearing house, similarly to circuits as Wir and Sardex. The second is that the possibility that clearing services could be offered by actors external to the bank channel, could push banks to conduct such activity for fear of a decrease in market shares.

As for the financial payments networks, the model is also suitable for the investigation of the dynamics which led to the formation of the first American clearing houses in the times of Free Banking. To this purpose, we adjusted the model accounting for some of the characteristics of that particular interbank market where the competition between bilateral and multilateral netting systems was born. As in the case of the real payments networks, we took into account a large set of variables and economic scenarios. The results point to a link between the birth of interbank clearing houses and the reduction of transaction and operational costs. The main contribution of this work consists in the addition of a model of endogenous formation of clearing houses to the extant literature on payments systems, whereas settlement methods, overall, have been often considered exogenous variables. With regard to the contexts of study, the paper also adds to the literature on trade credit, for clearing houses amongst companies can also be considered a form of financing in competition with trade credit and with standard banking methods. Finally, this work contributes to the literature on monetary policy, for the clearing houses arisen in the times of the American Free Banking can be seen as forerunner institutions of the contemporary central banking system.

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Bilateral netting and systemic liquidity shortages in banking networks*

Edoardo Gaffeo[†] Lucio Gobbi[†] Massimo Molinari[‡]

Abstract

The cross holding of interbank deposits represents an optimal ex-ante co-insurance arrangement whenever the uncertainty concerning banks' liquidity needs is idiosyncratic and imperfectly correlated. When a shock to aggregate liquidity demand occurs, however, such an arrangement could be detrimental – depending on the topological structure of interlinkages - as financial exposures become a means to spread risk. If the *ex-post facto* is an excess demand for liquidity, therefore, regulators could sever potential channels of contagion by forcing banks to net their mutual debt obligations. Starting from these premises we employ simulation techniques with simple interbank structures to obtain two results. First, a state-contingent mandatory policy to bilaterally net mutual interbank exposures comes with a trade-off between the benefits of thwarting the channels of contagion and the harms of a greater concentration of the remaining netted expositions. Second, the balance between the two prongs of the trade-off depends on the metric used by regulators to define financial stability and the topological structure characterizing the interbank market.

1. INTRODUCTION

Due to their peculiar role in the financial plumbing and a vast availability of ready-to-use data, interbank lending markets have recently become a hot topic for theoretical and empirical research. Among the many facets explored by the literature[7], the one we select as our starting point is the role of lending and borrowing in interbank markets as a form of liquidity co-insurance. We shall argue that the implications of this theoretical view are conducive to a clear-cut crisis management prescription aimed at curbing a contagion, namely to mandate banks to net their mutual lending exposures on a contingent basis. Exploring the pros and cons of such a proposal is the main goal of this paper.

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[*] The authors benefitted from the comments and suggestions by participants to the 22[nd] Annual Workshop on Economic Science with Heterogeneous Interacting Agents, the 5[th] International Workshop on Computational Economics and Econometrics and 5[th] Finest Anniversary Conference. The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Bank of Italy. The usual waiver of liabilities applies.

[‡] Bank of Italy.

[7] A search on Google Scholar for the keyword "interbank market" limited to the time span following the global freeze of mid-2007 returns more than 23.000 items. We refer the reader to the surveys by Hüser (2015), Glasserman and Young (2016) and Green *et al*. (2016) for discussions focused on different aspects

In doing so, we offer two contributions to the research exploiting numerical simulation techniques to model financial systems as interacting networks[8]. First, we complement the stream of papers dealing with liquidity cascades (Cifuentes et al., 2005; Gai et al., 2011; Lee, 2013; Chinazzi et al., 2015) by exploring how the bilateral netting of mutual claims affects the resilience of banking systems to funding runs. Second, our work centers on the crisis management strategy a regulator can activate in fostering financial stability. While the standard approach in the network literature (Aldasoro et al., 2017; Erol and Ordoñez, 2017) is to analyze how regulation at the level of nodes – through the imposition of capital and liquidity ratios, or the periodic submission of "living will" plans – can implement a safer financial environment by focusing on crisis prevention, we argue that the complementary goal of managing systemic liquidity crisis could be pursued by regulatory tools affecting the structure of links connecting intermediaries among themselves, along the lines discussed by Loepfe et al. (2013) with regards to asset-based overlapping risk exposures.

As shown e.g. in Allen and Gale (2000) and Freixas et al. (2000), by mutually exchanging funds, banks may hedge themselves against liquidity shocks arising from uncertainty in the timing or geographical destination of their depositors' consumption. Provided that the idiosyncratic risks they face are not perfectly correlated, the ex-ante swapping of claims allows banks experiencing liquidity deficits to be funded by banks experiencing liquidity surpluses ex-post[9]. The unpleasant side of the story is that, while such an arrangement turns out to be optimal whenever the total amount of liquidity to be reallocated inside the system is large enough (Castiglionesi and Wagner, 2013), it could prove deleterious when a negative shock to the aggregate demand for liquidity occurs. Interbank deposit exposures can now act as shock transmitters and lead to a spread of losses through contagion-like liquidity cascades over the whole net of lending/borrowing relationships. A major result – corroborated by means of diverse analytical methods – is that in this case the resilience of the system is a function of the topological structure characterizing interconnections[10]. Furthermore, the interaction between the severity of the systemic shock and market connectivity is crucial in determining the ability of the system to resist contagion: for small aggregate shocks higher connectivity helps to restrict the likelihood of default cascades, but for larger shocks the opposite holds true (Ladley, 2013; Cabrales et al., 2017).

A far less explored implication of the co-insurance motive for reciprocal exposures in interbank markets is that once an aggregate liquidity shock has occurred, the ex-ante rationale to exchange deposits may lose significance. As emphasized by Haldane (2009), a sensible approach to avoid contagion is in this case to "hide". In addition to liquidity hoarding (Gai and Kapadia, 2010; Heider et al., 2015), a possible "hiding" strategy when mutual deposits exist is to net them bilaterally. This helps a safe node to insulate itself from negative spillovers mediated through a withdrawal of the claims infectious banks have deposited in its coffer. Furthermore, bilateral netting should not imply any welfare loss besides that which is generated by the failure of the institutions originally affected

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[8] See, among the others, Nier *et al*. (2007), Gai *et al*. (2011), Krause and Giansante (2012), Anand *et al*. (2013) and Gaffeo and Molinari (2015).

[9] An optimal co-insurance scheme can be obtained also in a circular arrangement in which each bank is allowed to make an interbank deposit only in the adjacent bank, so that all relationships are unidirectional. See the incomplete market structure represented in Figure 2 of Allen and Gale (2000). However, note that in this case an interbank market ensures optimal risk-sharing only if the chain of lending relationships is such that banks receiving liquidity shocks of different signs are located alternately. The informational requirement for this arrangement to work is much more demanding than the one required when banks exchange claims mutually. A bank has not only to be informed about the distribution of the idiosyncratic liquidity shocks in each state of the world, but also on the exact position of any single bank in that distribution. The point goes normally unnoticed.

[10] Allen and Gale (2000), Battiston *et al*. (2012), Elliott *et al*. (2014), Acemoglu *et al*. (2015).

by an exogenous disturbance given that, from a post-shock point of view, the reciprocal exchange of deposits is essentially counterproductive.

Starting from these premises, in this paper we study the implications of mandating banks swapping deposits to enter into on-balance-sheet (OnBS) netting agreements[11], in order to assess whether this tool could successfully complement the lending-of-last-resort measures a central bank customarily deploys to contain a systemic financial crisis (Cecchetti and Disyatat, 2010).[12] An OnBS netting deal is a bilateral arrangement that allows two parties to offset the mutual debt obligations they have registered on their books. By signing the new agreement, the two pre-existing gross claims are contractually replaced by the single net amount that the net-debtor owes to the net-creditor.

Under the regulatory framework outlined in the Basel Accords, such arrangements are permitted on a voluntary basis as privately stipulated contracts, while the rulebooks currently in place focus on the manner in which the net exposure resulting from a close-out netting effective upon a default[13] has to be accounted for, how liquidity and capital requirements must be adjusted once the triggering event occurs, and under what conditions regulators are allowed to delay automatic close-out and termination rights to facilitate the orderly resolution of a distressed bank. While from a micro-prudential perspective it can be easily envisaged that OnBS netting arrangements end up affecting the seniority structure of debt, the implicit insurance against risks associated to borrowing and lending, and the drawing of internal limits for counterparty credit risk, their possible macro-prudential implications represent an almost uncharted territory.

In fact, the literature investigating the effect of OnBS bilateral netting on the resilience of interbank lending networks is not only small, but it also returns discordant outcomes. Upper and Worms (2004) use data on the German interbank market to argue that the bilateral netting of mutual exposures could shrink the aggregate loss in asset values during an episode of crisis from 76% to 10%. However, results for Belgium (Degryse and Nguyen, 2007) and the U.K. (Duan and Zhan, 2013) suggest a much weaker effect in reducing contagious defaults, whilst Eisinger et al. (2006) show that for Austria the consequence of introducing a bilateral netting arrangement is virtually irrelevant. As argued by Upper (2011), it is not clear whether this combination of mixed results have to be traced back to undetected differences in market microstructures, alternative hypotheses on the conditioning events, or the methodologies with which missing data on balance-sheet variables and unobserved linkages are obtained.

In order to explore the aptitude of interbank netting arrangements to tame systemic disruptions, we develop a financial network model suitably amended to make room for the possibility that mutual claims could indeed be netted on a bilateral basis. Building upon the algorithm for calculating total changes in liquidity proposed by Lee (2013), we use the model to study how funding shocks spread and get amplified through interbank linkages under alternative settlement arrangements. Our contribution considers shocks large enough to trigger a liquidity crisis. Given the size of the shock we try to investigate whether its degree of dispersion has an impact on the severity of the crisis. In our framework, liquidity shortages first arise when banks experience an unforeseen run on their

[11] We use this expression to distinguish our topic from the much more explored issues of netting interbank off-balance-sheet exposures associated to OTC derivatives on the one hand (Bliss and Kaufman, 2006; Cont and Kokholm, 2014; Garratt and Zimmerman, 2017), and netting settlement agreements in payment systems on the other one (Angelini *et al*., 1996; Freixas and Parigi, 1998)).

[12] As will become clearer below, in our model the central bank counteracts a major liquidity turmoil through outright purchases of illiquid assets in the open market.

[13] In a close-out netting agreement two parties agree that in the case of default of one of them, the obligations they have on each other are immediately terminated, the termination values are calculated and netted to arrive at a net single amount.

deposits. This exogenous funding shock forces buffeted banks to recall their interbank assets and deploy their liquid reserves (if available) to meet their depositors' demand for funds. A randomly chosen bank can hence be hit directly by a deposit shock or indirectly by the additional funding shock(s) imposed on it by other banks seeking to recover some additional liquidity in the interbank market. For this reason, ultimate liquidity needs might exceed the initial exogenous shock and bring about systemic liquidity shortages that could pose a threat to the whole system. Illiquidity driven disruptions are recorded whenever a bank faces liquidity needs in excess of its liquid assets (i.e. the sum of its interbank assets and liquid reserves set aside to comply with regulatory requirements).

We admit the possibility for a supervisory authority to regulate these operations by selecting the banks forced to activate OnBS netting procedures. Five experimental treatments are explored. The first is a benchmark case in which netting is simply prohibited, so that contagion dynamics unravel in a network where all interbank exposures are gross. A full netting process characterizes the second case, where all banks are required to net their mutual claims regardless of whether they are experiencing a run on their deposits or not. We then test three hybrid treatments, in which a partial netting process is implemented so that netted and gross interbank claims coexist. In our third case, OnBS netting is required only among the banks that do face a run on deposits. In the fourth case, the banks hit by an exogenous liquidity shock are forced to activate a netting settlement with safe counterparts only. Finally, in the fifth treatment a netting of mutual exposures is compulsory for non-affected banks only. The three latter policy options respond in varying degrees to alternative policy strategies aimed at limiting contagion, that is the attempt to isolate infected or susceptible nodes, respectively.

We evaluate how a mandatory enforcement of these netting protocols affect the performance of the interbank market under different scenarios on the size and distribution of exogenous liquidity shocks, as well as on the topological structure of interlinkages. As regards shocks, we first assume that only a small number of banks experience a hefty run on their deposits. We then take the very same shock in aggregate terms and gradually spread it across a larger number of banks (each therefore suffering a smaller deposit withdrawal), ending with the case in which the run affects uniformly all banks. To accommodate the possibility that results could be driven by the topological architecture of interbank relationships, in addition to random Erdős–Rényi networks, we perform simulations also for small-world and core-periphery structures. Finally, recognizing that the very notion of financial stability is multifaceted, the post-shock resilience of the system is evaluated along several – potentially conflicting – dimensions, namely the fraction of banks involved in the contagion, the ensuing systemic shortfall of liquidity, the distance to a potential condition of illiquidity of the banks that during the crises succeeded in preserving a positive liquidity position, and the volume of interbank transactions when the crisis comes to a halt.

Our simulations make clear that the relative benefits and disadvantages of the netting regulatory option are firmly rooted on a typical "risk-sharing vs. risk-spreading" trade-off. On the one hand, it decreases the number of channels through which first- and higher-round infections spill over through the network. This implies that the potential for contagion can be reduced in proportion to the total value of the mutual obligations that are offset. On the other hand, it contributes to concentrate losses not only on the banks hit by the original shock, but also on the ones whose original stock of unbalanced interbank positions disallow them to insulate. In other words, the benefit of reducing a spreading of contagion comes with an increase of the risk absorbed by the banks unable to fully disconnect themselves from the system.

We find that the balance between these two forces depends critically on the topological structure of the market. If the interbank market is characterized by a core-periphery architecture, substituting the standard gross settlement mode with a mandatory netting one does not provide on average significant advantages – although it does not do worse either – when the supervisory welfare criterion is defined in terms of the number of banks recording a liquidity strain, the total amount of liquidity shortage in the system, or the depth of the interbank market. The netting option works systematically better, however, when the policy target is that of assuring that the system comes out of the crisis in the best possible shape to face future negative shocks. Once characterized as a multiobjective problem on a suitably defined criterion space, therefore, a contingent-based mandatory netting scheme represents an optimal solution to pursue the multidimensional goal of financial stability. The actual magnitude of the Pareto improvement depends on the size and the distribution of the initial shock.

For core periphery and small-world topological structures, however, the netting option represents an efficient crisis management tool as soon as the central bank targets the distance to illiquidity of survived banks or the post-contagion trading volume in the interbank market, but it performs significantly worse than the gross settlement mode whenever the policy goal is that of minimizing the shortfall of aggregate liquidity. In these cases, a policymaker faces a conundrum that can be resolved only if the several dimensions substantiating the broad concept of financial stability are organized hierarchically.

The remainder of the paper is as follows. Section 2 offers a brief introduction to OnBS netting arrangements in light of the current regulatory environment and discusses how they could be related to the issue of financial stability at large. Section 3 introduces the model. Section 4 illustrates results for the case of random interbank networks. Section 5 extends the analysis to alternative topologies. Section 6 concludes.

2. NETTING AS A SYSTEMIC CRISIS MANAGEMENT TOOL

The plausibility of the theoretical explanation for the existence of interbank markets based on mutual insurance resides in the empirical salience of bilateral deposit exchanges among banks. In the parlance of network theory, the scope for co-insurance can be measured by assessing how many nodes in a directed graph are linked to each other by reciprocal linkages. This measure – called reciprocity - can be calculated at least in two ways. First, by brute force as the ratio between the number of links pointing in both directions $L^{\leftrightarrow}$ and the total number of links L, so that *r* represents the average probability a link is reciprocated.

$$r = \frac{L^{\leftrightarrow}}{L} \qquad (1)$$

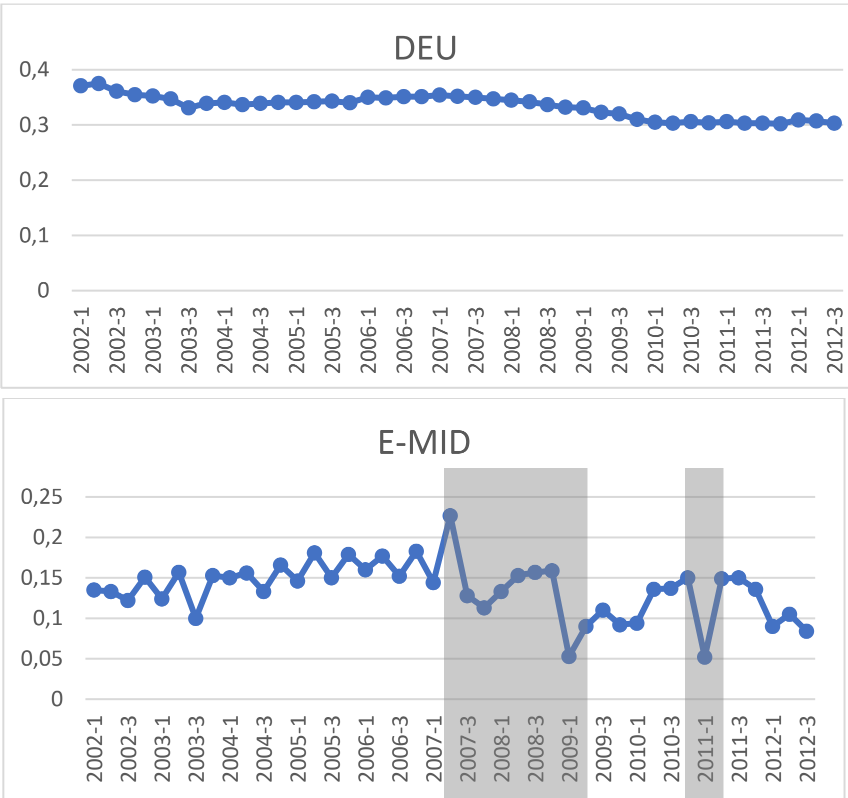
In order to overcome problems of scale and double-counting of self-loops potentially affecting *r*, a second metric developed in the literature is the correlation coefficient ρ between the entries (given by *aij* = 1 if a link from i to j exists, and *aij* = 0 otherwise) of the associated adjacency matrix. Is definded by (2)

$$\rho = \frac{\sum_{i\neq j}(a_{ij}-\bar{a})(a_{ji}-\bar{a})}{\sum_{i\neq j}(a_{ij}-\bar{a})^2} \qquad (2)$$

where $\bar{a} = \frac{\sum_{i\neq j} a_{ij}}{n(n-1)}$ is the average density and n is the total number of nodes in the network (Garlaschelli and Loffredo, 2004).

Regardless of the measure one might employ, real interbank networks tend to display a substantial amount of reciprocity. Figure 2.1 reports the value of r for the Euro-area e-MID market (Brandi et al., 2016), and of ρ for a sample of large bilateral liability exposures among German banks (Roukny et al., 2014), in both cases measured at a quarterly frequency over the time span 2002:Q1-2012:Q3. While the degree of reciprocity observed in these interbank markets has decreased in the aftermath of the 2007-09 global financial crisis, the scope for closing down possible channels of contagion through the bilateral netting of mutual exposures appears significant. The point is generally recognized by practitioners and trade associations, and both communities have long advocated for an extensive use of bilateral netting agreements as a means to mitigate counterparty risks and improve the liquidity of financial markets (British Banking Association, 2002; Mengle, 2010).

Figure 2.1. Upper graph) Values of $\rho = \frac{\sum_{i \neq j}(a_{ij}-\bar{a})(a_{ji}-\bar{a})}{\sum_{i \neq j}(a_{ij}-\bar{a})^2}$ for the German interbank market (DEU) over the period 2002:Q1-2012:Q3. Lower graph) Values of $r = \frac{L^{\leftrightarrow}}{L}$ for the Euro-area e-MID market (E-MID) over the period 2002:Q1-2012:Q3. Sources: Roukny *et al.* (2014) and Brandi *et al.* (2016), respectively.



It is somehow surprising, therefore, that among the set of tools that regulators are envisaged to deploy to mitigate the cross-sectional dimension of systemic risk, the option to curb externalities by mandating some kind of netting between the cross-holdings of interbank deposits is virtually absent[14].

[14] See e.g. Bank of England (2009), van der Zwet (2011), Schoenmaker and Wierts (2011), Dewatripont and Freixas (2012) and Constâncio (2014), where the issue of netting is totally neglected.

Sections 401-407 of the United States' Federal Deposit Insurance Corporation Improvement Act of 1991, for instance, admit and recognize the benefits of netting the payment obligations that financial institutions hold on behalf of their clients, but remain silent on the admissibility and usefulness of netting the deposits that banks exchange among themselves on the Fed Funds market. In turn, the legal framework of the European Union guarantees a legal protection only to privately-signed interbank bilateral close-out netting agreements (Directive 2002/47/EC, as amended by Directive 2009/49/EC), but for specific provisions that give regulators the power to delay their enforcement in order to guarantee the well-ordered resolution of a distressed bank (Banking Recovery and Resolution Directive 2014/59/EU).

While close-out netting agreements can be conceived as a tool to limit the first round of a contagion[15], we argue that a generalized enforcement of bilateral netting also among initially unaffected banks could help in mitigating higher-round effects, and therefore increase the resilience of the whole system. Specifically, this could be achieved by allowing a regulator to mandate the use of a novation netting agreement – according to which two parties cancel their mutual obligations out, and simultaneously replace them with a new obligation for the net amount – to a pre-defined set of interconnected banks as soon as the risk of a liquidity cascade materializes.

A major problem confronting regulators in the wake of a systemic financial crisis is that of designing a coherent structure encompassing a multitude of agencies, instances and constraints. Defining the boundaries of the mandate to mitigate the systemic risk they received is probably the most compelling task. Since financial stability is a highly multidimensional concept (Schinasi, 2004; Borio and Drehmann, 2009) and there is a potential for conflict between different dimensions, any operational framework supporting the prevention and management of financial crisis delivers, in fact, an array of working objectives. For instance, the Directive 2014/59/EU is explicit in recognizing that the set of tools available for recovering and resolving unsound or failing banks should aim to "[…] *prevent insolvency or, when insolvency occurs*, […] *to preserve the systemically important functions of the institution concerned*, […] *while minimising the impact of an institution's failure on the economy and financial system*".[16]

A crucial point to be addressed below is therefore that of defining a list of policy objectives a contingent-based mandatory netting scheme should aim for. Taking stock of the goals major central banks around the world declare to pursue in promoting financial stability (Jeanneau, 2014), we will assess the diverse netting options against four different dimensions of social welfare: i) To limit the number of units involved in the contagion; ii) If a contagion is unavoidable, to limit the strain of liquidity for the system as a whole; iii) To safeguard the financial health of safe banks; iv) To ensure that the interbank market does not come to a freeze.

The following steps will outline a set of corresponding metrics – a task we shall leave to the next Section – and an evaluation of the trade-offs that the policymaker faces when different welfare objectives return contrasting results. Once defined in this way, in fact, the main goal of financial stability can be immediately translated into a multiobjective decision-making problem (Miettinen, 1999; Chankong and Haimes, 2008).

[15] Recall that close-outs come into force just upon a default triggering event and can thus be exercised only by the counterparties directly exposed to the defaulted entity.
[16] See Sections 1 and 5 of the BRR Directive.

In formal terms, the problem can be expressed as follows. Let a high-level performance criterion be served through the minimization of the function:

$$F^s(z) = min[f_1^s(z), f_2^s(z), \cdots, f_m^s(z) | z \epsilon Z], \quad (3)$$

where $m \geq 2$ is the number of possibly conflicting objective functions $f(.)$, $z \in Z$ defines the action space, and $s \in S$ indexes the topology/microstructure scenario in which the optimization takes place. In our case, the high-level performance criterion is financial stability at large, the individual objective functions are the four dimensions of social welfare recalled above, while the set of policy actions comprises the alternative schemes for regulating mutual claims a regulator can choose from.

For any given structural scenario, an action z' Pareto dominates another action z'' if the two following conditions are met:

1. For all objectives, $f_i(z') \leq f_i(z'')$ $\forall$ $i \in 1,2,\ldots,m$.
2. For at least one objective, $f_j(z') < f_j(z'')$ $\exists$ $j \in 1,2,\ldots,m$.

This situation is particularly fortunate, as an optimal solution for the high-level performance criterion exists irrespective of the relative importance the policymaker attributes to individual objectives. If this occurs, a financial crisis can be successfully managed by means of a single policy tool, ensuring that all the many facets of financial stability can be composed into a coherent picture.

If either of these conditions are violated two alternatives are possible. On the one hand, finding a Pareto-optimal solution requires the identification of ways to prioritize conflicting objectives, so that the multiobjective problem can be suitably transformed into a single-objective one admitting a Pareto frontier. On the other hand, the problem has to be solved by means of a multi-objective optimization method able to identify the most effective policy proposal without having to create a priority order. In this work we decided to opt for the second option. Among the popular mutli-objective maximization methods we chose the M.O.O.R.A method, where the acronym M.O.O.R.A stands for Multi-Objective Optimization on the basis of Ratio Analysis (for an economic application of this method see Brauers and Zavadskas 2006). Thanks to this methodology we managed to identify which netting policy proposal was the best one for the majority of the cases tested.

The use of this method has the merit of considering the financial stability as a complex problem, the main limit of the approach consists in the attribution of the same weights to each financial safeguard objective.

From the current regulatory context there are no clear indications in terms of priority of objectives, even if the passage from the Directive 2014/59/EU reported above seems to suggest that regulators are aware of the necessity to explicit their priorities when managing a crisis. In this normative contest the application of the M.O.O.R.A method seems to us to be the more appropriate. Moreover, we are sure that approaching the issue of financial stability as a multiobjective decision-making process yields a fresh new perspective to the study of the optimal regulation of financial markets, a point that will be taken up repeatedly below.[17]

[17] A similar view is advanced in Nier (2009), where the multidimensionality of financial regulation is referred to the coexistence of the two main goals of consumer protection and systemic risk mitigation. We claim that the even latter – i.e., financial stability writ large – is a multidimensional object in itself.

3. THE MODEL

In this Section we describe how artificial financial networks with different topologies are built, the various settlement arrangements we study, the algorithm employed to calculate systemic liquidity shortages when contagion occurs, and the metrics used to assess the post-shock behavior of the interbank network. Our simulations advance according to a standard sequential procedure. At the beginning we characterize the network of interconnections among banks, as well as the assets and liabilities booked in their balance sheets. Then, we hit one or more banks by means of an exogenous shock to the liability side of the balance sheet. Finally, we apply different settlement treatments and comparatively assess the network's performance along several dimensions.

3.1 Set-up of the network

The simplest way to computationally derive the network structure of an interbank system is to build a directed adjacency matrix using two main drivers: the number of links pointing out from each node, and the total number of banks. The first driver is captured by the probability p_{ij} that a credit obligation between bank i (depositor) and bank j (debtor) – defined as 1_{ij} - happens, with $i \neq j$ and $i, j = 1, \ldots, N$. This operation returns, for each node i, the set of its borrowers Λ_i. If also 1_{ji} occurs, the possibility to net the two mutual expositions arises. The benchmark structure we consider is an Erdős–Rényi random network, where each node has a uniform attachment probability. In order to assess the role of market interconnectedness in propagating liquidity shocks, in what follows we will allow p to vary. In turn, the size of the network is invariably given by $N = 25$.[18]

Once the non-weighted network of relationships among banks has been shaped, the second step consists in assigning to each intermediary a stylized balance sheet consistent with a double-entry bookkeeping system (Table 3.1).

Table 3.1 Stylized balance sheet and identification analytic.

Assets	**Liabilities**
Interbank loans il_i	*Interbank borrowings* ib_i
External liquid assets q_i	*External deposits* d_i
External illiquid assets z_i	*Equity* e_i
Total assets= $il_i + q_i + z_i$	*Total liabilities* $ib_i + d_i + e_i$

[18] This is the number used in the pioneering work of Nier *et al*. (2007). We refer to Gaffeo and Molinari (2015) for a discussion aimed at justifying this choice.

Identification analytic
Number of banks, n
Total Interbank assets, IA
Total assets, $TA = IA/(1 - \omega)$; $(0.5 < \omega < 1)$
Total External assets, $EA = TA - IA$
Interbank loans of Bank i, $il_i = IA/n$
Number of borrowers of bank i, n_i
Loan of Bank i to j, $x_{ij} = il_i / n_i$
External assets of Bank i, $EA_i = EA/n$
Equity, $e_i = (EA_i + il_i) * \gamma$
Interbank borrowings $ib_i = \sum_{J \in n} x_{j,i}$
External deposits $d_i = (EA_i + il_i) - e_i - ib_i$
External liquid assets, $q_i = \delta * d_i$
External illiquid assets $z_i = EA_i - q_i$

On the asset side, bank *i* is endowed with an amount of illiquid external assets z_i, liquid assets q_i and interbank lending il_i. On the liability side, in turn, each bank counts interbank borrowing ib_i, deposits d_i and tangible equity e_i. By construction, the sum of the components on the asset side and of those on the liability side are equal ($il_i + q_i + z_i = ib_i + d_i + e_i$). We will assume that banks, when facing withdrawals, will recover funds by selling liquid external assets and interbank assets in constant proportion first. Only after all these assets are fully depleted, they can recur to illiquid external assets.

These latter can be liquidated at face value by selling them to the central bank, who consequently expands its balance sheet to accommodate the banking sector's demand for extra liquidity by boosting the level of cash reserves. In the wake of the crisis, therefore, the central bank operates as a lender of last resort to avoid fire sales of assets and backing market functioning. The actual tool used to inject liquidity, which we left un unmodelled in what follows for simplicity's sake, is assumed to be an unsterilized outright asset purchasing program assimilable to the *quantitative easing* policy adopted by several central banks around the world in response to the 2007-09 global financial turmoil and the ensuing Euro-area sovereign debt crisis.The presented dynamic is the same as that developed by Lee (2013). In this way our work fits into that line of literature that uses fixed point algorithms to investigate crises in static credit networks. The functioning Lee's algorithm is presented in section 3.3. Given that the actual monetary policy of many central banks is still far from normalized and that the history of financial crises teaches us that the crisis management tools used tend to enter into the range of policy options also for subsequent crises, we consider Lee's framework as employable although we are aware of the fact that this algorithm is not able to map all the economic contexts.

In order to populate the balance sheet of each bank, we start by exogenously fixing the amount of interbank assets *IA* and we assign an equal portion of them to each bank, $il_i = IA/n$. We then obtain the amount of total assets as $TA = IA/(1 - \omega)$, where $0.5 < \omega < 1$, as well as the corresponding amount of external assets as $EA = TA - IA$. The value of each single loan that bank *i* extends to a counterpart *j* (x_{ij}) is finally achieved by dividing il_i by the number of borrowers it is linked to. We can thus represent the interbank network by means of a weighted matrix X, where the horizontal summation returns $il_i = \sum_{J \in n} x_{i,j}$ and vertical summation $ib_i = \sum_{J \in n} x_{j,i}$. Note that interbank loans are slightly heterogeneous, given that the number of interbank debtors each bank has is potentially different. The stock of tangible equity of each bank is computed by multiplying the corresponding total assets by a parameter $0 < \gamma < 1$ measuring its leverage ratio. By difference we can then compute the amount of deposits, subtracting from the amount of total assets the value of each interbank borrowing plus the amount of equity. Finally, to obtain the level of liquid external assets (q_i) we multiply the volume of deposits by a parameter $0 < \delta < 1$, and we consequently decrease external assets to make sure that the double-entry bookkeeping holds. This guarantees that the network is fully identified, and individual balance sheets are consistent with aggregates.

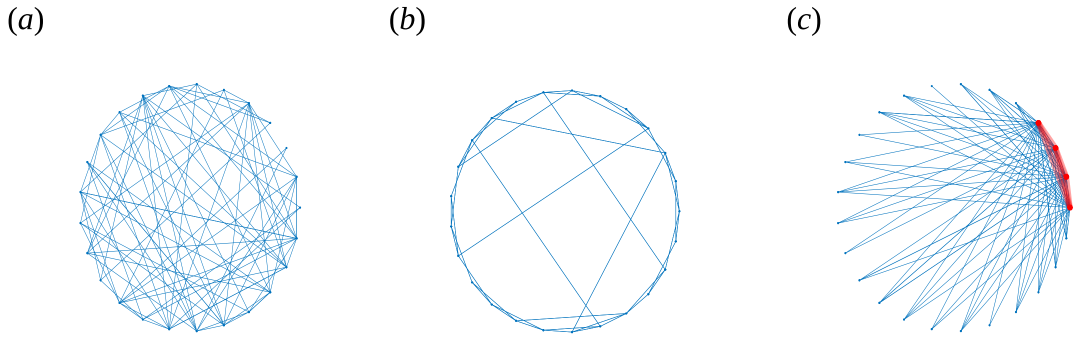
3.2. Alternative topologies

A quickly expanding empirical literature has pointed out that real interbank markets tend to display small-world and disassortative mixing structures in which low degree nodes form links with high degree nodes (Iori *et al*., 2008; Craig and von Peter, 2014; Fricke and Lux, 2015). As a robustness check we therefore extend our analysis to two topological architectures different from the purely random Erdős–Rényi case. Figure 3.1 illustrates comparable instances of the three structures we will deal with in simulations, from which the main features of the degree distribution characterizing each one of them can be appreciated.

The first alternative topology we take into consideration is a network exhibiting small-world properties. This topology is characterized by the fact that the nodes belonging to the financial system can be reached by others by means of a small number of intermediate steps. The generating procedure of small world network is based on the operation called re-wiring. Starting from a circular structure, the re-wiring consists in deleting some links within the network and then reassign them randomly in order to reduce the number of average steps to reach other nodes of the network. Here, starting from a lattice ring characterized by two connections per node to which we apply a β re-wiring level. The β parameter assumes a range of values between 0 and 1, in the first case the re-wiring is absent and therefore the starting lattice ring does not undergo variations, in the second case all the links undergo re-wiring and therefore the topology is identical to an Erdos Renyi network.

An additional disassortative structure is obtained by dividing the bank population into two samples, namely a core and a periphery. The banks in the core (*hubs*) are not only more connected, but also bigger than the peripheral ones. We model this architecture by choosing the number of hubs (N_{large}), so that the number of banks in the periphery is obtained by difference ($N_{small} = N - N_{large}$). We then assign a probability $p_s = 0$ to the links among peripheral banks, a probability p_l measuring how a hub connects to all kinds of banks, and a probability p_{sl} regulating the attachment between small and hub banks.

Figure 3.1 Different topologies for the lending (summation over rows) portion of the interbank market. (*a*): Erdős–Rényi network, with a common probability of attachment $p = 0.2$; (*b*): Small-world network; Lattice ring with 2 connections per node and β=0.3 (*c*): Core-periphery network, with probabilities of attachment for core and periphery banks equal to $p_l = 0.8$ and $p_{sl} = 0.2$, respectively. Core banks are red coloured.



In order to allocate a balance sheet to each node we exogenously fix the benchmark size of three different types of interbank loans. The bigger one characterizes lending contracts among hubs, the medium size corresponds to the lending from peripheral to hub banks, and the smaller one represents the lending between hub and small banks. Given the interplay between attachment probabilities and loan sizes, the network is populated by highly interconnected big banks which can be net borrowers or lenders, depending on the value of the parameters, and small banks belonging to the periphery which are not interconnected among themselves. We divide the benchmark values of interbank loans by the number of links each bank has, in order to keep the size of total lending constant as the attachment probability characterizing hubs varies. In this way we are able to obtain credit networks that do not change the relative size of banks as the connection levels change. We do this to have comparable credit networks. Once the structure of interbank expositions has been shaped, we apply the same procedure described for the Erdős–Rényi case to fill in the remaining parts of individual balance sheets.

3.3 Contagion

We define the total liquidity of bank i as the sum of its interbank assets plus the total amount of its external liquid assets:

$$l_i = \sum_{j \in N} x_{i,j} + q_i \qquad (4)$$

Subsequently, we identify the proportion of interbank loans over the total amount of liquid assets as $\varphi_{i,j} = x_{i,j}/l_i$, for $i, j \in N$, and the amount of external liquid assets over the total of liquid assets as $\varphi_{i,n+1} = q_i/l_i$, for $i \in N$. This allows us to define the system in terms of a relative liquidity assets matrix Φ, and to use the latter in modelling contagion.

The algorithm we employ to calculate the complete sequence of knock-on effects arising from a liquidity shock is the one developed by Lee (2013). As shown in Hurd (2016), this is formally identical to the fictitious default algorithm developed by Eisenberg and Noe (2001) to derive the equilibrium clearing vector of payments over a contagion, the only differences being the nature of the shock that triggers a crisis and the direction of the contagion. More precisely, while in the Eisenberg-Noe's algorithm the propagation process begins with a loss on the asset side – so that it spreads from a debtor to a creditor bank - in the Lee's algorithm the cascade begins with a deposit withdrawal on the liabilities side, and then it runs from a creditor to its debtors.

Operationally, the contagion process begins when an exogenous deposit run hits a given sample of intermediaries. In order to understand how the relative magnitude and distribution of shocks affect results, we will consider three different scenarios. In the first one, an arbitrarily chosen small number of banks $k \in N$ is disturbed by a major drawing of deposits, a situation we label "concentrated shock". In the other two scenarios, we spread the same aggregate amount of deposits' withdrawal over a larger sample of banks $k' > k$ ("dispersed shock") and over the almost whole population N ("generalized shock"), respectively. While keeping constant the absolute size of the shock allows an immediate comparison between the three cases, spreading it over different samples returns situations characterized by funding runs of different relative magnitude and distribution.

A bank affected by the shock needs to collect an adequate amount of liquidity to cover the unforeseen withdrawal of deposits by using its liquid assets and, once liquid ammunitions have been exhausted, by selling illiquid assets to the central bank at face value. It follows that the shock, as it hits, can be defined as:

$$\Delta d_i = \sum_{j \in N} \Delta x_{i,j} + \Delta q_i + \Delta z_i \quad . \qquad (5)$$

Each affected bank responds by recalling an amount of interbank assets $\Delta x_{i,j} = \varphi_{i,j} \cdot \min(\Delta d_i, l_i)$, an amount of liquid external assets $\Delta q_i = \varphi_{i,n+1} \cdot \min(\Delta d_i, l_i)$, and one of illiquid external assets $\Delta z_i = \max(0, \Delta d_i - l_i)$. As stated before, the chosen algorithm assumes that liquid assets are utilized before the liquidation of external illiquid assets. An improvement of this algorithm should insert fire selling on external assets. In this way Lee's algorithm would be more realistic. The total change in liquid assets for a given bank i can exceed the exogenous shock Δd_i, while the exact value of such change can be efficiently computed by solving the system of equations:

$$\Delta l_i = \min[\, l_i, \sum_{j \in N} \varphi_{j,i} \cdot \Delta l_j + \Delta d_i \,], \;\; \forall i \in N. \qquad (6)$$

As soon as the mapping describing the liquidity knock-on mechanism converges, the Knaster-Tarski Fixed Point theorem guarantees that at least one exact solution to (6) exists and it can be calculated in a finite number of iterations. This is verified if the mapping is monotone and nondecreasing in the outflow of liquidity experienced by banks, a condition that holds true due to the seniority structure we have assumed before. Furthermore, if the relative liquid exposure matrix Φ is irreducible substochastic, the equilibrium clearing vector of funding withdrawals is unique.

The liquidity need of bank i is thus given by:

$$l_i^n = \sum_{j \in N} \varphi_{j,i} \cdot \Delta l_j + \Delta d_i , \qquad (7)$$

corresponding to the sum of the deposit withdrawal it originally faces from its depositors and the request of liquidity by the set of its creditors needing liquidity. Every time $l_i^n > l_i$, a bank experiences a liquidity shortage computed as $l_i^s = l_i^n - l_i$. If $l_i^s > 0$, the bank is forced to enter into the outright asset purchase program deployed by the central bank. We define a *systemic liquidity shortage* (SLS) as the sum of the liquidity shortages suffered by all the banks belonging to the network.

3.4. Settlement procedures

The potential of a mandatory OnBS netting scheme as a crisis management policy is assessed by comparing the performance of five settlement treatments. The first one is the standard *gross settlement* (*G*) mode, which represents our benchmark. In this case we left unchanged the weighted matrix of interbank loans X obtained through the two initiation stages discussed above. The second treatment is a *full netting* (*FN*) scenario, where each bank is allowed to offset his bilateral credit/debit obligations. The matrix X is consequently rearranged by applying an algorithm that computes the net mutual exposition between any two banks $(x_{i,j} - x_{j,i})$. To exemplify, if the original 2x2 matrix X is:

$$X = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$$

when the regulator forces an OnBS netting it becomes:

$$X' = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Then we test three intermediate cases of *partial netting*. In the first case (*P*-I) we assume that the bilateral netting of mutual expositions is mandatory only for the banks that receive the external shock. In the others two cases the netting treatment is applied between distressed and non-distressed banks (*P*-II), and among non-distressed banks only (*P*-III), respectively. For example, consider the network be composed of four banks interlinked by mutual exposures as in the matrix X below. Suppose that the first two intermediaries are hit by a run. The weighted matrix X now becomes X , X'' and X''' for the three partial netting schemes just recalled, respectively.

$$X = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 2 & 2 & 0 \end{bmatrix};\ X' = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 2 & 2 & 0 \end{bmatrix};\ X'' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 0 \end{bmatrix};\ X''' = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \end{bmatrix}.$$

This example also makes clear that any kind of netting reinforces the strict substochasticity of the associated relative exposures matrix *Φ*, that represents a necessary and sufficient condition for the uniqueness of the payments clearing vector solving the fixed point problem (6).[19] In other words, if the network X is such that a clearing vector exists and is unique, the same holds true for X', X'' and X''' as well. Table 3.2 summarizes the treatments.

Table 3.2. Summarizing treatments

Settlement	Description
Gross	Neting is not applied.
P-I	Bilateral netting among the shocked banks
P-II	Bilateral netting among shocked and unshocked banks
P-III	Bilateral netting among the unshocked banks
FN	Bilateral netting among all banks of the networks

3.5. Metrics for policy objectives

The settlement options described above are assessed comparatively against four metrics measuring different dimensions of financial stability. The first is the troubled banks rate, defined as the fraction of banks forced to resort to the assistance of the central bank to sell their illiquid assets as the contagion proceeds. Operationally, we define a binary variable tb_i, for $i \in N$, which takes the value 1 if the i^{th} bank goes short of liquidity at some point, and 0 if the bank withstands the contagion process without the need to liquidate any of the illiquid assets it has on its books. The troubled banks rate Tb is computed as:

$$Tb = \sum_{i \epsilon N} \frac{tb_i}{N}. \qquad (8)$$

The second metric measures the dearth of systemic liquidity, obtained by adding up the individual liquidity shortages l^s relative to the size of individual external illiquid assets z.

$$ls = \frac{\sum_{i \epsilon N} l_i^s}{\sum_{i \epsilon N} z_i} \qquad (9)$$

[19] See Theorem 2.2 in Hurd (2016, pp.25-27).

Complementary to the previous metric, which focuses on the extensive margin, this one allows us to measure the intensive margin of the liquidity shortfall experienced by the whole market. By combining the two indicators, one can gain information on the latitude and magnitude of the exceptional liquidity injection the central bank is bound to extend in managing the liquidity crisis. As a result, our analysis permits to appreciate how alternative options on the way interbank exchanges are settled might interact with standard lender-of-last-resort functions deployed by monetary authorities.

The third index measures the percentage erosion of liquid means faced by the banks withstanding contagion without having to prematurely liquidate illiquid assets. As such, it can be read as an average distance to illiquidity in the case a new shock occurs, which we take as a proxy for the health of the banking system in its post-contagion state. The distance to illiquidity for viable banks is therefore computed as the ratio between the total amount of liquid external assets employed ($\sum_{i\epsilon N} Eq_i$) to confront the unexpected withdrawal of external or interbank deposits and the total amount of initially available liquid assets ($\sum_{i\epsilon N} q_i$) , where summations are taken over the set of banks that during the crisis episode never resorted to the lender-of-last-resort facility made available by the central bank. The Distance to illiquidity is then given by:

$$Dti = \frac{\sum_{i\epsilon N} Eq_i}{\sum_{i\epsilon N} q_i} \qquad (10)$$

The domain of this variable is by construction defined in the interval [0, 1]. The upper bound is reached when almost all the liquidity of untroubled banks is employed during the contagion process. On the contrary, the metrics tends to zero when their aggregate stock of liquidity has been preserved despite the crisis.

The final metric deals with the post-contagion depth of the market expressed in terms of the volume of interbank deposits. It is computed as the difference between the total interbank assets before the shock occurs ($\sum_{i\epsilon N} IA_i$) and the interbank assets employed during the contagion or in the netting process $\sum_{i\epsilon N} EIA_i$ all divided by $\sum_{i\epsilon N} IA_i$. Higher values of this index signal that the scope for co-insurance in managing idiosyncratic liquidity needs under post-crisis normal conditions has not been impaired during the turmoil generated by the funding run. So, we define the the *Depth* metric as:

$$Depth = \frac{\sum_{i\epsilon N} IA_i - \sum_{i\epsilon N} EIA_i}{\sum_{i\epsilon N} IA_i} \qquad (11)$$

3.6 M.O.O.R.A method analytic

The M.O.O.R.A is a multi-objective maximization method. This means that this methodology allows to identify the best alternative where there are a discrete number of objectives to be achieved. From the operational point of view this method consists of three phases.

In the first phase we define the i goals and the j alternatives to be tested ($i \in N$, $j \in$ M). Once objectives and alternatives are identified, the values assumed by each alternative with respect to each objective are assigned. We define $s_{i,j}$ the value of the j-th alternative to the i-th objective.

The second phase consists in determining the *ratios* through a normalization of the values of each alternative with respect to each objective. This procedure transforms each $s_{i,j}$ into $Ns_{i,j}$, and it consisits in dividing the value $s_{i,j}$ for the square root of the sum of the squares of the values obtained from all the different alternatives as expressed by the equation (12).

$$Ns_{i,j} = \frac{s_{i,j}}{\sqrt{\sum_{j}^{m} s_{i,j}^{2}}} \qquad (12)$$

The normalization process is necessary in order to be able to proceed with the optimization since it transforms the values reported by each alternative for each objective to a scale from 0 to 1.

The third and last phase is composed by two parts. The first consists in the choice of the objectives that must be maximized and those that will be minimized. So, we named g the number of variables to be maximized and consequently *(g-N)* the number of variables to be minimized. The last step of the procedure consists in identifying the Ny_j normalized aggregate score of each alternative. This value is analytically obtained through the (13).

$$Ny_j = \sum_{i=1}^{i=g} Ns_{i,j} - \sum_{i=g+1}^{i=n} Ns_{i,j} \qquad (13)$$

In the (13) the g variables that must be maximized appear with a positive sign while the (N-g) variables to be minimized appear with a negative sign. Once the Ny_j value is obtained for each alternative it is possible to identify the optimal value through the sorting.

In our case the optimal policy that the regulator should undertake results from the combination of the alternatives, the 5 settlement methods, and the objectives, indicated by the 4 metrics. As we have defined the metrics in the previous section, the variables to be minimized are the number of banks that register an illiquidity position, the systemic liquidity shortage and the distance to default. The only variable to be maximized is the depth of the interbank market given that the greater the value of this metric the greater the level of safeguarding of this market will be. Table 3.3 summarizes the objectives setting.

Table 3.3. M.O.O.R.A objectives setting.

Metrics	Maximization /Minimization
Troubled banks (Tb)	Minimize
Total liquidity shortages (ls)	Minimize
Distance to illiquidity *(Dti)*	Minimize
Depth of the interbank market *(Depth)*	Maximize

3.6. Overview

The framework we offer to weigh in the pros and cons of forcing the netting of interbank bilateral exposures when a liquidity crisis occurs is rather complex. It seems worthwhile to summarize in Table 3.4 the various instances we consider for: *i*) the two key variables driving the dynamics of contagion, i.e. the topological structure of the market and the type of shock hitting the system; *ii*) the crisis management option chosen by the regulator; *iii*) the goal used to evaluate the social welfare of any given policy prescription.

Table 3.4. Overview of the simulation framework.

Topological Scenarios	Distribution of shocks (degree of dispersion in terms of number of banks shocked)	Settlement Treatments (Alternatives)	Welfare criteria (Objectives)
Erdős–Rényi	Concentrated	Gross	Troubled banks rate ($\sum_{i\epsilon N}\frac{tb_i}{N}$)
Small-world	Dispersed	Full netting	Liquidity shortfall ($\frac{\sum_{i\epsilon N} l_i^s}{\sum_{i\epsilon N} z_i}$)
Core-periphery	Generalized	Partial netting I	Distance to illiquidity ($\frac{\sum_{i\epsilon N} Eq_i}{\sum_{i\epsilon N} q_i}$)
		Partial netting II	Depth of the market ($\frac{\sum_{i\epsilon N} EIA_i}{\sum_{i\epsilon N} A_i I}$)
		Partial netting III	

In what follows, for each topology we will first present a specific case where we will report profiles corresponding to the value registered by the welfare system at the end of the contagion process - calculated as the fixed point of the cascade mapping generated by the fictitious default algorithm () - for different levels of interconnectedness. This presentation will allow us to indicate the main dynamics with respect to each metric taken into consideration. Then, for each topology, we provide the results of the M.O.O.R.A process which indicates the best policy option for different shock dispersion degrees and connectivity levels.

4. RANDOM NETWORKS

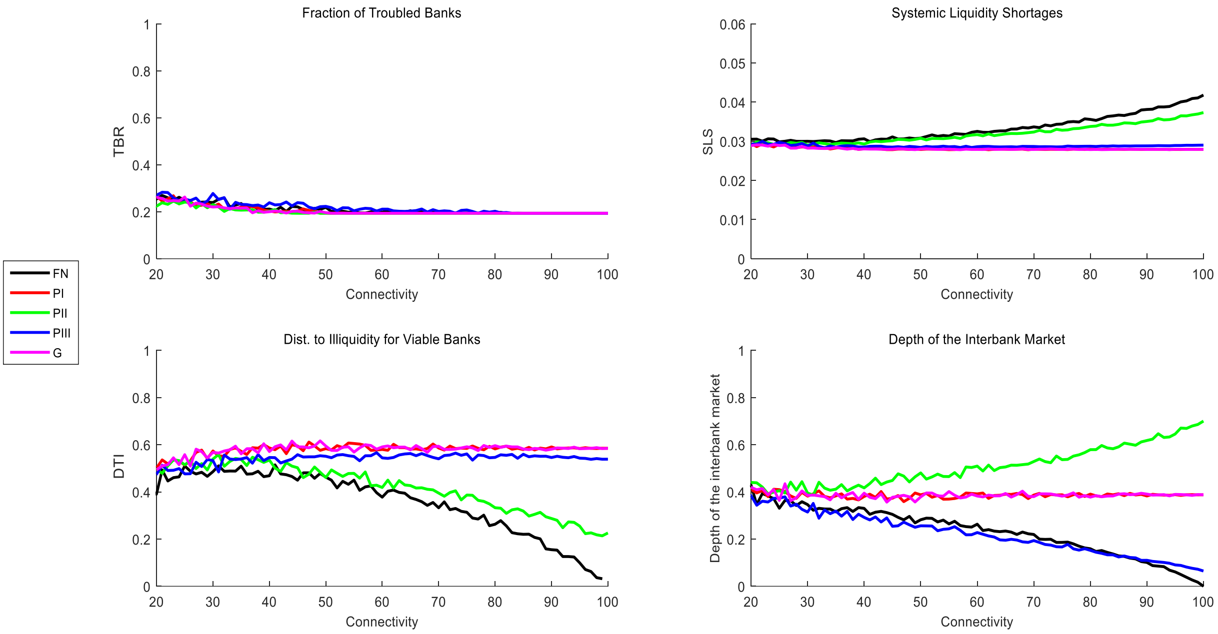
We start our analysis from the Erdős–Rényi topology. Table 4.1 summarizes the baseline simulation parameters. Since our scope is purely explorative, none formal external validation has been performed. Nevertheless, calibration values are fully consistent with the ones customarily used in the literature we are building upon (Niet *et al*., 2007; Gai and Kapadia, 2010; Lee, 2013).

Table 4.1. Parameters and calibration for the Erdős–Rényi topology.

Parameter	Description	Baseline calibration
N	Number of banks	25
ω	External assets ratio	0.83
γ	Leverage ratio	0.04
δ	External liquid assets ratio	0.03
P	Interconnectedness probability (range)	(0.2, 1)
Δd	Size of the funding shock as a % of deposits	25%
K	Number of shocked banks (each bank is shocked equally)	4

Figure 4.1 reports results for a concentrated shock scenario, in which the depositors of four randomly chosen banks suddenly withdraw 25% of their deposits. The upper-left panel of the figure shows that the five types of interbank settlement perform in a fully comparable fashion as far as the troubled banks rate is concerned. Systemic liquidity shortages are slightly stronger under the *FN* and the *P*-II treatments (upper-right panel), however, especially at the high end of the connectivity spectrum. The situation is completely reversed along the distance to illiquidity dimension (lower-left panel), where both *FN* and *P*-II engender a considerable advantage with respect to the *P*-I, *P*-III and *G* cases, respectively.

Figure 4.1. Performance profiles over the interconnectedness spectrum for the different crisis management options, when the topology of the interbank market is Erdős–Rényi. The case analysed is for 4 distressed banks due to a 25% deposit withdrawal each. In the first quadrant the Troubled Banks metric, in the second the Liquidity Shortages, in the third the Distance to Illiquidity, and in the fourth the Depth of Interbank Market



These findings are the consequence of two distinct effects. The first one implies a typical balancing between a risk-sharing and a risk-spreading effect. On the one hand, OnBS netting schemes cut several potential channels of contagion by isolating the banks with interbank positions that can be netted almost in full. On the other hand, any netting settlement operates to disconnect the interbank market, lowering the risk-sharing associated with a mutual exchange of interbank deposits. In particular, the decreased scope for sharing liquidity risks comes with a stronger concentration of liquidity calls to those debtors who cannot fully net their position towards distressed banks. From this point of view, netting settlements contribute to increase financial instability. Moreover, there is a second effect weighing the liquidity shortage registered at the aggregate level against the distance to illiquidity of untroubled banks, that depends on the relative magnitude of the external run disturbing intermediaries. For a funding shock large enough, the netting of mutual claims increases the internalized portion of the unexpected depositors' withdrawal. As a result, it tends to increase the liquidity shortfall of shocked banks on the one hand, while lowering the liquidity calls towards the remaining part of the network on the other one. This is the reason why the distance to illiquidity tend to be structurally lower in the netting treatments including shocked banks. In fact, the internalization of a larger portion of knock-on losses implies a lower liquidity need during the whole contagion process. Whilst the two effects tend to offset each other in terms of the faction of banks incurring into liquidity troubles, this is not the case for the other welfare metrics. In the *FN* and *P*-II treatments, the concentration effect increases systemic liquidity shortages if compared to the standard *G* treatment, as well as to the P-I and P-II ones. Symmetrically, the losses' aggregation process allows *FN* and *P*-II to outperform all other treatments as far as the index of financial health after contagion is concerned.

The lower-right panel of Figure 4.1 illustrates the post-crisis depth of the interbank market under the five settlement protocols. In the four treatments considering the netting of mutual exposures, results are driven by two components. The first is the internal dynamics associated to novation wiping out interbank contracts, while the second is the contagion process forcing intermediaries to call back interbank credits. For the *G* treatment, on the contrary, the only reason for the shrinkage of the interbank market is due to the standard dynamics of the liquidity cascade.

From this quadrant emerges the excellent performance of the Partial Netting II which is able to preserve the interbank market better than all other treatments, significantly reducing the contagion process.

We note that the disaggregated analysis by metric allows to identify the trade offs in the field and consequently the strengths and weaknesses of each policy proposal. What is very difficult to identify from this perspective is the combined output obtained by each policy option. We therefore move to the M.O.O.R.A analysis to get this type of information. Table 4.2 shows the parameters and the calibration of the shocks for the simulations undertaken with the M.O.O.R.A method.

Table 4.2. M.O.O.R.A parameters and calibration for the Erdős–Rényi topology.

Parameter	Description	Baseline calibration
N	Number of banks	25
ω	External assets ratio	0.83
γ	Leverage ratio	0.04
δ	External liquid assets ratio	0.03
P	Interconnectedness probability (range)	(0.2, 1)
Δd	Size of the funding shock as a % of deposits (range)	25-5%
K	Number of shocked banks (each bank is shocked equally. (range)	4-20

Figure 4.2. M.O.O.R.A outputs. Each treatment is tested against the standard gross settlement when the topology of the interbank market is Erdős–Rényi. The three axis dimensions are the number of the shocked banks (range 4-20), the percentage probability of connection between nodes and the M.O.O.R.A overall performance.

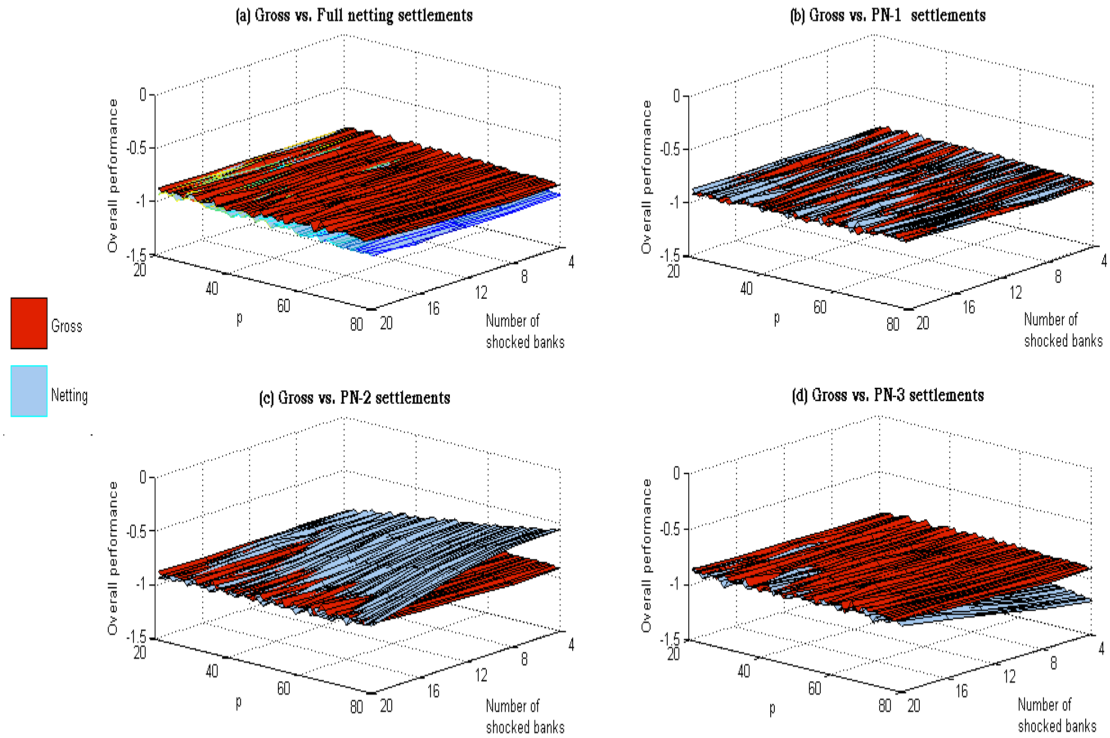


Figure 4.2 is divided into 4 quadrants. In each quadrant we show the comparison in terms of M.O.O.R.A output between one of the netting proposals and the standard gross treatment.

The figure shows the P-II is the best treatment for most levels of connection probability and shock distribution. Only for highly distributed shocks the G settlement performs in line or better than the P-II. This happens because the positive effect due to the isolation between the affected and non-affected banks fades away as the number of shocked banks tends to almost the entire system. As seen for the disaggregated analysis, the level of connection of the network shows a positive relation with respect to the performance of the P-II. This dynamic is a consequence of the fact that as the number of links between nodes increases the netting opportunities between agents increase too. In this way the isolation effect of the P-II becomes stronger.

For what concerns the other policy proposals we note how the G treatment dominates for almost the whole domain defined by the parameters. This is due to different reasons. In the case in which the FN is taken into consideration, the negative performance that this treatment scores in terms of reduction of depth of the interbank market seems to be the main discriminant for its underperformance. Analyzing the performance of the P- I we note that this treatment shows results in line with the gross settlement generalizing what was seen in the disaggregated analysis. With regard to the P-III, a particularly negative performance is recorded for concentrated shock. This happens because the P-III is strongly affected by a fallen of depth in the interbank market inherent in the netting procedure. In fact, concentrated shocks correspond to a large number of non-affected banks that carry out netting. On the contrary, for generalized shocks the reverse dynamics that tends to drive a convergence between the two treatments.

5. DISASSORTATIVE NETWORKS

The random architecture analysed so far represents a natural benchmark to consider. As already recalled, however, real-world interbank networks have been shown to possess strongly disassortative mixing degree distributions, with a few banks having substantially more connections than the average degree. Since the key idea at the root of a crisis management strategy based on a mandatory OnBS netting is that of cutting links to contain contagion, it seems interesting to explore how its implementation could perform in topologies alternative to the random one.

5.1 Small-world networks

The first robustness check we fulfil refers to a small-world topology. Table 5.1 summarizes the baseline simulation parameters characterizing this experiment.

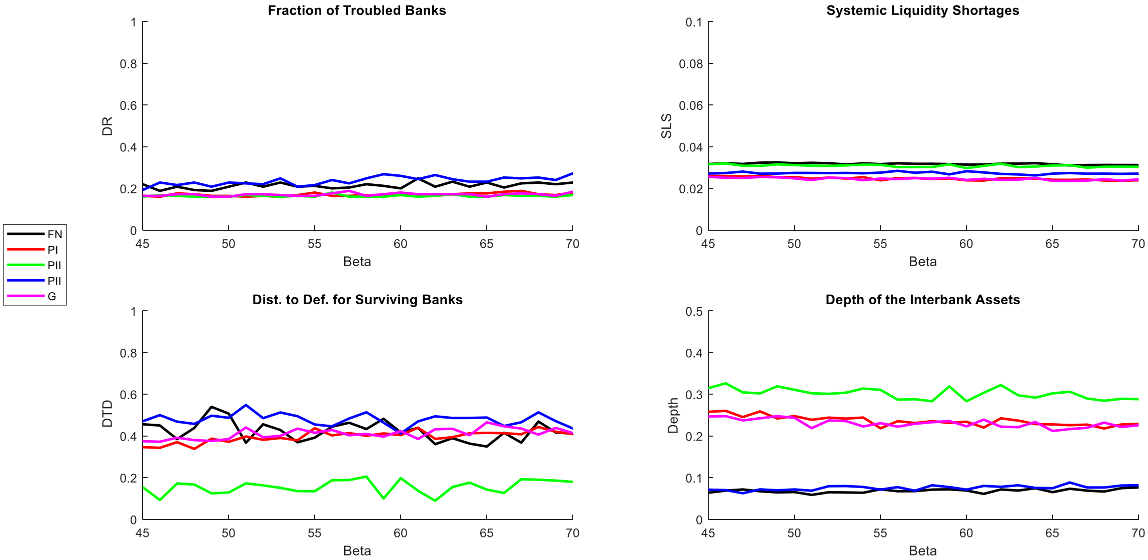
Table 5.1. Parameters and calibration for the small-world topology. Parameters not reported are equal to the ones employed in the Erdős–Rényi case.

Parameter	Description	Baseline calibration
N	Number of banks	25
D	Node degree of the starting lattice ring	2
β	β re-wiring parameter (range)	0.45-0.70
Δd	Size of the funding shock as a % of deposits	25%
K	Number of shocked banks (each bank is shocked equally. Range)	4

Figure 5.1 reports the results for the concentrated shock scenario, in which the depositors of 4 banks suddenly withdraw 25% of their deposits. As usual, we apply the four netting treatments and the benchmark gross settlement mode in a comparative fashion. The first noteworthy feature is that, in line to what we observe for random networks, the different profiles over the whole interconnectedness range for the fraction of troubled banks are essentially undistinguishable for the Gross, Partial Netting I and II, signalling that the contagion-preserving and the loss-concentration effects tend to offset each other. FN and P-III scored a bad performance compared to the other treatments showing a dominance of the loss concentration over the contagion-preserving effect. The balance between the two prongs of the trade-off drives the slight bad performance of the *P*-II treatments as regards the metric measuring liquidity shortages, as well as is good score in the distance to illiquidity metric. On the contrary, the FN performs badly as regard as liquidity shortage without scoring a good performance on the distance to default. This is probably due to the negative effect in term of higher number of distressed banks scored by FN respect to P-II. Due to their relative inability to isolate the shock affecting the stressed banks, in turn, *P*-I and *P*-III turn out to be systematically less effective than the

other netting treatments. Moreover, the *P*-III treatment due to the large number of netted contracts , lowers the depth of the interbank market without being able to score an improvement along the other welfare dimensions. All in all, the *P*-II treatment is effective in the distance to illiquidity dimension, and it is also the best treatment in preserving the deepness of the interbank market.

Figure 5.1. Performance profiles over the interconnectedness spectrum for the different crisis management options (β parameter on x axis), when the topology of the interbank market is small-world. The case analysed is for 4 distressed banks due to a 25% deposit withdrawal each. In the first quadrant the Troubled Banks metric, in the second the Liquidity Shortages, in the third the Distance to Illiquidity and in the fourth the Depth of Interbank Market



Now, we procede with M.O.O.R.A investigation. Table 5.2 indicates the values of the set of parameters employed.

Table 5.2. M.O.O.R.A parameters and calibration for the Small World topology.

Parameter	Description	Baseline calibration
N	Number of banks	25
D	Node degree of the starting lattice ring	2
Β	Beta re-wiring parameter (range)	0.1-0.7
Δd	Size of the funding shock as a % of deposits (range)	25%-5%
K	Number of shocked banks (each bank is shocked equally. Range)	4 -20

Figure 5.2. MOORA outputs. Each treatment is tested against the standard gross settlement when the topology of the interbank market is Small-World. The three axis dimensions are the number of the shocked banks (range 4-20), the beta re-wiring probability between nodes and the MOORA overall performance.

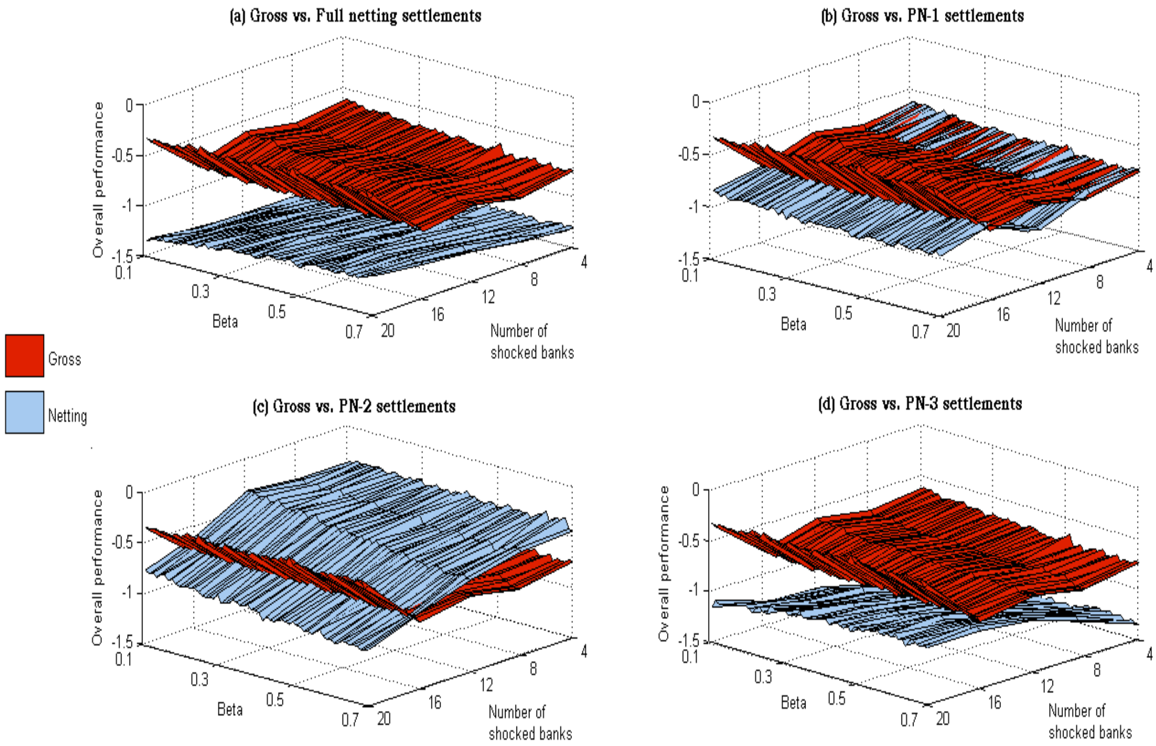


Figure 5.2 presents the results of the MOORA procedure for the 4 treatments of netting versus the standard gross settlement mode. Compared to the homogeneous topology analyzed in the previous section, we observe that there are marked differences between treatments. As far as the FN is concerned, for all the domain defined by the parameters the netting treatment underperforms the gross settlement. This output is mainly due to the very low performance that the FN treatment scores in the metric concerning the depth of the interbank market without being able to get an improvement along the other welfare dimensions. The P-I treatment shows a performance in line and sometimes higher than the G treatment for concentrated shock. On the contrary, as the level of shock dispersion increases the two treatments tend to diverge with the prevalence of the G treatment. This is due to the fact that for generalized shocks the number of netted contracts surges, inducing in that way a poor performance of the P-I treatment in the metric dealing with the depth of the interbank market.

The P-II netting is the best treatment for highly and medium concentrated shocks. The dominance of this treatment is mainly due to the ability of P-II in preserving liquidity and ensuring the depth interbank market depth during contagion. Nevertheless, when the shock is generalized, the number of banks performing netting is very limited and therefore the netting benefits on the reduction of contagion are not significant.

Finally, the P-III treatment. The netting between the unaffected banks is below the standard gross settlement for all parameter levels. This treatment tends to burn too much interbank obligations for concentrated shocks, inducing in this way a prevalence of the concentration effect on the isolation effect. With regard to generalized shocks, the P-III treatment is penalized by the poor application of the netting.

5.2. Core-periphery networks

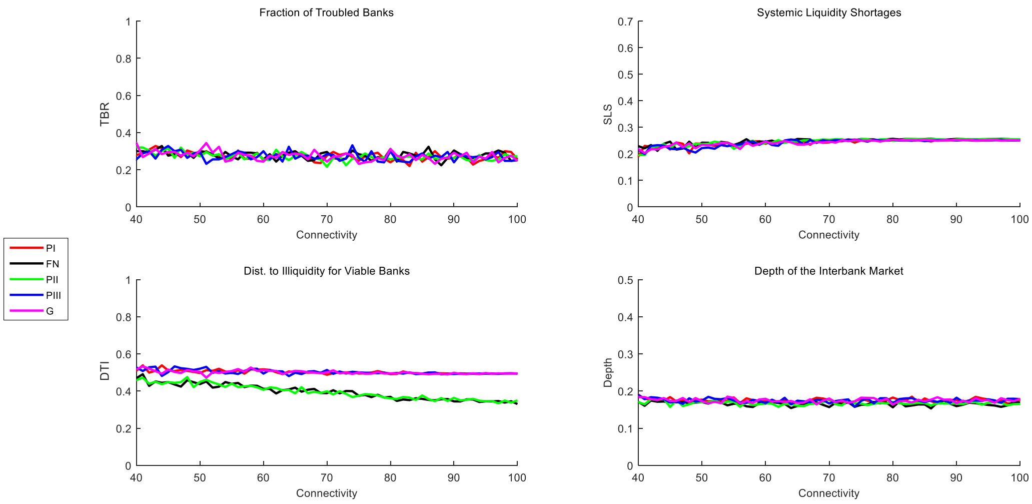
As a last experiment, we test our model on a core-periphery topology. Table 5.2 summarizes the baseline simulation parameters. This topology is characterized by a few large banks that are flanked by many small ones. As for the previous topologies we begin with the disaggregated analysis. We first shock 4 core banks by means of a 25 % withdrawal of its deposits. Table 5.3 shows the value of the parameters employed in our simulations.

Table 5.3. Parameters and calibration for the core-periphery topology. Parameters not reported are equal to the ones employed in the Erdős–Rényi case.

Parameter	Description	Baseline calibration
N_{large}	Number of hubs	4
p_{sl}	Small-to-hub interconnectedness probability	0.4
p_l	Hub interconnectedness probability (range)	(0.4, 1)
l_p	Small-hub, hub-small and hub-hub loan size	1 – 3 – 5
Δd	Size of the funding shock as a % of deposits	25%
K	Number of shocked banks (each bank is shocked equally)	4

In this case, all the partial netting treatments previously described can also be tested. As shown in Figure 5.3, *FN* and *P*-II outperform the other treatments in preserving financial stability after contagion. The other three metrics remained approximately equal, a finding that can be explained as follows. Since safe banks – i.e., those not hit by the exogenous shock – belong to the periphery and are not connected to each other, *P*-III collapsed to *G*. Moreover, *P*-I generates a contagion dynamic similar to those of *G*. Although shocked banks are forced to net their mutual exposures, the very fact that all the hubs are hit by the same shock left their liquidity needs unchanged. *FN* and *P*-II score a better performance with respect to the distance to illiquidity, due to the fact that the loss-concentration effect associated to netting acts to limit the volume of liquidity drained from the system by shocked banks.

Figure 5.3. Performance profiles over the interconnectedness spectrum for the different crisis management options, when the topology of the interbank market is core-periphery. The case analysed is for 4 hub banks disturbed by a 25% deposit withdrawal each. In the first quadrant the Troubled Banks metric, in the second the Liquidity Shortages, in the third the Distance to Illiquidity and in the fourth the Depth of Interbank Market.

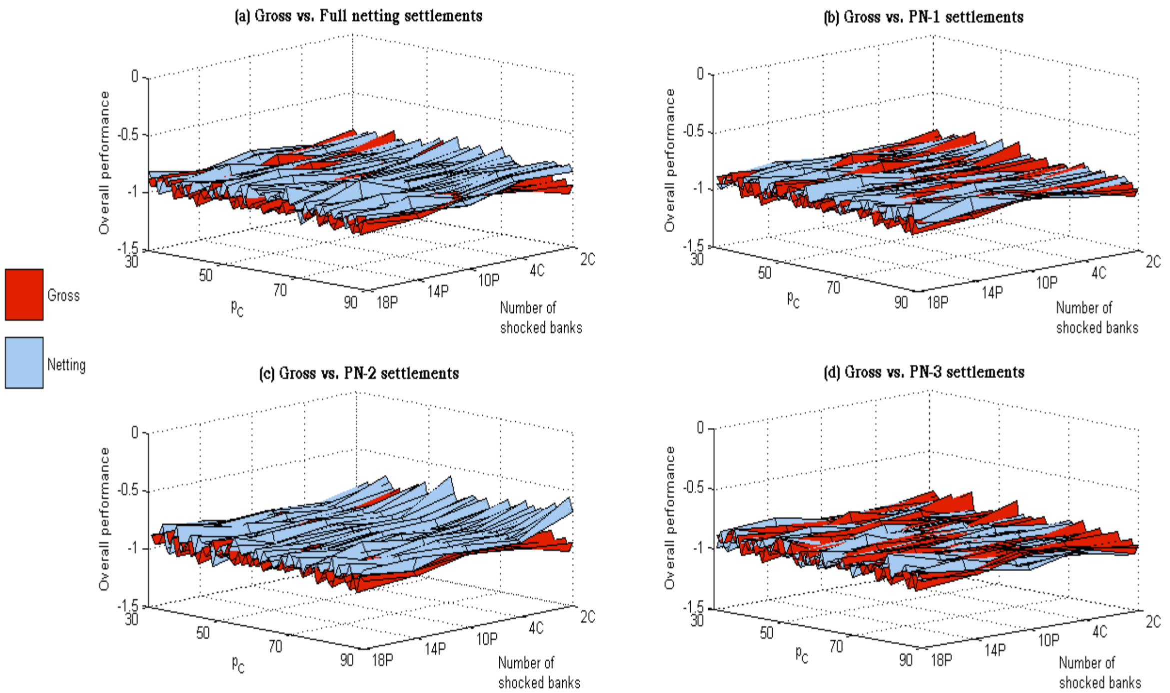


Now we procede to M.O.O.R.A investigation, Table 5.4 shows the parameter values employed in our simulations.

Table 5.4 M.O.O.R.A. parameters and calibration for the core-periphery topology. Parameters not reported are equal to the ones employed in the Erdős–Rényi case.

Parameter	Description	Baseline calibration
N_{large}	Number of hubs	4
p_{sl}	Small-to-hub interconnectedness probability	0.4
p_l	Hub interconnectedness probability (range)	(0.3, 0.9)
l_p	Small-hub, hub-small and hub-hub loan size	1 – 3 – 5
Δdh	Size of the hub bank funding shock as a % of deposits (range)	25-50%
Δds	Size of the small bank funding shock as a % of deposits (range)	8-18
Kh	Number of shocked hub banks (each bank is shocked equally)	2-4
Ks	Number of shocked peripherical banks (each bank is shocked equally)	6-18

Figure 5.4. M.O.O.R.A outputs. Each treatment is tested against the standard gross settlement when the topology of the interbank market is Core Periphery. The three axis dimensions are the number of the shocked banks (C=core bank, P=peripherals bank), the connection probability of the hub nodes and the MOORA overall performance.



Observing Figure 5.4, a positive overall netting performance is recorded for core periphery topologies. The analysis of the performance of the FN in the first quadrant shows an almost structural dominance of this treatment compared to the G settlement standard. This outcome seems to be driven mainly by the good performance of the FN in preserving liquidity during the contagion process. The result is robust for concentrated shocks that hit the hub banks and for the shocks dispersed on peripheral banks. Considering the PN-I, we note how this treatment registers a positive performance when the shock is dispersed on small banks. This is due to the fact that the PN-I treatment tends to disconnect peripherals banks from each other enhancing the isolation effect. In this way the liquidity requests from the shocked peripheral banks are directed towards the large hub banks able to meet these needs. With regard to the shocks concentrated on hub banks, there is a substantial equality between the two treatments.

For what concerns the P-II treatment, there is a clear dominance of netting compared to the G settlement driven mainly by its excellent performance in terms of liquidity savings during the contagion process. The result is robust for different levels of hub banks' connection and for the degree of shock dispersion. Finally, we consider the treatment P-III which registers a performance slightly lower than the standard G settlement for shock concentrated on the hub banks, while a performance that tends to be comparable to the G treatment as regards generalized shocks.

6. CONCLUDING REMARKS

The key objective of this paper was to shed light on the effects exerted by a contingent-based OnBS netting prescription on the mitigation of systemic financial disruptions. In particular, we have employed numerical simulations to investigate whether a deposit run could trigger a systemic liquidity crisis more easily in an environment in which a standard gross settlement mode is applied in settling interbank lending exposures, in comparison to an environment in which alternative OnBS netting schemes are used as a crisis management tool. The impact of OnBS netting has been assessed against several metrics capturing the multidimensionality of financial stability, such as the fraction of banks asking for a liquidity support to the central bank, systemic liquidity shortages, the distance to illiquidity for the banks that never went short of liquid means during the whole crisis, and depth of the interbank market after contagion. Once the values of these metrics have been identified we proceeded using the MOORA multi-objective optimization method in order to identify the most appropriate policy proposal.

Our simulations highlight that the scope for forcing banks to net their mutual claims in an attempt to defuse possible channels of contagion implies that regulators face a crucial trade-off, whose balance varies depending on the inner dynamics of clearing arrangements. A bilateral netting settlement mode tends to concentrate interbank liquidity calls towards a lower number of counterparties. This effect weakens the potential to share liquidity risks associated to mutual interbank loans, worsening the prospect to be exposed to a financial pain for all the banks who are not allowed to fully disconnect themselves. In turn, given that novation agreements reduce the interbank depth, a netting of bilateral exposures contributes to cut several channels of contagion. The interplay between these two forces determines the final outcome.

As regards the Erdos Renyi and small world topologies, the M.O.O.R.A method identifies the P-II as optimal treatment. The advantage of this treatment compared to the others is to be found in its ability to preserve the aggregate liquidity of the system and the depth of the interbank market during the process of contagion. As we proceed to more realistic core-periphery and small world structures, M.O.O.R.A method outlined a clear policy prescription for disperse and generalized shocks. The *P-II* treatment outperforms the other crisis management options even for these topologies. Summarizing our analysis, we found the best policy option is to isolate the banks affected by the shock from those not affected. In this way netting acts as a “sanitary belt” which is able to preserve financial stability.

By allowing banks to participate in an outright asset purchase program orchestrated by the central bank, the present model neglects completely market illiquidity. This assumption was retained in order to keep liquidity runs detached from the dynamics generated by failure cascades affecting the assets’ side. Future improvements should relax this hypothesis. Our intuition is that, in this case, another important trade-off would enter the scene as soon as an OnBS netting scheme was made mandatory to face a systemic liquidity shortfall, in that the higher fire selling losses generated in a more interconnected market should be weighed against the relative financial position of those banks that (partially) succeed in insulating themselves from contagion at the cost of bearing more concentrated liquidity shortages.

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The economics of netting in financial networks

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Abstract

Can on-balance-sheet (OnBS) netting of interbank deposits be employed to prevent financial contagion and improve the resilience of a banking system? The aim of this paper is to investigate whether a collective enforcement of this kind of agreements could reduce the systemic risk during a crisis. To this end, we compare contagion dynamics under OnBS netting and a standard gross settlement through the lenses of a mean-field model and computer simulations. We find that OnBS netting tends to reduce the number of defaults, preserve the aggregate amount of bank capital but takes its toll on retail deposits. Hence, our analysis provides support for a policy mix that combines a bilateral netting enforcement with a national insurance scheme for retail deposits. The desirability of netting increases when the system is highly connected, susceptible to large shocks especially when vulnerabilities are first detected in banks located in the core of the network.

1. INTRODUCTION

This paper focuses on the application of on-balance-sheet (OnBS) bilateral netting agreements in interbank networks. At odds with the typical offsetting of OTC derivative transactions, OnBS netting is a contractual arrangement that allows two financial institutions to offset mutual debit-credit relationships that are booked on their balance sheets, by signing a novation agreement such that a single net amount is contractually substituted for the previous individual gross amounts they owed to each other or by stipulating a contract such that the former procedure is conditional on the default of the counterparty. The aim of the paper is to investigate whether a mandatory enforcement of this kind of settlement contracts could control or even prevent default cascades in financial networks and boost their systemic resilience in the run-up of a crisis. We first clarify the system-wide mechanics of OnBS netting agreements in minimal-scale network topologies, in order to assess intuitively under which circumstances a policymaker mandating the offsetting of gross interbank exposures by means of OnBS netting arrangements could succeed in maintaining financial stability. The variables we track in our comparative exercise as measure of systemic performance are the aggregate stock of capital which is wiped out by a crisis contagion process, the number of defaults occurring through contagion, and the portion of the shock absorbed by deposits of the whole system. We show that the enforcement

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of a comprehensive obligation to net interbank claims represents an effective crisis-management policy whenever the initial shock is big enough, while for smaller disturbances the consequences for contagion are mixed. Moreover, we find that the degree of heterogeneity of interbank loans plays a key role in driving these outcomes. We then offer results from a mean-field approximation and agent-based simulations performed within more complex structures, in order to provide a comparison among the two types of settlement in more realistic scenarios as regard the number of banks, the network topology and the level of capitalization. Following a standard approach in the literature, we characterize different instances of the financial network by means of two key drivers - the connectedness probability and the number of nodes in the system - and we allow for the possibility that a contagion cascade could be triggered by a targeted shock wiping out a large portion of a bank's external assets. In Erdös-Rényi networks, mandatory OnBS netting definitely dominates gross settlement for default and capital metrics for different levels of leverage ratios, interbank market sizes, connectedness and degrees of heterogeneity in loans. As we move to core-periphery structures, however, we find that the magnitude of the triggering shock is a crucial variable in determining which arrangement dominates. In particular, while a strong dominance of OnBS netting occurs for default and capital metrics in almost all the cases tested whenever a big shock occurs, for smaller shocks the final output depends in a non-linear way on the network topology, the banks' size, the depth of the interbank market and the leverage ratio. The metrics relating to the portion of the shock absorbed by deposits displays a structurally better performance in the case of gross settlement for both tested topologies. This is mainly due to the fact that bilateral netting, by reducing the amount of interbank debts, fosters the bank hit by the shock to absorb a greater share of the losses through its deposits in contrast to the gross settlement case. Our investigation returns clear policy implications. As the risk of a systemic disruption looms large, the option to mandate a generalized novation of interbank gross exposures through OnBS netting agreements falls only conditionally within the set of crisis-management tools aimed at preserving financial stability. The prompt availability of data on the scope, size and functioning of real-world financial networks is a thus a key prerequisite for its deployment. Furthermore, since the offsetting of mutual interbank exposures tends to concentrate the effects of an external shock on the stakeholders of the buffeted bank, the presence of a deposit insurance scheme is a compelled complementary policy measure that has to be deployed. The remainder of the paper is organized as follows. Section 2 presents a literature review. Section 3 illustrates with elementary examples the intuition at the basis of netting mutual exposures in interbank markets. Section 4 is devoted to a more detailed analysis by means a mean-field approximation and agent-based simulations in random networks. An extension to core-periphery topologies is offered in Section 5. Section 6 concludes.

2. LITERATURE REVIEW[20]

In this work we analyze the impact of OnBS netting in contexts of financial crisis engendered by shocks that hit an intermediary's assets. Our contribution adds to the strand of the literature which employs network analysis as a methodology for the study of the dynamics of contagion within interbank credit networks (Nier et al 2007; Gai et al 2010; Battiston et al. 2012; Krause and Giansante 2012).[21] Such literature includes several clear-cut findings on the relationship between the topology

[20]For a broader review, please refer to paper one.

[21] For a literature review see Chinazzi and Fagiolo (2015) and Huser (2015).

of the interbank network and the contagion. One key finding is the non-linear relationship between the degree of connectivity and the number of defaults due to failure cascades occurring in an interbank network, usually called U or M shaped relationship. According to the latter, the increase of the connectivity of an interbank network tends to multiply the channels of contagion to a point where the risk sharing effect, which is due to the high connection level, prevails against the risk spreading effect (for the generalization of this finding to dynamic modelling, see Georg 2013). Nier et al. (2007) have highlighted the importance of the capitalization level, of the concentration level of the topology, and of the depth of the interbank market for the probability of a default cascade. Other works have put emphasis on fire selling as an amplifying factor of the contagion (Chifuentes 2007) as well as on financial accelerators (Battiston et al. 2012b), liquidity shocks (Gai and Kapadia 2011), resolution procedures (Gaffeo and Molinari 2015), and bank-size heterogeneity (Caccioli et al. 2012). In the present work, many of the findings emerged from the aforementioned literature will be confirmed. In particular, we will analyze the effect of the OnBS from three perspectives. As for the first perspective, we focus on the qualitative analysis of small-size networks with the aim of identifying the basic dynamics underpinning bilateral netting. The analysis of small credit networks has been performed by Allen and Gale (2000) and Lee (2013) to study the processes of contagion engendered by liquidity shocks. The advantage of focusing on small networks consists in the provision of an immediate understanding of the contagion process; the main disadvantage, on the other hand, is that moving from a simple structure to a complex one does not only change the scale but, in many cases, also the nature of the phenomenon under scrutiny. In this light, we consider the analysis of small networks as a starting point to be followed by mean field analyses and, eventually, simulations of networks composed by heterogeneous agents. As for the second perspective, thus, we focus on mean field methodology, which has been employed by May and Aranaminphaty (2010) to analyze default cascades in relation to the shocks that hit an intermediary's assets. Horst (2007) resorts to the same methodology to develop a credit rating model analyzing the contagion process of a network of companies. Borovykh et al. (2017) develops a mean field approximation model with the aim of studying the effect of financial accelerators and of fire selling on system risk. In general, the main strength of the mean field approach consists in the ability to analytically identify the conditions under which cascade phenomena can occur (see Barucca et al. 2017 for a review). The main limitation, on the other hand, consists in the strong assumptions of homogeneity imposed by such approach. As for the third and last perspective of our analysis, therefore, we built artificial credit networks populated with banks by following a procedure similar to that one of Nier et al. (2007). By doing so, we were able to compare the performance of bilateral netting with a theoretical benchmark supported by the extant and then analyze more realistic networks characterized by core-peripheries structures closer to the instances of the empirical work considered (Boss 2004; Mistrulli 2005; Fricke and Lux 2012; Craig and Von Peter 2014; Langfield et al. 2014).

Few works have investigated the relationship between OnBS netting and system risk. Unfortunately, these works delivered inconsistent results. The empirical analyses of Degryse and Nguyen (2007) and Duan and Zhan (2013) highlight a modest effect of deposit netting on the reduction of systemic risk, for what respectively concerns the interbank systems of Belgium and the United Kingdom. On the other hand, Upper and Worms (2004) note a strong effect of bilateral netting on the decrease of systemic risk, whereas Esinger et al. (2006) show a substantial irrelevance of bilateral netting to the data concerned with the Austrian interbank market. The inconsistency of these results is primarily due to the different topologies of the interbank markets of the countries considered and to the modes of estimation of the interbank loans, other than to the level of the shocks taken into account. Here we seek to provide a theoretical framework capable of accounting for such a heterogeneous set of results. Bilateral netting is also relevant to the distribution of systemic risk across the classes of creditors of the financial intermediaries. Emmons (1995) shows how the OnBS netting works as a transfer tool of the credit risk from non-senior to senior creditors. The model analyzed by Emmon leaves out default cascades; our work can be seen as complementary to his contribution, for we analyze the risk shifting effect of the netting and the distribution of the losses engendered by a shock on the assets among different classes of creditors in contexts of default cascades.

3. HOW BILATERAL NETTING WORKS

In this Section we provide an introductory discussion on how OnBS netting can affect the network topology by constraining interbank market transactions. We took into consideration four simple topologies, with the explicit aim of assessing whether this kind of settlement agreements could be reasonably tested in more complex environments. We limit ourselves to a qualitative proof-of-principle analysis by focusing on three variables: *i*) topology; *ii*) magnitude of the shock; *iii*) interbank loans' size.

3.1 Complete network

The first network we analyse is a complete one, as shown in Figure 3.1. This type of topology is characterized by the fact that each bank is linked to all the others by means of reciprocal interbank credit and debit obligations. The equity of each bank is defined as e_i, $i \in (1, n)$, n is set to 4, the interbank debt that bank i has towards any of the other intermediaries of the network is ib_i while the interbank asset is il_i. Deposits and non-interbank assets are labelled respectively as d_i and a_i. At the beginning we thus assume that every loan has an equal size. We will relax this hypothesis later on. Suppose that $B1$ is buffeted by an idiosyncratic shock f. If the default condition (1) holds true:

$$f > e_1 \qquad (1)$$

$B1$ defaults. In this case, $B1$ spreads out towards the whole system an amount of losses equal to l, as defined in equation (2):

$$l = \min\,[t, ib_i] \qquad (2)$$

where $t = f - e_1$. Now $B2$, $B3$ and $B4$ import an amount of losses equal to $m = \frac{l}{n-1}$ each, and their default condition is given by:

$$m \geq e_{i\neq 1} \qquad (3)$$

It easy to grasp that the magnitude f of the shock is the key parameter to understand the severity of contagion. Considering the gross settlement treatment first, we observe that for sufficiently small f the losses imported by $B_{i\neq 1}$ are capable to hurt them but not high enough to force them to default ($m < e_{i\neq 1}$). In this case the "risk sharing effect" – defined as the possibility to split l among a high number of counterparties – prevents a default cascade. Figure 3.2 shows how a netting procedure isolates $B1$ from the others by cutting all the links of the network. This happens because we assume loans equality. In this case the shock is totally absorbed by $B1$'s depositors and shareholders only, $f = \Delta e_1 + \Delta d_1$. It is important to note that if the shock f is large enough, netting is able to prevent three defaults at the expenses of *Bank1* depositors. In other words, the possibility to isolate a given number of nodes from contagion – lethal or not – ends up concentrating losses on the stakeholders of the buffeted bank.

Figure 3.1. Complete network with homogenous loans.

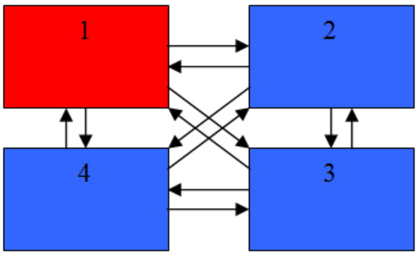
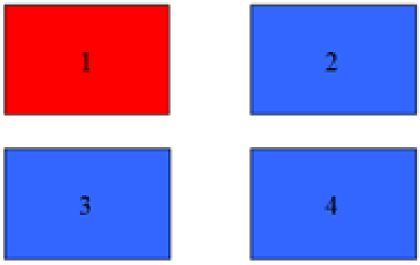
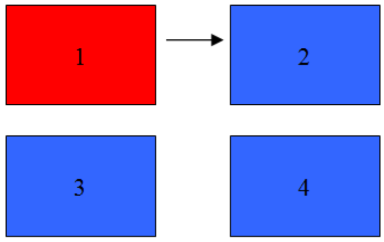


Figure 3.2. Complete network with homogeneous loans after netting.



As we relax the assumption of homogeneous loans, a fraction of the original shock can be transmitted from the defaulting bank to its net creditors even for a netted settlement mode. The effect on defaults is uncertain and depends critically on the variance of the deposit loans' size. For the sake of simplicity, we consider the case in which all loans are identical and equal to 5, but for the expositions between the first two banks. In particular, we assume that $B1$ owes 15 to $B2$, while $B2$ holds a debt towards $B1$ of 5. Equity is set at 5 for every bank. Figure 3.3 represents the interbank market after the netting treatment. Since $B1$ is still a net debtor towards $B2$, the offsetting of bilateral expositions cannot be complete. If the shock f is equal to 12, the amount of losses imported by $B2$ is 7. For this reason, $B2$ defaults. Note that if the netting had not been applied, risk sharing would have succeeded in avoiding the failure of $B2$, given that under the gross settlement mode the loss imported by each intermediary would have been equal to 7/3.

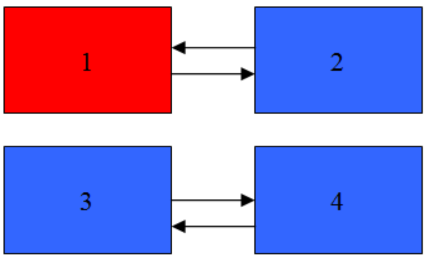
Figure 3.3. Complete network with heterogeneous loans after netting.



3.2. Disconnected network

In Figure 3.4 we represent a disconnected network with two independent subsamples of banks. In this context, independence means that there are not cross-sample credit-debit obligations. In this particular case, netting reduces or equalizes the amount of losses imported by the un-shocked bank for all possible values of f. Moreover, this happens independently of the loan size and the magnitude of the shock. The reasons are mainly two. First, the magnitude of the losses spread out by $B1$ thins during the netting procedure and it is zero for homogeneous loans. Second, there is none possibility of sharing risk, due to the dis-connectedness of the interbank market. When netting is applied and the shock f is high enough depositors of Bank1 score higher losses.

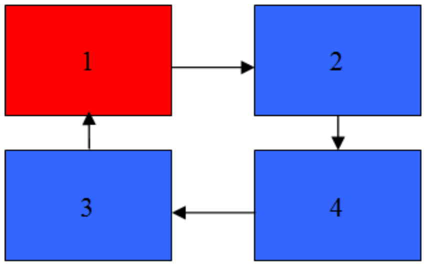
Figure 3.4. Disconnected network.



3.3. Incomplete network

A circular incomplete network is represented by an interbank market where each bank is at the same time net creditor and debtor towards two different intermediaries (Figure 3.5). In this particular topology, netting has no role given the absence of reciprocal obligations. So, when the shock f hits $B1$, netting could not be the solution for avoiding a financial contagion which inevitably fades away moving along interbank market.

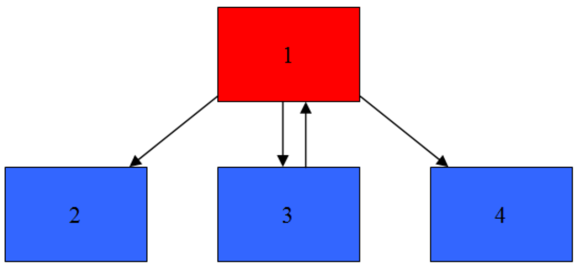
Figure 3.5. Circular network.



3.4. Tiered network

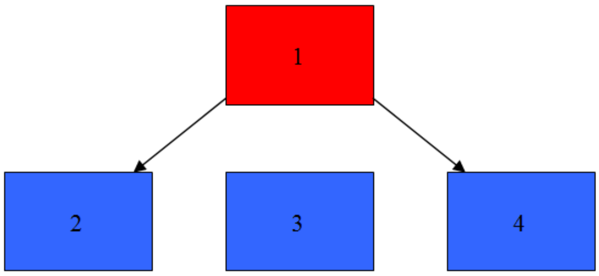
Real financial networks usually show a core-periphery structure. This means that interbank markets are characterized by a few big banks with high degree of connection linked to many small banks with a lower one. We portray this kind of topology in Figure 3.6, where $B1$ is a net debtor towards $B2$ and $B4$ and holds a balanced position towards $B3$. Considering homogeneity in loans and a gross settlement scenario, when the hub is buffeted by a big shock f we score a default for all the small banks linked to it. In the case in which f is small enough, the risk sharing effect spreads the smaller losses across lenders, preventing a default cascade.

Figure 3.6. Core-periphery network.



We observe in Figure 3.7 that a netting procedure isolates one of the three small banks. If the magnitude of the shock f is small, the concentration of the losses towards $B2$ and $B4$ is not sufficient to wipe out all their equity buffer. On the other hand, whenever f looms larger the imported losses by $B2$ and $B4$ could be sufficiently high to force them to default. In this case, a netting enforcement would not be appropriate.

Figure 3.7. Core-periphery after netting.



Summarizing, for this topology the magnitude of the shock should be the leading indicator triggering a "netting policy". Loan heterogeneity, in turn, affects our analysis by presenting the same traits explained for the complete topology.

4. NETTING IN RANDOM NETWORKS

The simple examples discussed above have allowed us to highlight how the interplay of shocks' size, heterogeneity of mutual expositions as well as their topological architecture can shape the impact of a regulatory policy mandating the close-out netting of interbank loans. We now extend this analysis to more complex market structures. This task is accomplished by recurring to both analytical and computational methods. Let us start from the case of a purely random network.

4.1 A mean-field approximation

In order to assess analytically the impact of bilateral netting in a large web of interconnected banks, we first apply the mean-field approximation technique proposed by May and Arinaminpathy (2010) to study contagion in interbank networks characterized by an Erdős-Rényi topology $G(n, p)$, where n is the number of nodes and p is the common probability of forming a link. For the sake of simplicity, we set the number of banks to $N = 25$. For each bank, we normalize total assets to 1. The gross value of interbank assets is defined as θ, so that external assets are given by $a = 1 - \theta$. Structured in this way the network is populated by banks of equal size and composition of the accounting balance sheet. Finally, e represents tangible net worth and z the average number of interbank lenders. In what follows a randomly chosen bank is hit by an exogenous shock that wipes out a fraction f of its external assets. To make the chain of defaults clearer, we split the analysis in subsequent phases of contagion.

4.1.1. Phase 1

In phase 1, an idiosyncratic shock:

$$S(1) = f(1 - \theta) \qquad (4)$$

causes the bankruptcy of a randomly chosen bank if:

$$S(1) > e. \quad (5)$$

Given the Erdős-Rényi topology, under the standard gross settlement mode the average degree of each node reads as:

$$z = p\,(N - 1). \quad (6)$$

Netting can be applied only if two nodes are linked by reciprocal edges. Since in a random network the average probability of finding a reciprocal link between two connected vertices is equal to the average probability of finding a link between any two vertices, the netted average degree of a representative node is equal to:

$$\hat{z} = z(1 - p) = p(N - 1) - p^2(N - 1) = \hat{p}(N - 1) \quad (7)$$

where $\hat{p} = p(1 - p)$. Equation (7) contributes to highlight the trade-off that regulators face when trying to tame contagion by imposing close-out netting agreements. In this case, the average per-capita loss that the defaulting bank transmits to each one of its lender is:

$$\widehat{Loss} = \frac{[S(1) - e]}{\hat{z}} > \frac{[S(1) - e]}{z} = Loss. \quad (8)$$

In other terms, while the closing of bilateral exposures causes a reduction of possible channels of contagion, the total loss imported by any lender who is not allowed to disconnect itself from the defaulted bank is larger. At a systemic level, this generates a trade-off between a risk-shielding effect and a loss-concentration effect. As we will show below, the weigh between the two prongs of the trade-off depends on the interconnectedness of the interbank market. Given that along the mean-field approximation all interbank loans are equal, if $\hat{z}$ is lower than z by a percentage $\phi > 0$, $\hat{\theta}$ decreases by the same proportion when compared to θ. Hence, we can adopt the following change of notation to redefine $\underline{\hat{z}}$ and $\hat{\theta}$ respectively as:

$$\hat{z} = z(1 - \phi), \quad (9)$$

$$\hat{\theta} = \theta(1 - \phi). \quad (10)$$

For the homogeneous Erdos Renyi network we are studying, $\phi = p$.

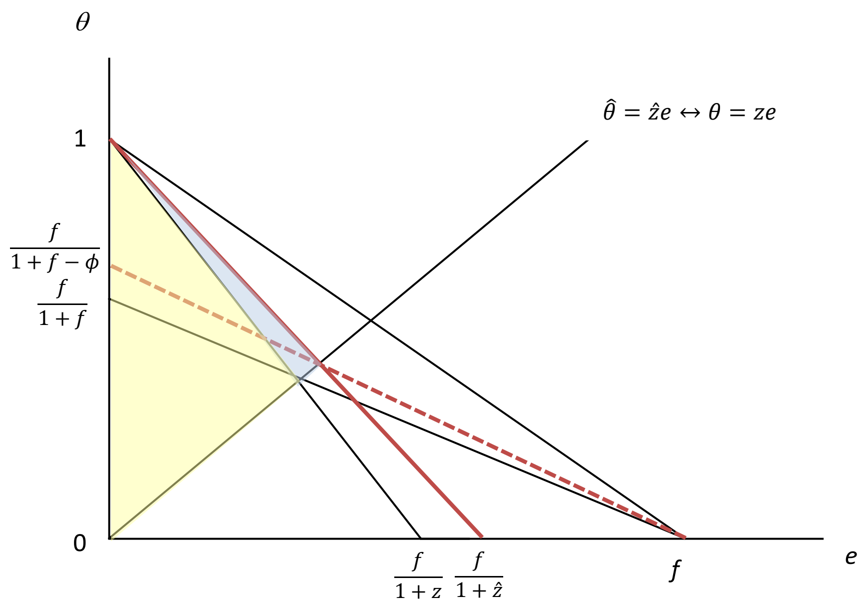
4.1.2. Phase 2

We now proceed to determine the impact of the first wave of contagious losses imported by the lenders of the initially shocked bank. When close-out netting is allowed, on average this amount is given by:

$$S(2) = \frac{min[\hat{\theta},\ S(1)-e]}{\hat{z}} \qquad (11)$$

that, as soon as $S(2) > e$, causes a cascade of additional defaults. Figure 4.1 illustrates the average "phase 2" propagation of the original shock over the whole network by applying the same graphical solution used by May and Arinaminpathy (2010). The yellow light-shaded area represents the region, expressed in the (e, θ) space, in which the creditors of the originally affected bank will receive a second-round disturbance causing them to fail if interbank loans are settled on a gross basis. The blue light-shaded area, in turn, returns the increase in imported losses when a close-out netting settlement is imposed.

Figure 4.1. The blue light-shaded area represents the increase of second-round imported losses associated to netting, expressed as a region of interbank lending θ and tangible capital e.



The mean-field approximation allows us to calculate exactly the number of banks involved in this phase of contagion. For a network composed of 25 banks and $p = 0.2$, for instance, while in the gross settlement mode second-round effects distress 5 banks, in the case of close-out netting – in which the "netted" $\hat{p}$ becomes 0.16 – only 4 banks are bound to import fatal losses.[22] Higher values of p widths the gap between p and $\hat{p}$, shrinking the number of creditor banks involved in the second-round propagation of the shock. At the same time, however, the increase in interconnectedness causes the dashed red line to rotate upward and the continuous red line to rotate rightward, meaning that each single bank affected by contagion faces a larger amount of losses.

[22] These numbers are the nearest integers of the exact values 4.8 and 3.8, respectively.

4.1.3 Phase 3

Higher-round effects are more difficult to calculate, since now the source of contagion is not univocal as in the previous phase but can be due to a multiplicity of nodes that became infected during the second-round stage. Furthermore, a single bank that possesses enough capital to survive a first attack may contribute to spread the contagion further, with the result that the cascading process can backfire and causes it to fail later on. In general terms, under the netting settlement the shock transmitted during "phase 3" by a "phase 2" distressed bank to one of its counterpart is:

$$S(3) = \frac{min\left[\hat{\theta}, \frac{min(\hat{\theta}, S(1)-e)}{\hat{z}} - e\right]}{\hat{z}} \qquad (12)$$

while the bank absorbing this shock goes bankrupt if $k_c\, S(3) > e$, where k_c is the number of times a bank is hit during this phase, under the assumption that contagious events are binomially distributed. According to the calculations reported in May and Arinaminpathy (2010), under the gross settlement mode the third-round stage of contagion evolves so that eight banks are hit once, four banks are hit twice, and one bank is hit three times. Using the same approach, it turns out that when defaulting banks are allowed to activate bilateral netting agreements eight banks are hit once, two banks are hit twice, while none bank is hit three or more times. As we count the total number of times at least a single node is involved in the default cascade during the second and third stages, the gross mode reports a score of 18 (5 nodes in phase 2 and 13 nodes in phase 3), that we can compare with a total score of 14 for the netting case (4 in phase 2 and 10 in phase 3, respectively). Once again, it is straightforward to show that as the probability of interconnectedness increases, the risk-sheltering advantage of allowing defaulting banks to net their mutual interbank exposures looms larger. By the same token, the loss-concentration effect magnifies the damages for all the banks that are not able to isolate themselves through the offset of mutual exposures.

4.2. Simulations

4.2.1 Initialization procedure

The results from the analytical model just discussed are now checked by means of the well-known computational framework developed by Nier *et al*. (2007), so that we can easily compare contagion dynamics under netting with those obtained in the standard case with gross interbank exposures. More in detail, we aim to assess how netting agreements affect the non-linear relationship between the number of insolvency-driven defaults and the degree of interconnectedness detected in an array of previous works. In what follows, we explain the network generating process, characterize the default cascade, introduce the three metrics employed to assess contagion dynamics, explain the netting algorithm and finally discuss the results of our Monte Carlo simulation exercises. We characterize a financial network through two main parameters, namely the number of banks (nodes) populating the network (N) and the level of connectivity defining the probability (p) that a credit-debit obligation arises between any two banks. Following *Nier et al*. (2007), we begin our analysis with a random Erdős-Rényi topology, in which all nodes share the same attachment probability. Recognizing that real financial cobwebs are often described as core periphery structures, we shall afterward investigate the extent to which our benchmark results are robust as we consider to tiered networks made up by a few highly connected nodes (i.e. hubs with a high p) surrounded by many lowly connected ones (i.e. the periphery with a low p). Once the network of interconnections has been shaped, we assign to each intermediary a balance sheet consistent with a double-entry book keeping system. On the asset side, each bank i is endowed with an amount of external assets a_i and interbank lending il_i. On the liability side, each bank scores interbank borrowing ib_i, deposits d_i and its stock of equity e_i (see Table 4.1). By construction, the sums of asset side components (A) and of the liability ones (L) are equal ($L_i = ib_i + d_i + e_i$; $A_i = a_i + il_i$, $L_i = A_i$).

Table 4.1. Stylized balance sheet of a bank.

Assets	**Liabilities**
l_i	b_i
a_i	d_i
	e_i

In order to obtain stock consistency, we proceed by applying a three-step procedure. The first step consists in fixing the amount of total asset (TA) available in the network and, at the same time, the proportion ω of external assets (EA). Then, we set the value of the total amount of interbank assets $IA = TA - EA$ and compute the size of each single loan dividing IA by the total number of links of the network. Following this procedure all loans are of equal size, and later on we will relax this hypothesis allowing for heterogeneous nominal values for interbank contracts. We obtain the total amount of interbank lending and borrowing of each intermediary by multiplying the standardized size of a loan by the number of total number borrowers and lenders of each bank i. We represent the network cobweb of the interbank market by means of an adjacency matrix X where the sums over rows yield the $il_i = \sum_{j \in N} x_{i,j}$ and the sums over columns $ib_i = \sum_{j \in N} x_{j,i}$:

$$\boldsymbol{X} = \begin{bmatrix} 0 & \cdots & x_{1j} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & \cdots & 0 & \cdots & x_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{Nj} & \cdots & 0 \end{bmatrix}$$

We then recover the amount of total assets for each bank. As in Nier *et al*. (2007), we initially set the bank's external assets equal to $\overline{a_i} = max(0, ib_i - il_i)$. As we sum all the external assets $\sum_{i \in N} \overline{a_i}$, we are able to assign to each bank a common residual portion of total external assets computed as $\frac{(EA - \sum_{i \in N} \overline{a_i})}{N}$ on top of its idiosyncratic component $\overline{a_i}$, so that $a_i = \frac{(EA - \sum_{i \in N} \overline{a_i})}{N} + \overline{a_i}$. The stock of capital of each bank is computed by multiplying its total assets by the leverage ratio γ. We compute the amount of deposits of each bank by subtracting to the amount of total assets the value of each interbank borrowing and the amount of equity. Once completed, this array of sequential steps allows us to fully characterize a balance-sheet-consistent network. We exogenously trigger contagion dynamics by hitting the artificial financial system by means of an idiosyncratic shock that buffets all the external assets of a randomly chosen bank. The shocked bank covers the losses first with its capital. The residual shock (if any) in excess of the bank's equity base is then spread evenly among the interbank creditors of the defaulting bank and, if necessary, to retail depositors that act as the ultimate sink bearing the leftover loss (phase1). As explained e.g. in Furfine (2003), a default cascade can unravel as follows.[23] First, the loss imported by each creditor of the shocked bank is absorbed by its equity. Shall this not be enough to absorb the imported loss, the creditor bank will also default (phase 2) and the further losses are again distributed among his creditors, possibly bringing about an additional wave of defaults (phase 3). If the cumulated imported losses are high enough, they could lead to a systemic collapse of the whole system through a domino effect, which stops when the exogenous shock fully propagates to non-interbank liabilities (i.e. equity and retail deposits).

In order to assess the degree of disruption induced by the contagion cascade, we resort to three metrics that capture different dimensions of systemic risk. The choice of the metrics is a consequence of the type of shock we have modeled. Shock on external assets fuels the dynamics of contagion transmitted by the interbank obligations and absorbed, in a last resort, by capital or external deposits. Therefore, we highligths these two dimensions.

We focus first on the default variable defined as the total number of banks which holds zero capital at the end of the contagion. We set a default variable $Def = \sum_{i \in N} def_i$, where each *def* takes the value of 1 when the bank *i* becomes insolvent (i.e. when equity turns negative) and 0 when the bank survives to the contagion process. Notice that this is the foremost metric that the regulator observes in order to understand the deepness of a crisis. As a second distress metric, we look at capital erosion measured at the end of the contagion. This gives us a measure of post-contagion financial system's health status, from which one can gauge its capacity to face future turmoil. Our metric of capital erosion (*CE*) is computed as $CE = \frac{(FC - IC)}{IC}$, where *IC* and *FC* are the sum of the net worth of each intermediary respectively before and after the crisis. The third and final index focuses on the deposit side of the story and is computed as $DD = \frac{DD}{S}$, where *DD* is total amount of deposits burnt during the contagion process, while *S* is the absolute value of external assets wiped out by the idiosyncratic shock. The Table 4.2 summarizes the set of metrics.

[23] Notice that this is equivalent to the dynamics developed in the mean-field approximation exercise of Section 4.1.

Table 4.2. Metrics overview.

Metrics	Analytic	Description
Default (*Def*)	$Def = \sum_{i \in N} def_i,$	Total number of banks which holds zero capital at the end of the contagion process.
Capital Erosion (*CE)*	$CE = \frac{(FC - IC)}{IC}$	Total percentage capital erosion measured at the end of the contagion process.
Distressed Deposit (*DD*)	$DD = \frac{DD}{S},$	Percentage portion of the shock absorbed by the external deposits.

To model the netting settlement, we resort to a netting operator *F* that re-arranges interbank credit-debit obligations. The procedure consists in allowing banks to offset their mutual exposures every time there is a bilateral reciprocal relationship between them. This occurs every time a bank *i* at the same time borrows from and lends to another specific bank *j* of the network. Formally, each element of the weighted adjacency matrix *X* is filtered through the netting operator as follows:

$$x_{ij}^{n} = F(x_{ij}) = max(0, x_{ij} - x_{ji}), \;\; \forall\, i, j = 1, \ldots, n \qquad (13)$$

When the network is populated by homogenous loans, the netting algorithm resets to zero reciprocal claims by severing both links between the two banks affected by the netting algorithm. In case of heterogeneous loans, the offsetting procedure still reduces bilateral exposures but it cuts only one of the two links. For example, let us consider the following weighted adjacency matrix *X*:

$$X = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 \\ 5 & 2 & 0 & 1 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

featuring some interbank exposures that are unidirectional, some that are bidirectional and homogeneous, and finally some bidirectional but heterogeneous ones. As one can see from the filtered matrix *X*', all unidirectional links remain unchanged, all homogenous reciprocal links disappear and among the heterogeneous reciprocal ones, only the smallest in size disappears, while the largest one is cut down to its netted amount:

$$X' = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

4.2.2 Test with homogeneous loans

Table 4.3. shows the values of the network parameters used in simulations.

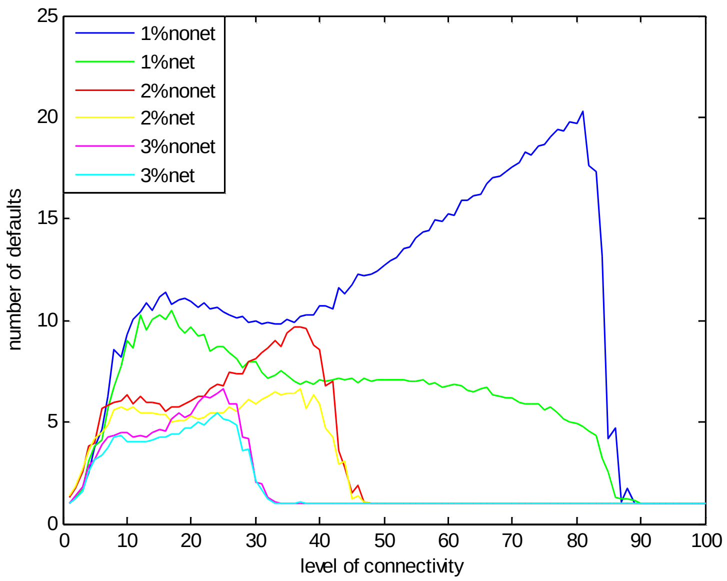
Table 4.3. Parameter values for benchmark simulations.

Interconnectedness	0-100
Network size (*N*)	25
Shock size as a % of external assets	100%
Interbank asset relative to total assets	20%
Bank Capitalization/Total Assets	1%-2%-3%

Figure 4.2. displays the average default rate as a function of *p*. The defaults profiles under the gross settlement (reported as "no net" in Figure 4.2 and those that follow) replicate the findings in Nier *et al*. (2007). The perfect replicability and comparability with a work well known in the literature is the main reason for choosing the size of the shock at 100%. Later, moving towards more realistic topologies, we model shocks of different sizes.

For example, when the system is undercapitalized at 1% a clear M-shape relationship between the default rate and connectivity is detectable. How do these profile change when netting is enforced during the contagion process?

Figure 4.2. Default profiles, homogeneous loans.



We observe that netting is more effective than the gross treatment in shielding the system from widespread default cascade. The contagion profiles observed when netting is at work are in fact consistently lower than those found under the standard rule. The advantage of netting over the gross settlement procedure increases when the bank capitalization decreases. More in detail, for lowly capitalized network, the usual M-shaped curve disappears and the global maximum of the default profile for netted networks is lower than the local minimum found in networks with gross exposures. Another important finding for scarcely capitalized network is that the higher the connectivity, the stronger is the benefit of netting. This is due to the inner dynamics of the trade-off emerging when netting is enforced, namely that between concentration effects and risk-sharing ones. More precisely, the netting treatment cuts the channels through with contagion runs while, at the same time, forcing the shocked bank to internalize a loss higher than the one it would score under the gross settlement mode. On the one hand, netting contributes to lower the "real shock" spread by the buffeted bank and hence reduces systemic risk. On the other hand, it can diminish the overall risk-sharing capacity of a fully interconnected system. In our simulations the concentration effect overwhelms the risk sharing one and netting ultimately outperforms the standard treatment under a wide range of scenarios within the parameters space spanned by our experiments.

Let us now examine how bank capital responds to netting during the contagion. Figure 4.3 displays the extent of capital erosion under gross and net settlements and shows that netting allows for a significant equity saving. Such advantage in terms of preserving capital is stronger for highly connected network and the result is consistent for every level of capitalization. In general, in this kind of model a shock runs in the network from debtors to creditors through interbank claims, and it is absorbed by capital and deposits, as explained earlier. It follows that we expect a negative correlation between the value of deposits and equity erosion.

Figure 4.3. Capital erosion profiles, homogeneous loans.

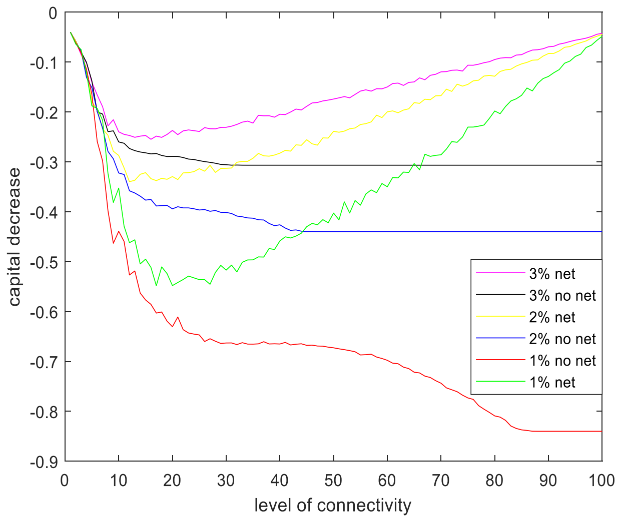


Figure 4.4 shows an increase in deposit erosion when netting agreements are employed. For all capitalization levels, there is a monotonic relationship between the connectivity level and the difference of deposit erosion between the two treatments. In other words, the higher the level of

connection among banks, the higher the portion of shocked charged on depositors at the initial stage for netting treatment. This can be rationalized recognizing that the number of netted contracts increases with the degree of interconnectedness. Therefore, the netting-induced shrinking of the interbank market depth tends to eliminate a class of junior debtors making depositors more vulnerable when a bank faces a consistent shock. There is also an effect due to the level of capital. The lower the level of capital of each bank, the greater the share of shock spread in the interbank market and, ultimately, the higher is the diffusion of contagion. The interplays between connectivity and capital effects seems to explain why the divergence between the two treatments is more pronounced for low levels of capital.

Table 4.4 shows instead how the losses absorbed by deposits are concentrated among the depositors of the bank hit by the shock in both treatments. At low connection levels of the network, more than 90% of the affected deposits is concentrated in the shocked bank, whereas at lower-middle, medium, and high connection levels, almost all of the affected deposits are detained by the shocked bank. The concentration of deposits losses is well-suited indicator to understand the diffusion level of the contagion.

Figure 4.4. Deposit erosion profiles, homogeneous loans.

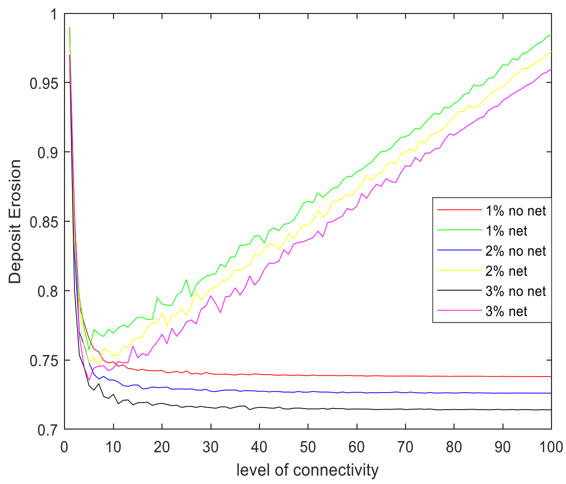


Table 4.4 Deposits Loss Concentration for homogeneous loans.[24]

Conn.	3% net	3 %gross	2% net	2% gross	1% net	1% gross
0-20	0,94	0,93	0,93	0,92	0,92	0,91
20-40	0,99	0,99	0,98	0,98	0,99	0,98
40-60	1	1	1	0,99	0,99	0,99
60-80	1	1	1	1	0,99	0,99
80-100	1	1	1	1	1	1

[24] Table reports DDf/DD, where DDf is the amount of deposit eroded at the shocked bank. Level of connectivity *p* in the rows (mean of the range), treatments and capital level in the columns.

4.2.3 Test with heterogenoeus loans

Next, we introduce heterogeneity in loan size for the same kind of network we have just tested. Heterogeneity is introduced slightly twisting the interbank asset assignment procedure. More precisely, after having fixed the value of interbank asset $IA = TA - EA$, we compute the total interbank lending for each bank as $il_i = IA/N$. In this way each bank is endowed with same amount of interbank asset. Finally, we divide il_i by the number of links randomly assigned to each bank. Since the number of activated links is potentially different for each bank, individual loans can change size from one bank to another. It must be pointed out that, given that the attachment probability p is equal for every bank, the implied variance of the nominal value of loans is not high, which is appropriate for this type of topology characterized by banks of similar size and equal degree of connection.The main difference compared to the previous procedure is that we set a homogeneous loan size for each bank made up of non-homogeneous contracts. On the contrary, previously, banks were built with a non-equal value of interbank credit, but each single loan was of the same value.

The outputs of our simulations are reported in the three figures (4.5, 4.6, 4.7, respectively) hereunder.

Figure 4.5. Default profiles, heterogeneous loans.

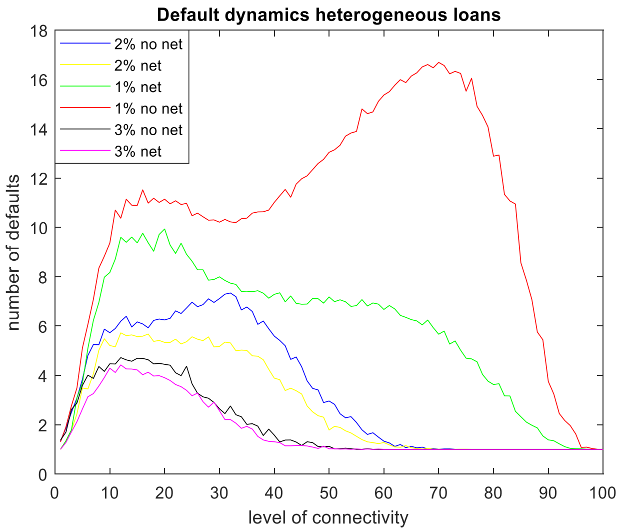


Figure 4.6. Capital erosion profiles, heterogeneous loans.

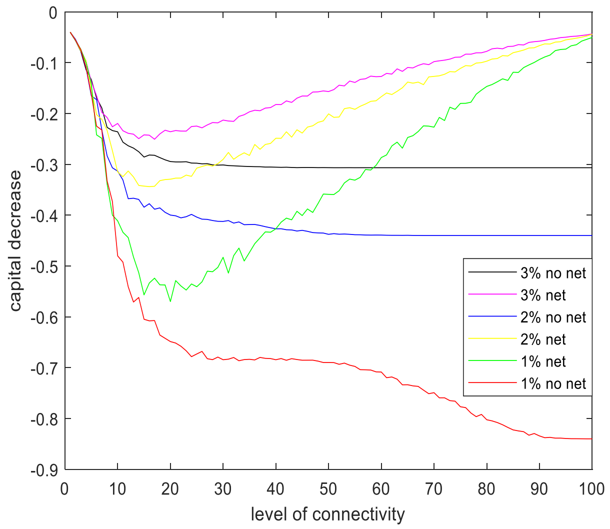


Figure 4.7. Deposit erosion profiles, heterogeneous loans.

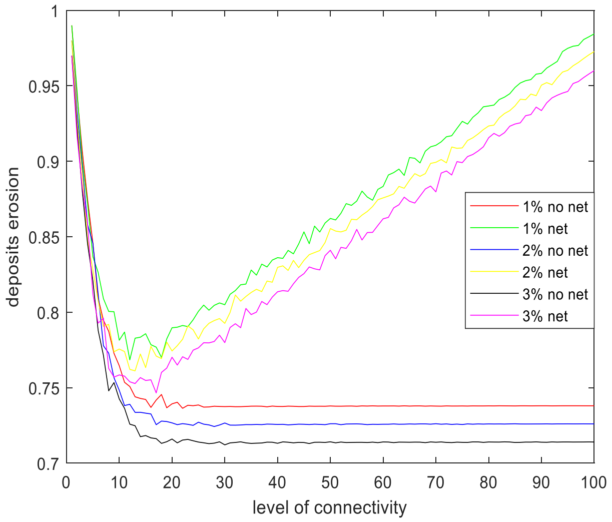


Table 4.5 Deposits Loss Concentration for heterogeneous loans. Table reports DDf/DD, where DDf is the amount of deposits eroded at the shocked bank. Level of connectivity *p* in the rows (mean of the range), treatments and capital level in the columns.

Conn.	**3% net**	**3 % gross**	**2% net**	**2% gross**	**1% net**	**1% gross**
0-20	0,95	0,95	0,95	0,95	0,96	0,94
20-40	0,99	0,99	0,99	0,99	0,99	0,99
40-60	1	0,99	1	1	1	0,99
60-80	1	1	1	1	1	0,99
80-100	1	1	1	1	1	0,99

For what concern default dynamics, we observe that outputs are very similar to those of the homogeneous treatment, even if the M-shape relationship is less pronounced for less capitalized networks. Moreover, the average number of defaults is lower with respect to the homogeneous cases for the gross settlement, while it is almost unchanged for the netting treatment. Netting supremacy is robust to the introduction of heterogeneity for almost all Monte Carlo simulations. In Figure 4.6 we show the capital preservation at the end of contagion process. Also for this variable the outputs are very similar to the homogeneous cases, showing a strong netting capital preservation. More precisely, we detect an increase in the gap between the two treatments once the connection probability increases. The last graph shows the deposits erosion. As illustrated before, the output remains the same one observed with homogeneous loans while the concentration of losses charged over the deposits remains unaffected as shown in Table 4.5.

5. CORE-PERIPHERY NETWORKS

As a robustness check we move towards a more realistic network structure by addressing the case of a core-periphery topology. Furthermore, the capitalization level is assumed in line with the one registered in real data. In particular, we compare results for levels of leverage equal to 3%, 4.5% and 6%. Tiered networks are shaped by modifying the procedure described above in a minimal way. In particular, we divide the population of N banks into two subsets, one composed of 4 *hub banks* and the other one of 21 *normal banks*.

Loans are not homogeneous, as we identify four types of loan sizes. Two specific intra-group types of loan, respectively on contracts among large banks only (*lb)* and contracts among small banks only (*sb*). Cross-subset interbank markets are characterized by loans from large to small banks (*ls*), and by loans from small to large ones (*sl*). We obtain this type of heterogeneity as follows. First, we assign a proportion μ of interbank and external assets to small banks, and 1- μ to the larger ones. Then, we took a proportion α from the amount of the small banks' interbank assets, and we divide it by the number of links among small banks, obtaining the value of *sb.* Similarly, we proceed multiplying the amount of small interbank assets by (1- α) and then dividing by the number of small banks interbank credits towards large banks in order to quantify *sl.* The value of *lb* and *ls* are computed in the same way multiplying the amount of large banks interbank assets by η *and* (1- η) and then dividing again for the number of links among large and large to small banks. Subsequently, the identification procedure of the network proceeds like in the case of Erdös-Rényi; the only exception lies in the fact that the proportions of external assets attributed to large and small banks are respectively 1- μ e μ. Finally, we model heterogeneous shocks considering three types of sizes respectively: large (100% of external assets), medium (50% of external assets) and small (25% of external assets).

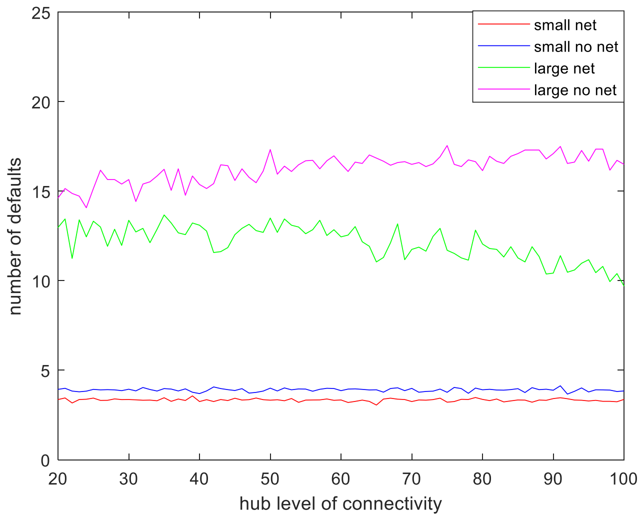
Table 5.1. Parameter values for simulations, core-periphery networks.

Network size *N*	25
Number of hub and small banks	4-21
Hub banks connectivity	0.2-1
Small banks connectivity	0.2
Shock size (portion of external assets)	1-0.5-0.25
Interbank asset proportion respect to total assets	0.3
Small banks proportion μ of interbank assets	0.5
Intra-group proportion α of small banks interbank assets	0.3-0.4-0.5
Intra-group proportion η of hub banks interbank assets	0.7-0.6-0.5
Inter-group proportion 1-α of small banks interbank assets	0.7-0.6-0.5
Inter-group proportion 1- η of hub banks interbank assets	0.3-0.4-0.5
General level of capitalization	3%-4,5%-6%

Considering the ample spectrum of parameters employed, here we focus on results for the case of a large shock affecting a hub or a peripheral bank, respectively.[25] More precisely, we separate the contagion dynamics generated by a shock on one small banks from the cascades originating by a shock disturbing one hub bank. The rational for this choice is that of emphasizing the different role that each type of bank plays in a contagion process. The key importance of the capitalization level in driving results will be touched upon as a latter point. The output of our simulations is rather clear. For what concerns the dynamics of defaults, we identify an overall strong dominance of the netting treatment (Figure 5.1). Looking at the graph, we observe that netting and gross treatments perform barely equally when an idiosyncratic shock hits an intermediary who belongs to the periphery. In this case the trade-off between the loss-concentration and the risk sharing effects is balanced. In turn, by looking at the default dynamics generated by a shock to the core we observe a consistent divergence between the two treatments, with a clear advantage associated to netting. This is due to the fact that the offsetting of mutual exposures ensures to isolate the big distressed hub not only from the periphery, but also from the other core banks. The gap between the two treatments is consistent when the magnitude of shock is large or medium, reaching a 40% difference. Lower gaps are scored for small shocks, even if the dominance of netting remains a constant for all the simulations. We find a positive relation between the advantage of netting and the level of connectivity, mainly due to the loss-internalization effect.

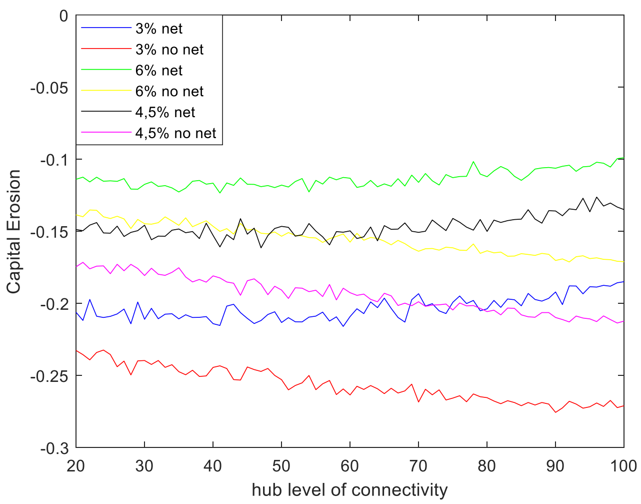
[25] For a comprehensive exploration of all the possible combinations see the Appendix.

Figure 5.1. The figure shows the default dynamics when a shock hits a large hub bank or a small peripheral one for the two treatments. Large shock. Parameters: α=0.5, *η=0.5* and *γ* =0.03.



For what concerns the metric measuring the depletion of aggregate capital we observe a picture fully consistent with the one just discussed (Figure 5.2). In this case we focus on the profiles emerging by averaging over shocks to large and small banks, but we explore different levels of capitalizations. The portion of capital burnt out during the contagion process is inversely related to the level of capitalization. Hence, a system characterized by an initial higher leverage is associated with a relative worst performance in terms of depleted capital. By charging a higher portion of the shock on depositors, the internalization effect lowers the amount of losses spread through the system. This effect becomes stronger the higher is the level of connectivity of the hub, since this is associated to a lower depth of the interbank market.

Figure 5.2. Capital erosion profiles for all level of capitalization and for all treatments. Large shock. Parameters: α=0,5 and *η=0,5*.



Finally, we look at deposits dynamics in Figure 5.3. For what concerns this metric the gross treatment performs better. As explained before, this is the logical consequence of the results observed for the other two metrics. We detect an inverse relationship between the amount of equity and the portion of the shock absorbed by depositors. This means that the higher is the level of capitalization, the lower is the burden charged on depositors. Table 5.2 shows how the losses absorbed by depositors in the case of the default of a hub bank are concentrated almost exclusively in the shocked bank. The finding is true for all capitalization levels, types of settlement, and connection levels.

Figure 5.3. Deposit erosion profiles. Large shock. Parameters: α=0,5 and *η=0,5*. The figure includes the dynamics for all levels of capitalization and for all treatments.

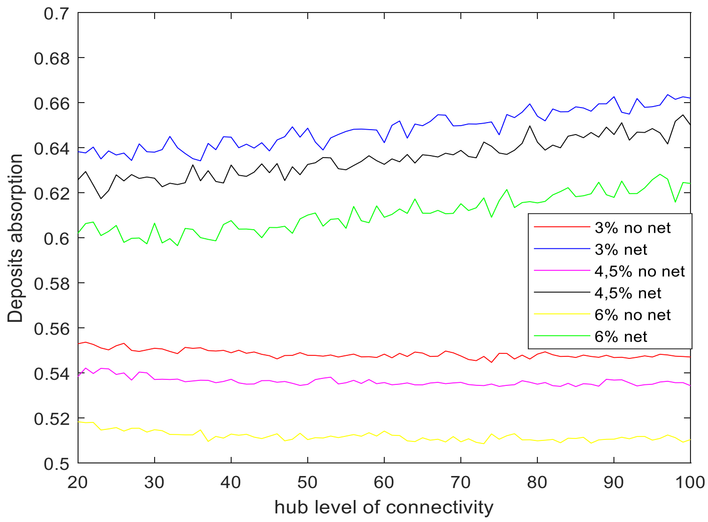


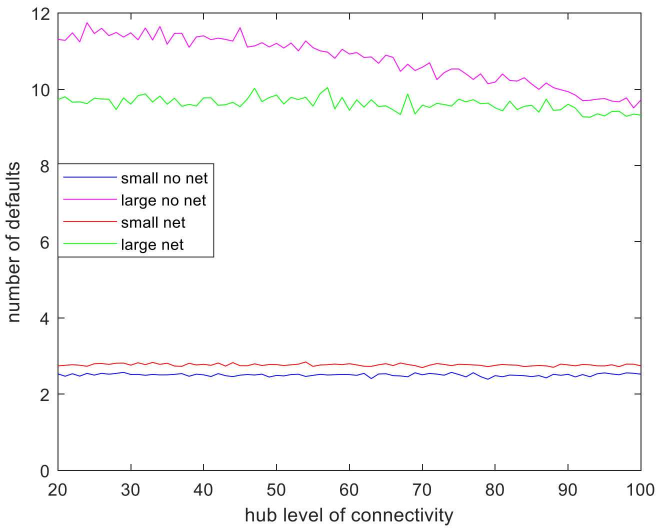
Table 5.2. Deposits Loss Concentration for core periphery.[26] Table reports DDf/DD, where DDf is the amount of deposit eroded at the shocked bank. Level of connectivity *ph* in the rows (mean of the range), treatments and capital level in the columns. The table considers only the shocks on the hub banks.

Conn.	**3% net**	**3 % gross**	**4,5% net**	**4,5%gross**	**6% net**	**6%gross**
20-40	0,98	0,98	0,99	0,98	0,99	0,99
40-60	0,99	0,99	0,99	0,99	0,99	0,99
60-80	0,99	0,99	0,99	0,99	0,99	0,99
80-100	0,99	0,99	0,99	0,99	0,99	0,99

As a final point, it is worthwhile to notice that the relative performance of netting in a core-periphery network is a function of the intensity of the shock. Figure 5.4 replicates the same experiment conducted so far, but for the magnitude of the disturbance hitting banks, that now amounts to 25% of external assets.

[26] Table reports DDf/DD, where DDf is the amount of deposit eroded at the shocked bank. Level of connectivity *ph* in the rows (mean of the range), treatments and capital level in the columns. The table considers only the shocks on the hub banks

Figure 5.4. The figure shows the default dynamics when a shock hits a large hub bank or a small peripheral one for the two treatments. Small shock. Parameters: α=0.5, *η=0.5* and *γ* =0.03.



When the small shock impacts a bank belonging to the core, the final number of defaults is lower, but the relative performances of the two treatments are reversed.

6. CONCLUDING REMARKS

The main aim of this paper was to investigate the impact of OnBS bilateral netting in improving systemic financial stability. In particular, this kind of agreements tends to isolate banks cutting interbank links off. This procedure has mainly two effects. On the one hand, it lowers the channels of contagion limiting the number of banks involved in the default chain. On the other hand, netting lowers the risk sharing ensured by interbank mutual expositions, concentrating losses towards a fewer number of intermediaries. The effect of these two opposite forces is uncertain. Our simulations have shown a dominance of the loss-concentration over the risk sharing effect for a wide range of parameters, different topologies and shock sizes. The dominance of netting over the default and equity metrics is mainly due to the fact that the concentration effect allows to charge a higher portion of the initial shock on the depositors of the hit bank. In this way, netting tends to preserve the rest of the financial system.

Our work corroborates several findings already known in the extant literature on standard settlement procedures and extend them to the topic of bilateral netting. In particular, we identify an inverse relationship between capitalization level of the network and financial contagion phenomena, other than a non-linear relationship of the U shape kind between connection level and default cascades. In the case of heterogeneity of the size of the banks populating the network, large-size banks confirm to be risk spreaders. As for the relationship between this work and the empirical literature, our results can provide an explanation for the discordant empirical results. In particular, our analysis finds a substantial irrelevance of the OnBS netting to networks which are adequately capitalized vis-à-vis homogeneous topologies and small-magnitude shocks. On the contrary, the effect of the clearing procedure demonstrates to be considerably relevant to the prevention of default cascades vis-à-vis medium and large shocks, network with lower-middle capitalization, and heterogeneity of the size of the intermediaries.

Some policy suggestions follow naturally. First, regulatory authorities should be able to identify the shock that the system faces as soon as it appears. Then, they have to evaluate the degree of interconnectedness of the distressed bank, his size and his location in the interbank network. Finally, in the case the shock hit a hub banks a netting enforcement combined with an intervention of national deposits insurance fund would be efficient in preserving market stability.

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A.1. Analytics of the mean-field approximation of Section 4

A.1.1. Phase 2 (early phase of contagion)

In phase two we detect the impact of the first wave of losses imported by the lenders of the bank defaulted in phase one. In the case of a gross settlement mode, the per-capita loss imported by z banks is represented by the condition:

$$S(II) = [\theta, S(I) - e] \min / z \qquad \text{(A.1)}$$

Considering the netting case, in turn, the loss imported by each loss the $\hat{z}$ banks $\hat{S}(II)$ becomes:

$$\hat{S}(II) = [\hat{\theta}, S(I) - e] \min / \hat{z} \qquad \text{(A.2)}$$

For two treatments, respectively, the z and $\hat{z}$ lenders fail if:

$$S(II) > e \qquad \text{(A.3)}$$

$$\hat{S}(II) > e \ . \qquad \text{(A.4)}$$

Solving for conditions (A.3) and (A.4), we obtain the threshold values:

$$\theta_c = (f - e)/(1 + f) \qquad \text{(A.5)}$$

$$\hat{\theta}_c = (f - e)/(1 + f - \phi) . \qquad \text{(A.6)}$$

If $\theta < \theta_c$ the default condition for the gross case is indicated by the following condition:

$$\theta > z * e . \qquad \text{(A.7)}$$

Having substituted $\hat{\theta} = \theta * (1 - \phi)$ for the netted case, we find that the default condition is the same as the gross case as shown in the following expression:

$$\theta * (1 - \phi) > z * (1 - \phi) * e , \qquad \text{(A.8)}$$

while if $\theta > \theta_c$ and $\hat{\theta} > \hat{\theta}_c$, the conditions for the two treatments become:

$$\theta < 1 - (e * (1 + z)) / f \qquad \text{(A.9)}$$

$$\theta < 1 - (e * (1 + z(1 - \phi))) / f \ . \qquad \text{(A.10)}$$

Finally, for the gross case the condition on capital for not defaulting is represented by:

$$e > f / (z * (1 + f)) + 1) \qquad \text{(A.11)}$$

while for the netted case the equivalent expression is:

$$e > f / (z * (1 + f) - (z * \phi) + 1) . \qquad \text{(A.12)}$$

A.1.2. Phase III (crisis propagation)

During the previous phase each bank could import losses only from the bank failed in phase I. In this phase, we must take into consideration also multiple imported losses, that is the imported losses coming from different defaulted banks. In the gross settlement mode, the imported losses for each bank are defined by:

$$S(III)=\{\theta,[\theta,\ S(I)\text{- e}]\ min\ /z\text{- e }\}min/z, \qquad \text{(A.13)}$$

while for netting treatment the condition is represented by:

$$\hat{S}\ (\mathrm{III})=\{\hat{\theta},[\ \hat{\theta},\ S(I)\text{- e}]\ min\ /\hat{z}\text{ - e }\}min/\hat{z}\ , \qquad \text{(A.14)}$$

The default conditions now read as:

$$k_c * S(III) > e \qquad \text{(A.15)}$$

$$k_c * \hat{S}(III) > e \qquad \text{(A.16)}$$

where k_c is the number of times a bank is hit in phase III. Solving (A.15) and (A.16) we obtain the default condition for the two treatments, respectively. For $\theta_c > \theta$ and $\hat{\theta}_c > \hat{\theta}$, where $z^* = z/k$ and $\hat{z}^* = \hat{z}/k$, we have:

$$\theta > (e*(1+z^*)*z) \qquad \text{(A.17)}$$

$$\theta > (e*(1+\hat{z}^*)*\hat{z})/(1-\phi) \quad \text{or} \quad \theta > e*(1+\hat{z}^*)*z\,. \qquad \text{(A.18)}$$

On the other hand, when $\theta_c < \theta$ the gross settlement default condition is:

$$\theta < 1-(e*(1+z+(z*z^*)))/f \qquad \text{(A.19)}$$

while for netting treatment:

$$\theta < 1-(e*(1+\hat{z}+(\hat{z}*\hat{z}^*)))/f\ . \qquad \text{(A.22)}$$

Even in this phase, the required capital for not failing is higher for the netting treatment:

$$e > f/(f*z+f*z*z^*+1+z+(z*z^*)) \qquad \text{(A.23)}$$

$$e > f/(f*z+f*z*\hat{z}^*+1+\hat{z}+(\hat{z}*\hat{z}^*))\,. \qquad \text{(A.24)}$$

A.2. Additional simulation results

The figures that follow report simulation results obtained under a large set of alternative parameterizations for the core-periphery network topology. The expression "large" refers to a shock on banks forming the core, while "small" to a shock on banks on the periphery. Table A.2.1 shows the order of presentation of the graphs. For completeness, the list reports also the graphs inserted in the main text (mt).

Table A.2.1. Graphs presentation summary.

Metric	General level of capitalization	Intra-group proportion of interbank assets for small (α) and hub banks (η)	Shock size	Figure
Default Number	γ = 0.03	α = 0.5; η = 0.5	Large shock (100%)	Pag 94
			Medium shock (50%)	A.2.1.
			Small shock (25%)	Pag 96
	γ = 0.045		Large shock (100%)	A.2.2.
			Medium shock (50%)	A.2.3.
			Small shock (25%)	A.2.4.
	γ = 0.06		Large shock (100%)	A.2.5.
			Medium shock (50%)	A.2.6.
			Small shock (25%)	A.2.7.
	γ = 0.03	α = 0.4; η = 0.6	Large shock (100%)	A.2.8.
			Medium shock (50%)	A.2.9.
			Small shock (25%)	A.2.10.
	γ = 0.045		Large shock (100%)	A.2.11.
			Medium shock (50%)	A.2.12.
			Small shock (25%)	A.2.13.
	γ = 0.06		Large shock (100%)	A.2.14.
			Medium shock (50%)	A.2.15.
			Small shock (25%)	A.2.16.
	γ = 0.03	α = 0.3; η = 0.7	Large shock (100%)	A.2.17
			Medium shock (50%)	A.2.18
			Small shock (25%)	A.2.19
	γ = 0.045		Large shock (100%)	A.2.20
			Medium shock (50%)	A.2.21.
			Small shock (25%)	A.2.22.
	γ = 0.06		Large shock (100%)	A.2.23.
			Medium shock (50%)	A.2.24.
			Small shock (25%)	A.2.25.
Capital Erosion	All levels of capitalization	α = 0.5; η = 0.5	Large shock (100%)	Pag 94
			Medium shock (50%)	A.2.26.
			Small shock (25%)	A.2.27.
		α = 0.4; η = 0.6	Large shock (100%)	A.2.28.
			Medium shock (50%)	A.2.29.
			Small shock (25%)	A.2.30.
		α = 0.3; η = 0.7	Large shock (100%)	A.2.31.
			Medium shock (50%)	A.2.32.
			Small shock (25%)	A.2.33.

… continue

Metric	General level of capitalization	Intra-group proportion of interbank assets for small (α) and hub banks (η)	Shock size	Figure
Deposits Erosion	All levels of capitalization	α = 0.5; η = 0.5	Large shock (100%)	Pag 95
			Medium shock (50%)	A.2.34.
			Small shock (25%)	A.2.35.
		α = 0.4; η = 0.6	Large shock (100%)	A.2.36.
			Medium shock (50%)	A.2.37.
			Small shock (25%)	A.2.38.
		α = 0.3; η = 0.7	Large shock (100%)	A.2.39.
			Medium shock (50%)	A.2.40.
			Small shock (25%)	A.2.41.

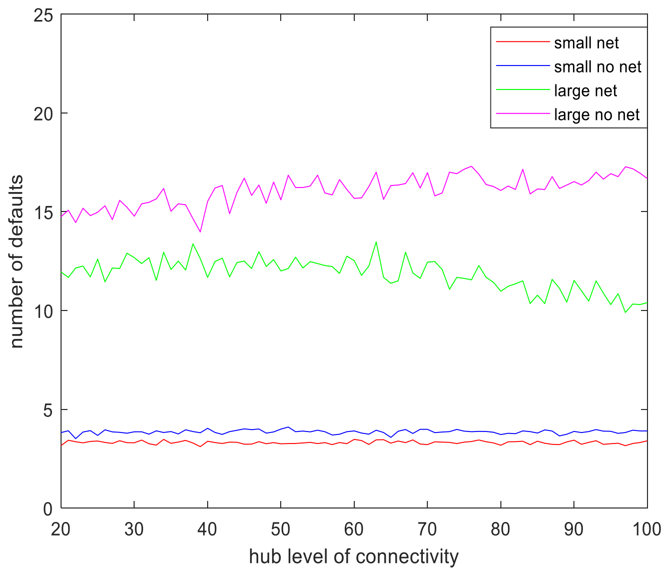


Figure A.2.1. Default dynamics, large and small banks. Medium shock (50%). Parameters: α = 0.5, η = 0.5 and capital γ =0.03.

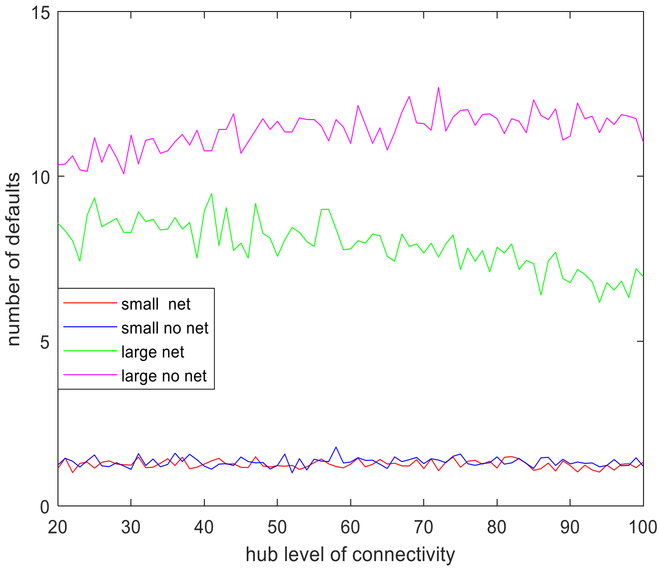


Figure A.2.2. Default dynamics, large and small banks. Large shock (100%). Parameters: α = 0.5, n_b = 0.5 and capital γ = 0.045.

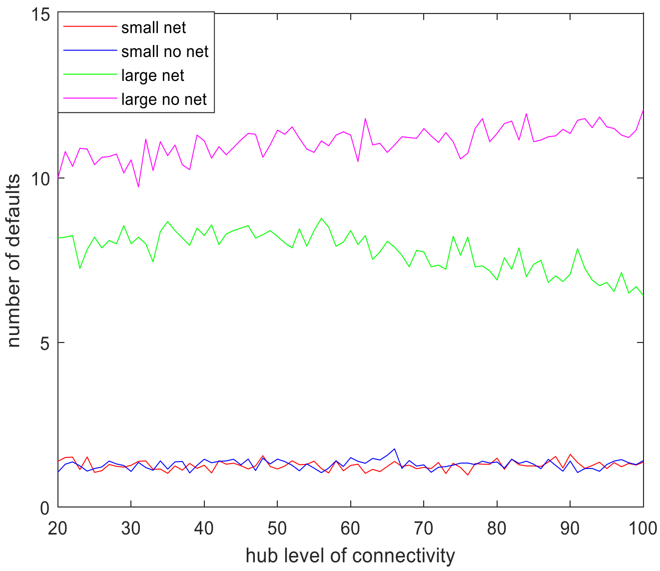


Figure A.2.3. Default dynamics, large and small banks. Medium shock (50%). Parameters: α = 0.5, n_b = 0.5 and capital γ = 0.045.

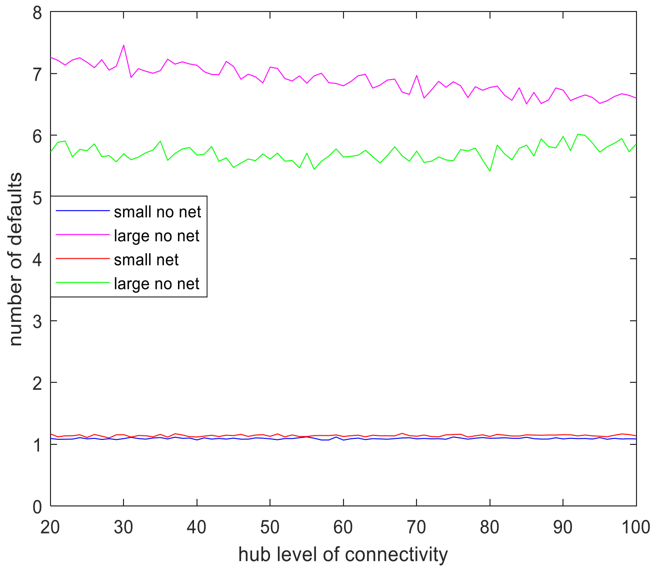


Figure A.2.4. Default dynamics, large and small banks. Small shock (25%). Parameters: α = 0.5, n_b = 0.5 and capital γ = 0.045.

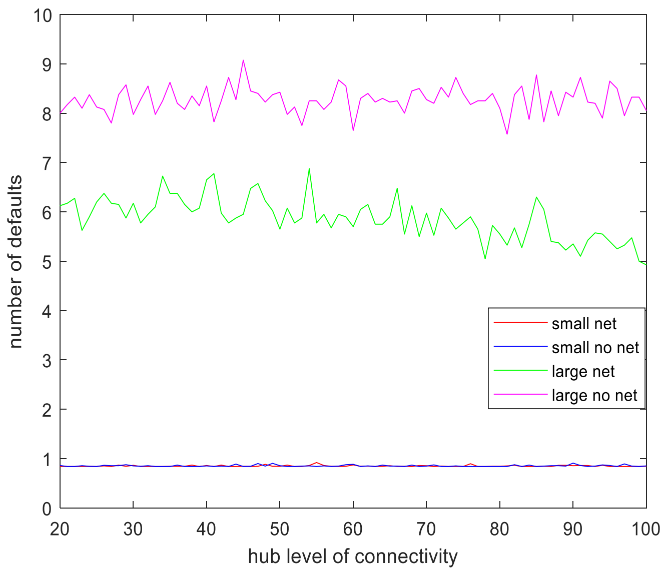


Figure A.2.5. Default dynamics, large and small banks. Large shock (100%). Parameters: α = 0.5, n_b = 0.5 and capital γ = 0.06.

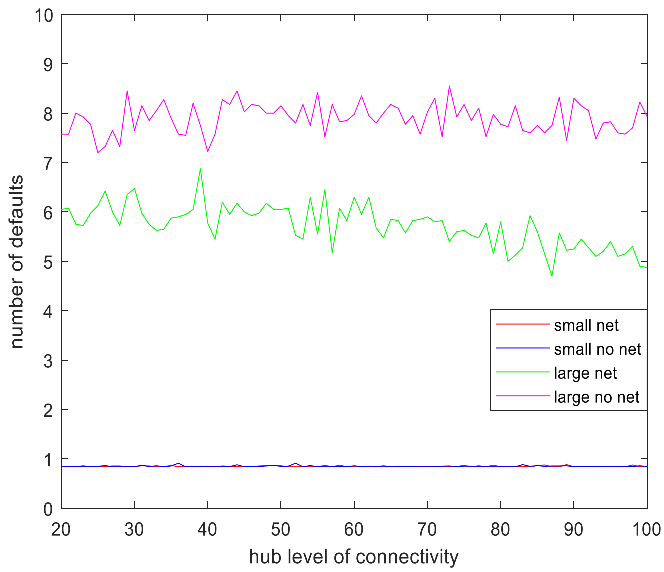


Figure A.2.6. Default dynamics, large and small banks. Medium shock (50%). Parameters: α = 0.5, n= 0.5 and capital γ = 0.06.

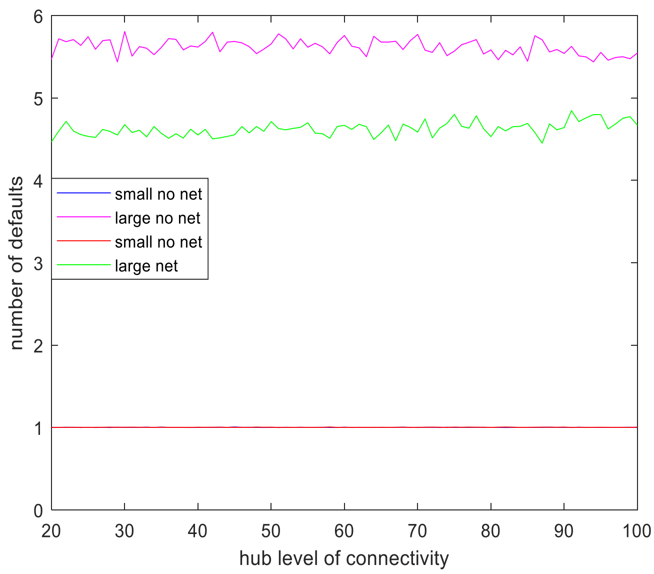


Figure A.2.7. Default dynamics, large and small banks. Small shock (25%). Parameters: α = 0.5 n = 0.5 and capital γ = 0.06.

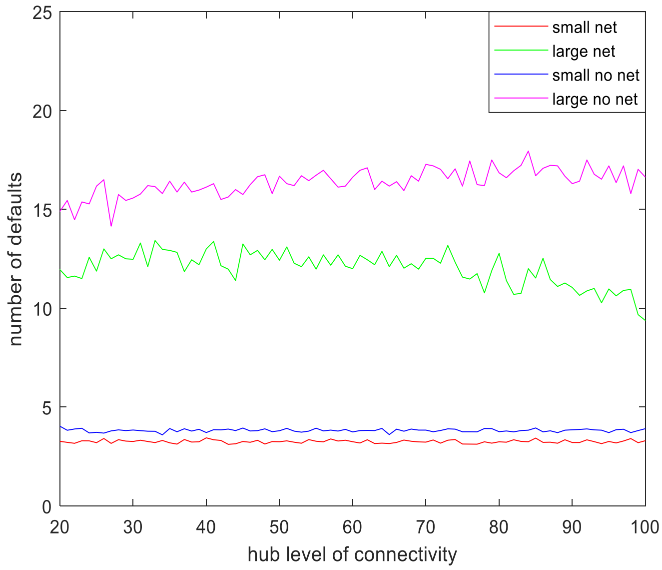


Figure A.2.8. Default dynamics, large and small banks. Large shock (100%). Parameters: $\alpha = 0.4$, $n_b = 0.6$ and capital $\gamma = 0.03$.

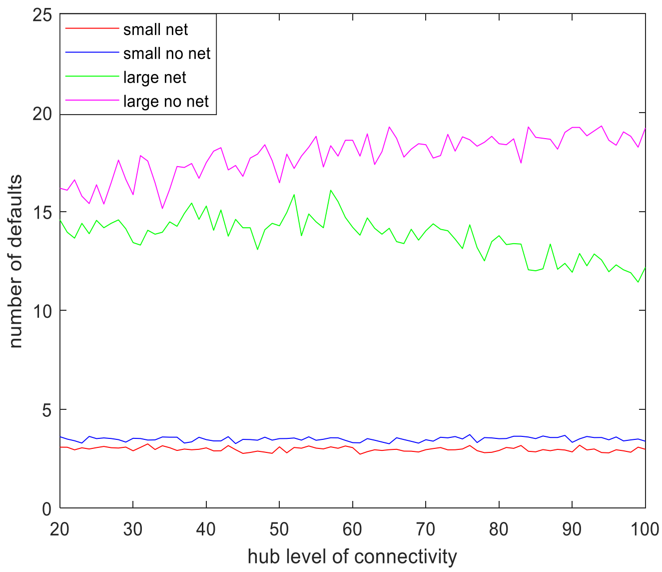


Figure A.2.9. Default dynamics, large and small banks. Medium shock (50%). Parameters: $\alpha = 0.4$, $n_b = 0.6$ and capital $\gamma = 0.03$.

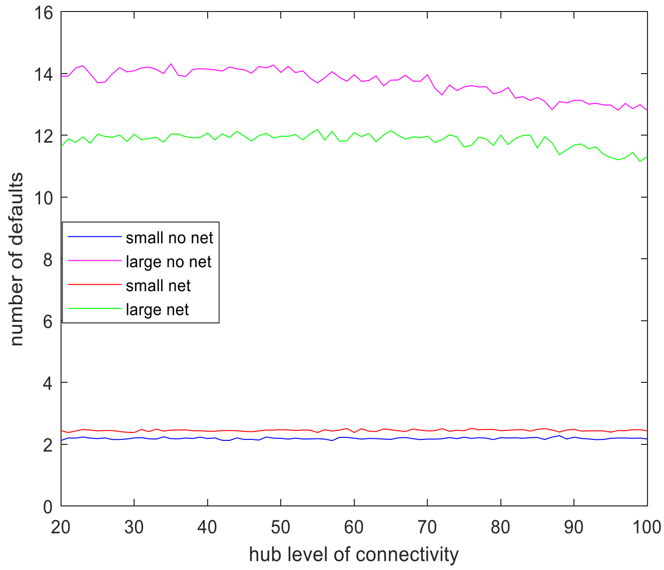


Figure A.2.10. Default dynamics, large and small banks. Small shock (25%). Parameters: α = 0.4, n_b = 0.6 and capital γ = 0.03.

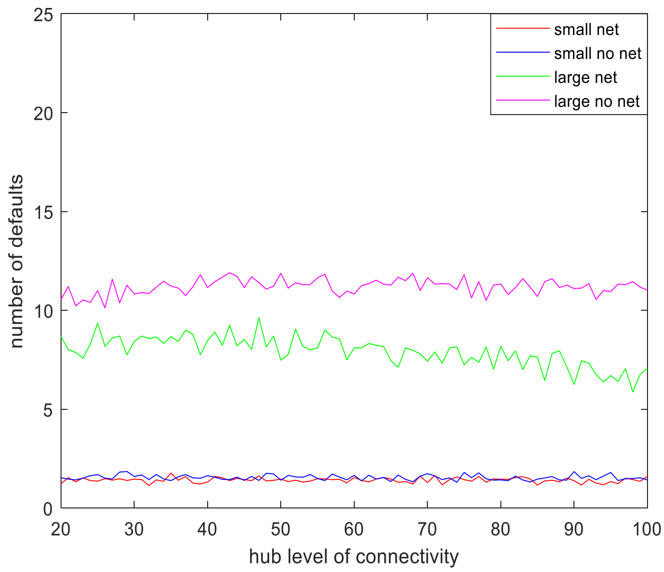


Figure A.2.11. Default dynamics, large and small banks. Large shock (100%). Parameters: α = 0.4, n_b = 0.6 and capital γ =0.045.

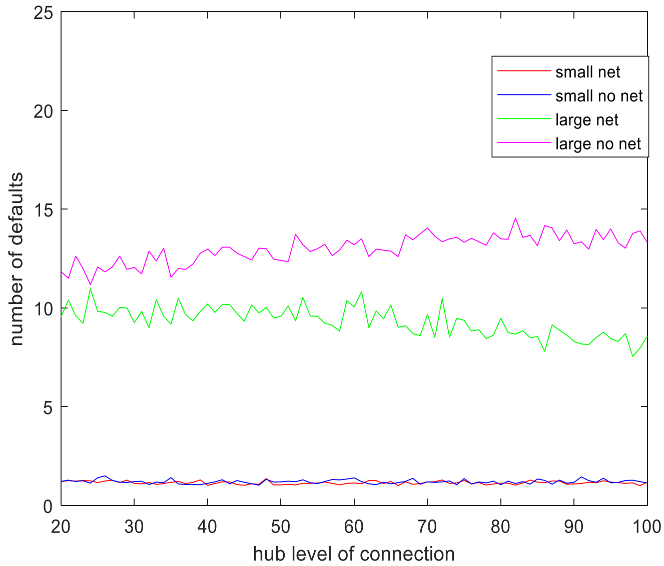


Figure A.2.12. Default dynamics, large and small banks. Medium shock (50%). Parameters: α = 0.4, n_b = 0.6 and capital γ = 0.045.

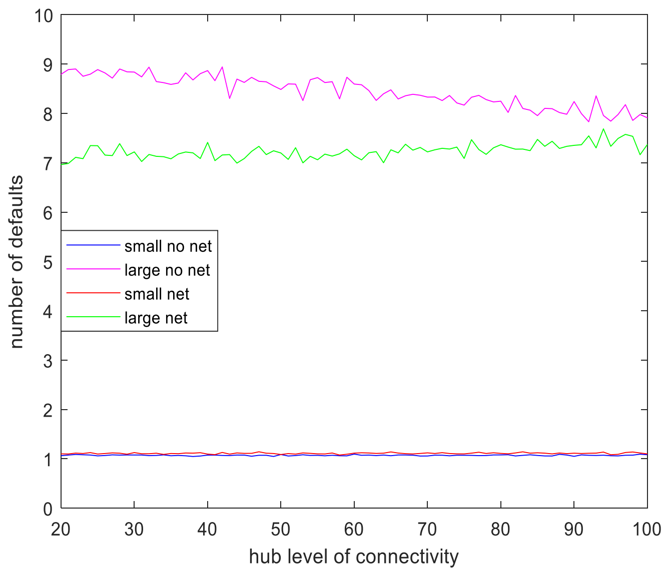


Figure A.2.13. Default dynamics, large and small banks. Small shock (25%). Parameters: α = 0.4, n_b = 0.6 and capital γ = 0.045.

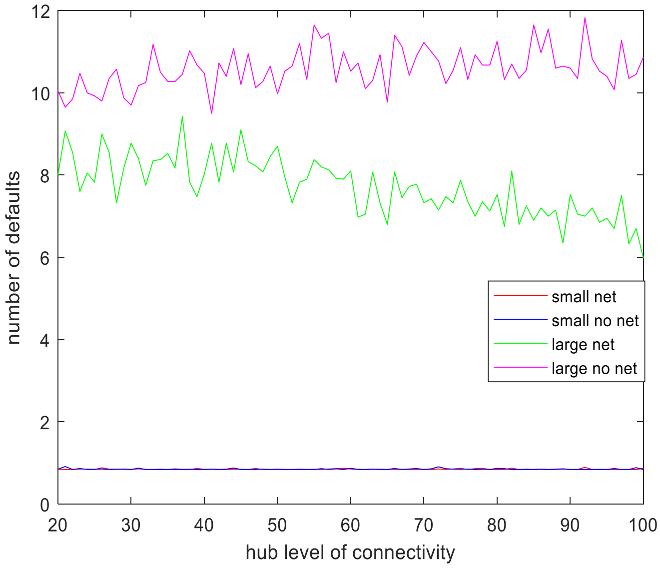


Figure A.2.14. Default dynamics, large and small banks. Large shock (100%). Parameters: $\alpha = 0.4$, $n_b = 0.6$ and capital $\gamma = 0.06$.

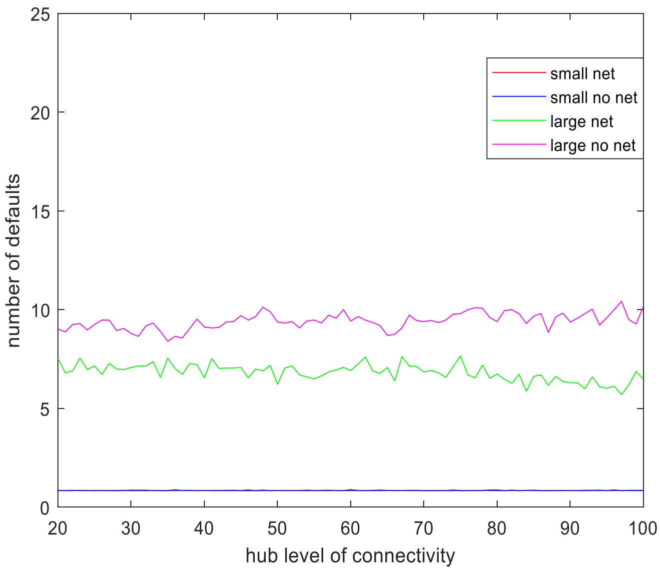


Figure A.2.15. Default dynamics large and small banks. Medium shock (50%). Parameters: $\alpha = 0.4$, $n_b = 0.6$ and capital $\gamma = 0.06$.

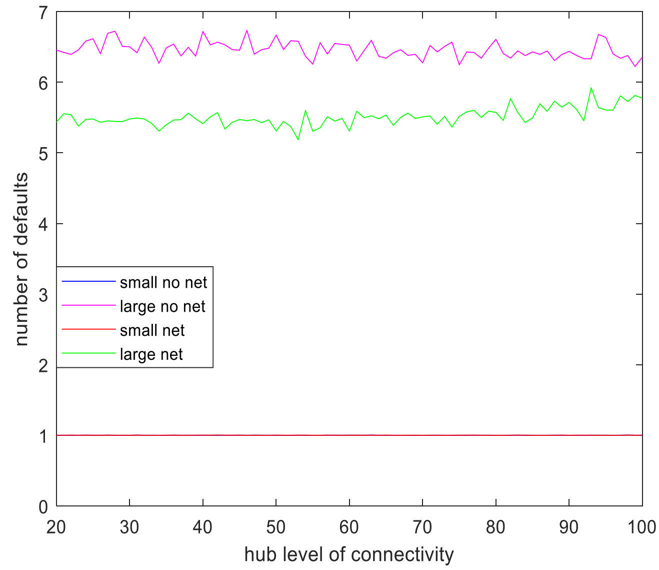


Figure A.2.16. Default dynamics, large and small banks. Small shock (25%). Parameters: $\alpha = 0.4$, $n_b = 0.6$ and capital $\gamma = 0.06$.

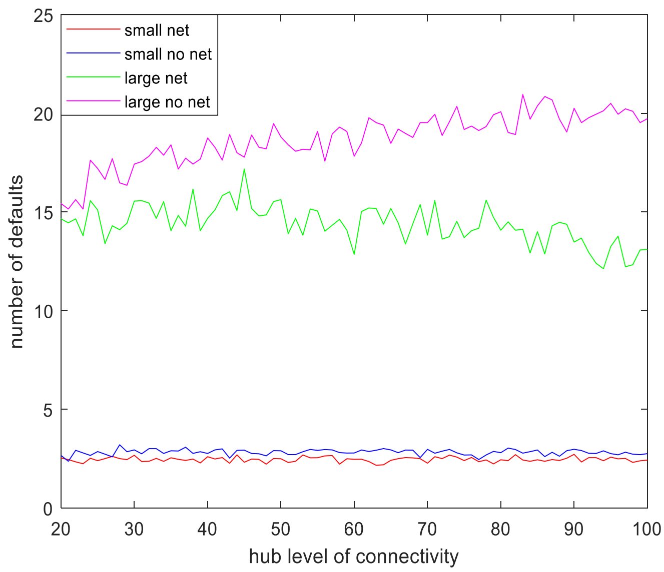


Figure A.2.17. Default dynamics, large and small banks. Large shock (100%). Parameters: $\alpha = 0.3$, $n_b = 0.7$ and capital $\gamma = 0.03$.

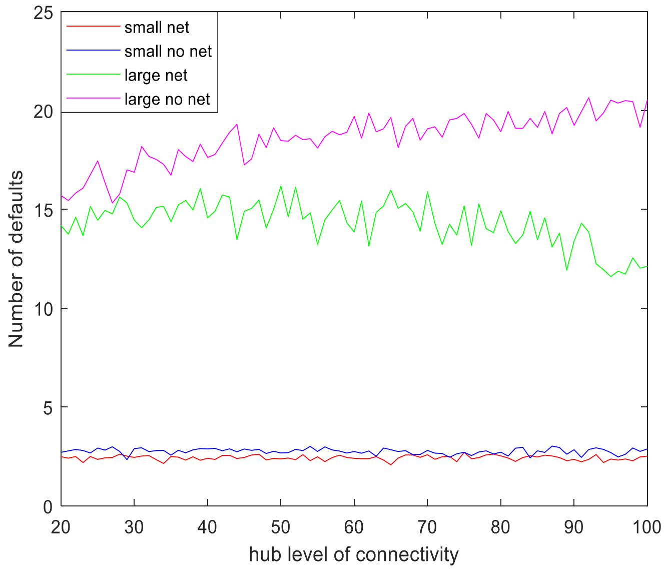


Figure A.2.18. Default dynamics, large and small banks. Medium shock (50%). Parameters: α = 0.3, n_b = 0.7 and capital γ = 0.03.

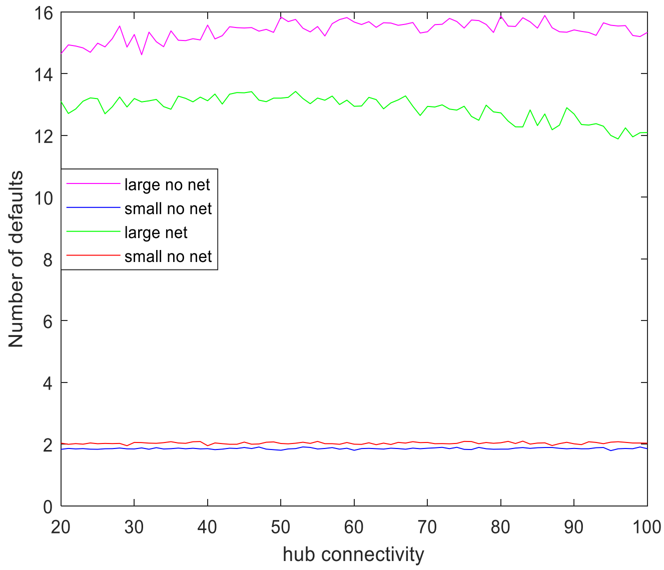


Figure A.2.19. Default dynamics, large and small banks. Small shock (25%). Parameters: α = 0.3, n_b = 0.7 and capital γ = 0.03.

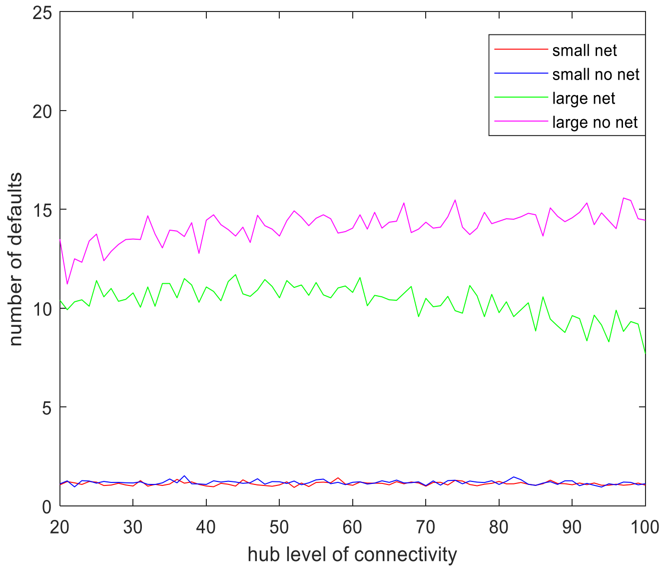


Figure A.2.20. Default dynamics, large and small banks. Large shock (100%). Parameters: α = 0.3, n_b = 0.7 and capital γ = 0.045.

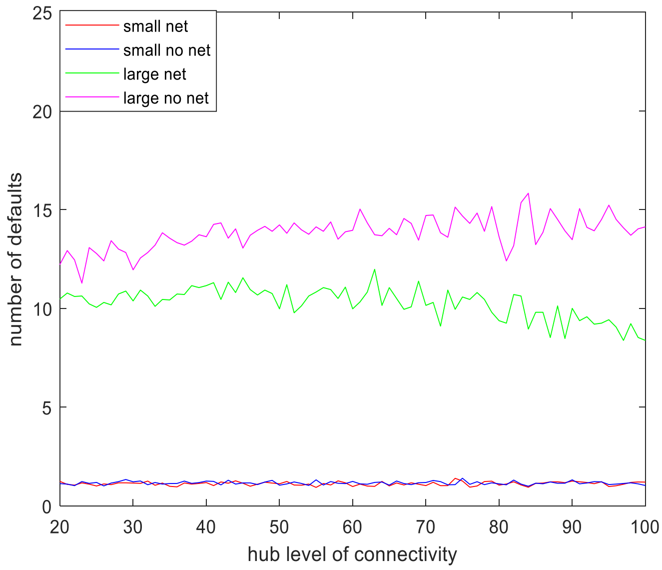


Figure A.2.21. Default dynamics, large and small banks. Medium shock (50%). Parameters: α = 0.3, n_b = 0.7 and capital γ = 0.045.

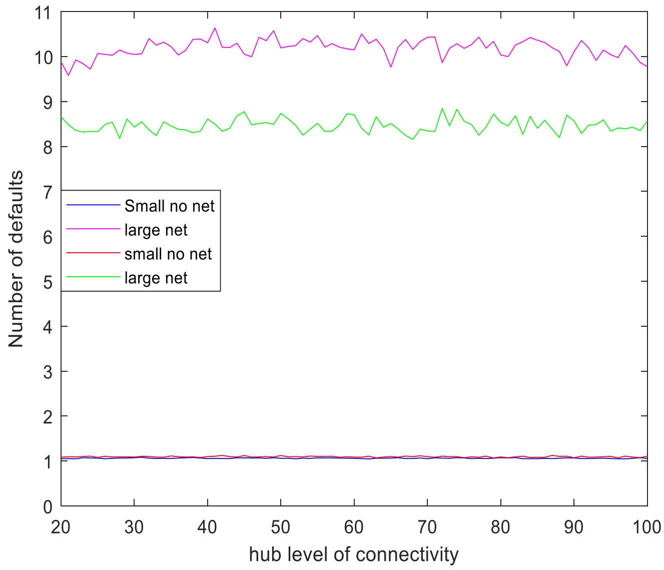


Figure A.2.22. Default dynamics, large and small banks. Small shock (25%). Parameters: $\alpha = 0.3$, $n_b = 0.7$ and capital $\gamma = 0.045$.

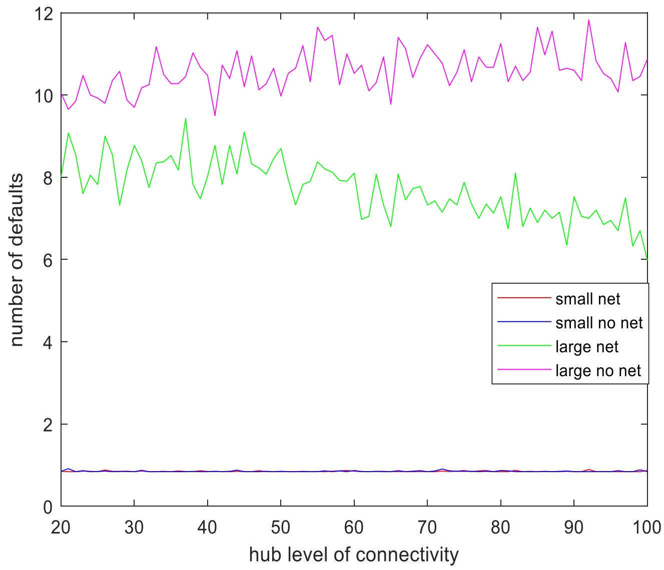


Figure A.2.23. Default dynamics, large and small banks. Large shock (100%). Parameters: $\alpha = 0.3$, $n_b = 0.7$ and capital $\gamma = 0.06$.

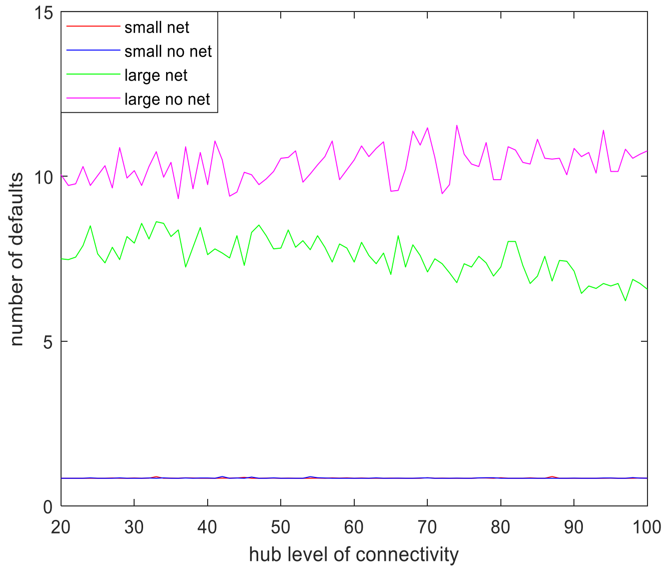


Figure A.2.24. Default dynamics, large and small banks. Medium shock (50%). Parameters: α = 0.3, n_b = 0.7 and capital γ = 0.06.

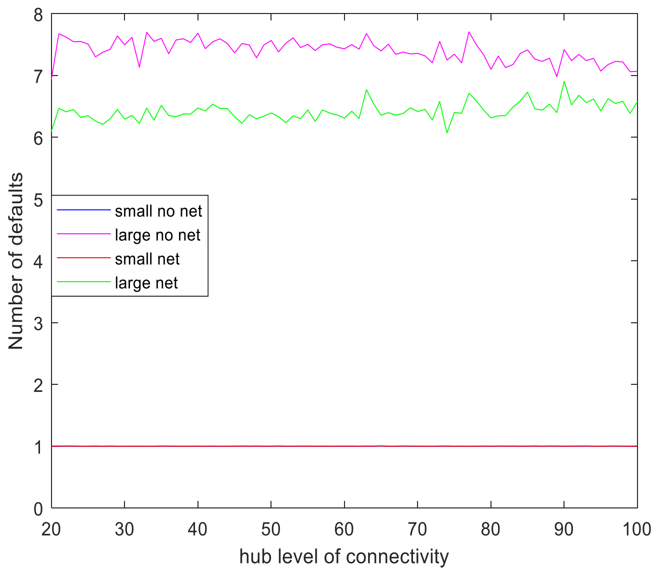


Figure A.2.25. Default dynamics, large and small banks. Small shock (25%). Parameters: α = 0.3, n_b = 0.7 and capital γ = 0.06.

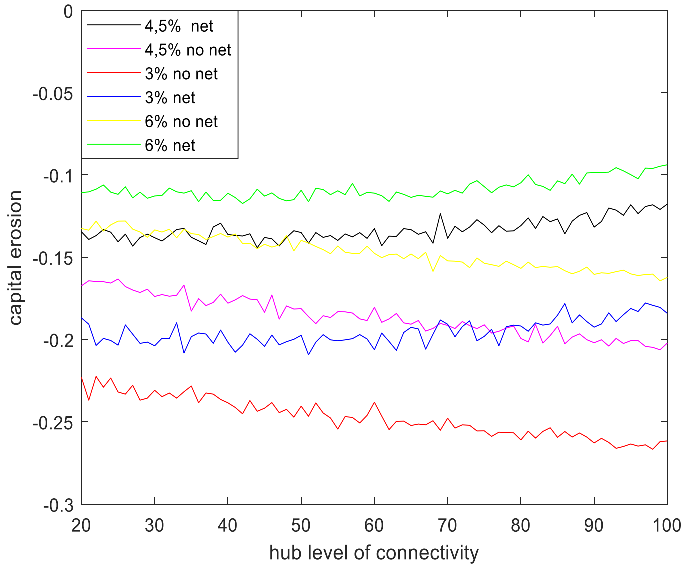


Figure A.2.26. Capital erosion. Medium shock (50%). Parameters: α = 0.5, n_b = 0.5.

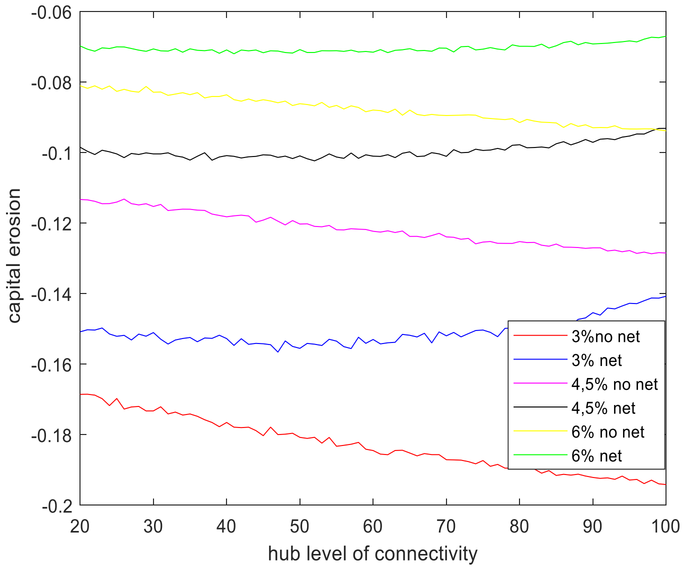


Figure A.2.27. Capital erosion. Small shock (25%). Parameters: α = 0.5, n_b = 0.5.

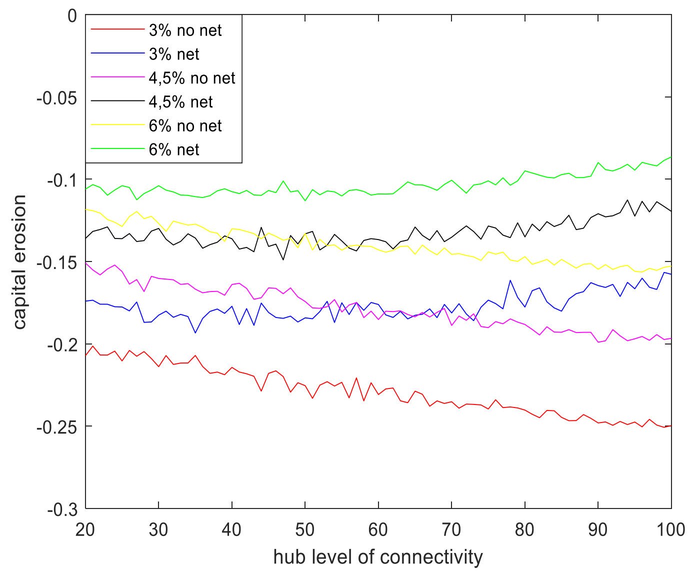


Figure A.2.28. Capital erosion. Large shock (100%). Parameters: α = 0.4, n_b = 0.6.

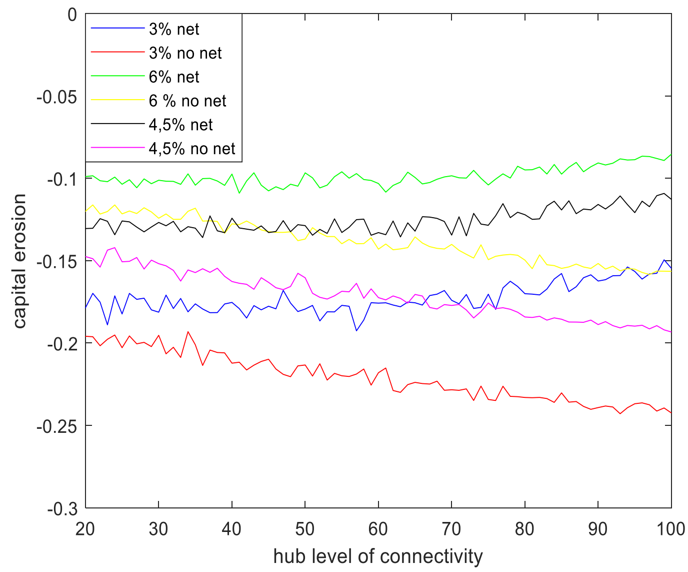


Figure A.2.29. Capital erosion. Medium shock (50%). Parameters: α = 0.4, n_b = 0.6.

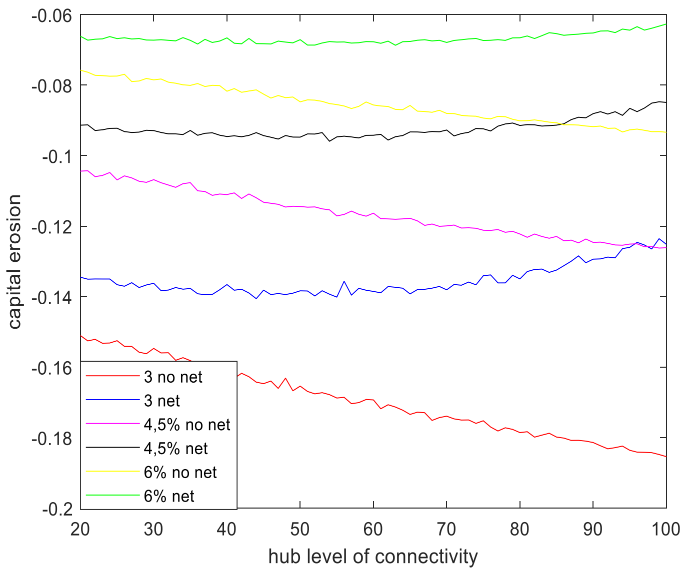


Figure A.2.30. Capital erosion. Small shock (25%). Parameters: α = 0.4, n_b = 0.6.

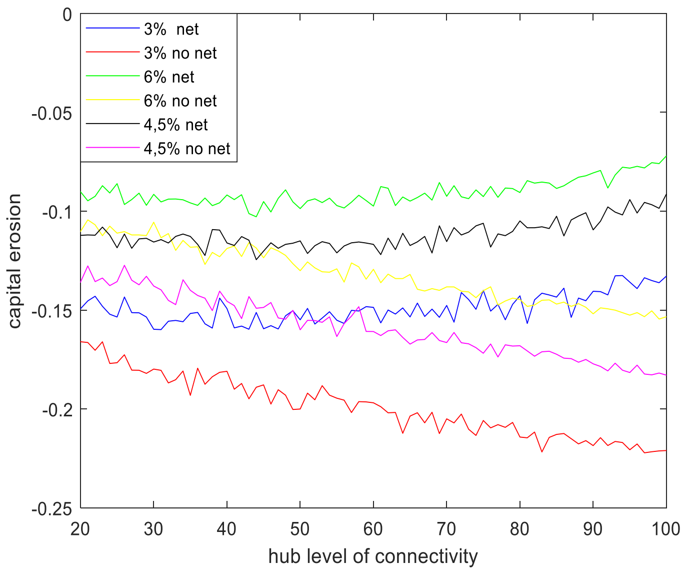


Figure A.2.31. Capital erosion. Large shock (100%). Parameters: α = 0.3, n_b = 0.7.

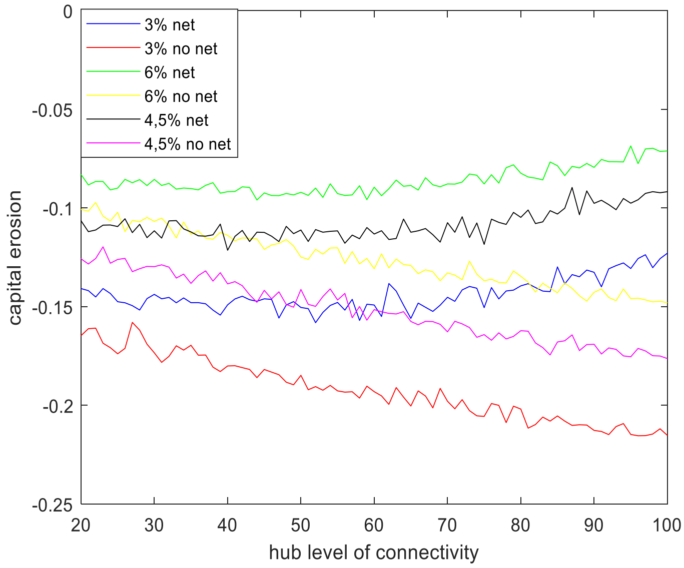


Figure A.2.32. Capital erosion. Medium shock (50%). Parameters: α = 0.3, n_b = 0.7.

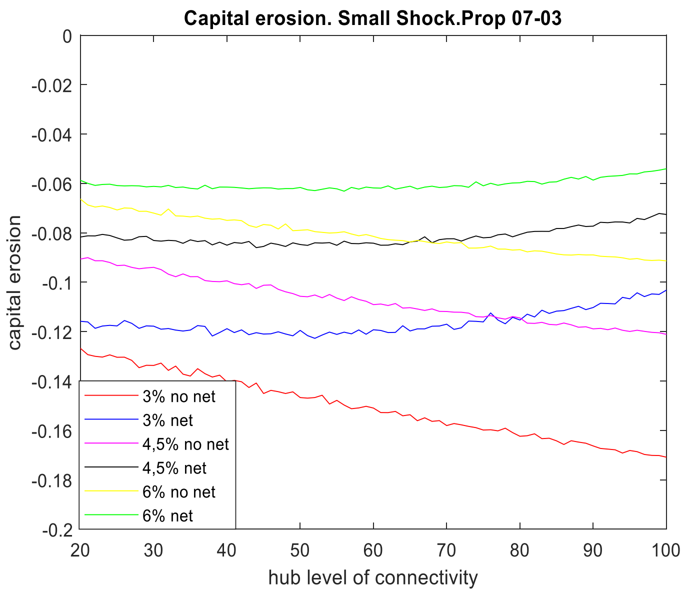
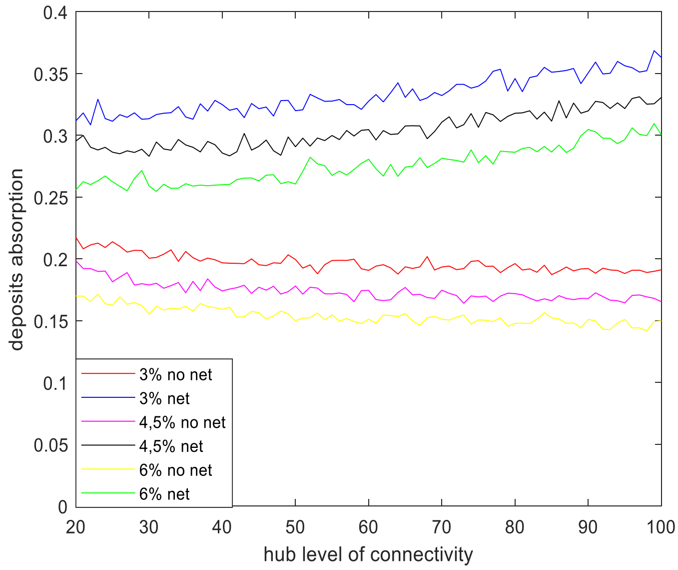


Figure A.2.33. Capital erosion. Small shock (25%). Parameters: α = 0.3, n_b = 0.7.



FigureA.2.34. Deposit erosion. Medium shock (50%). Parameters: $\alpha = 0.5$, $n_b = 0.5$.

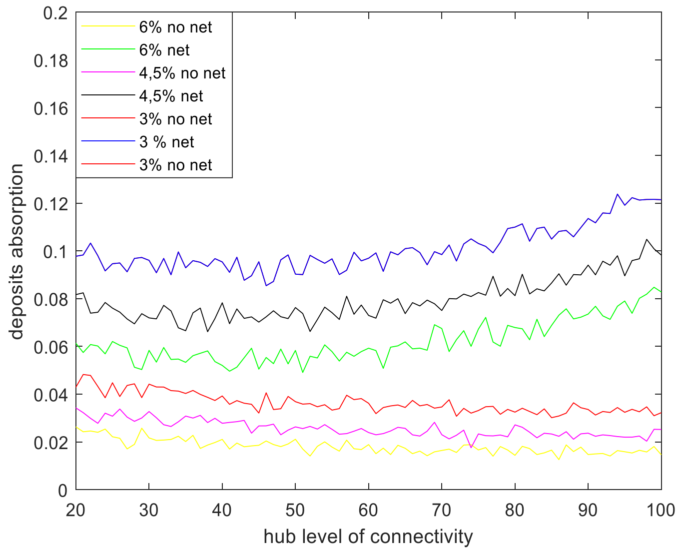


Figure A.2.35. Deposits absorption. Small shock (25%). Parameters: $\alpha = 0.5$, $n_b = 0.5$.

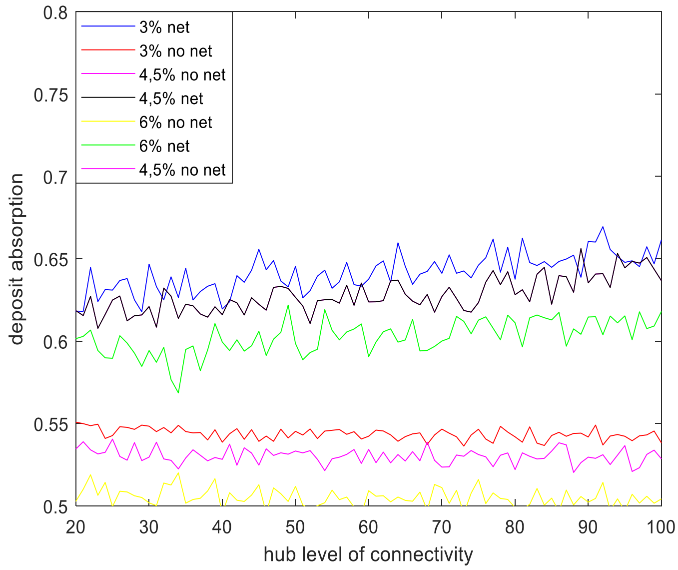


Figure A.2.36. Deposits absorption. Large shock (100%). Parameters: α = 0.4, n_b = 0.6.

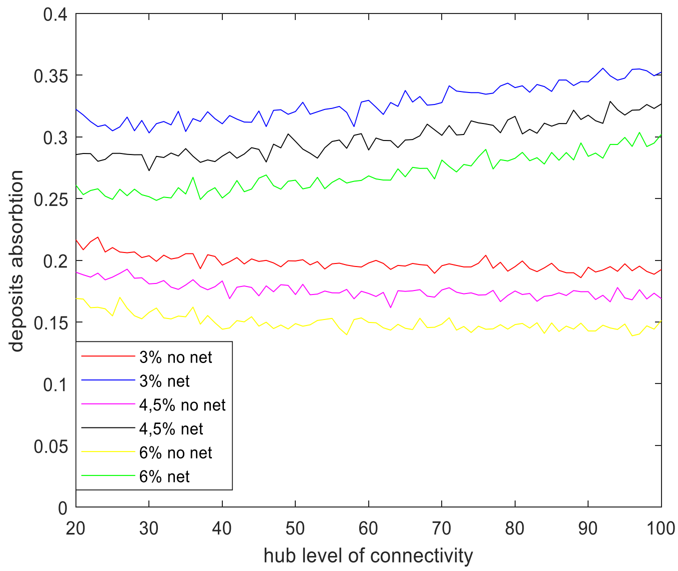


Figure A.2.37. Deposit erosion. Medium shock (50%). Parameters: α = 0.4, n_b= 0.6.

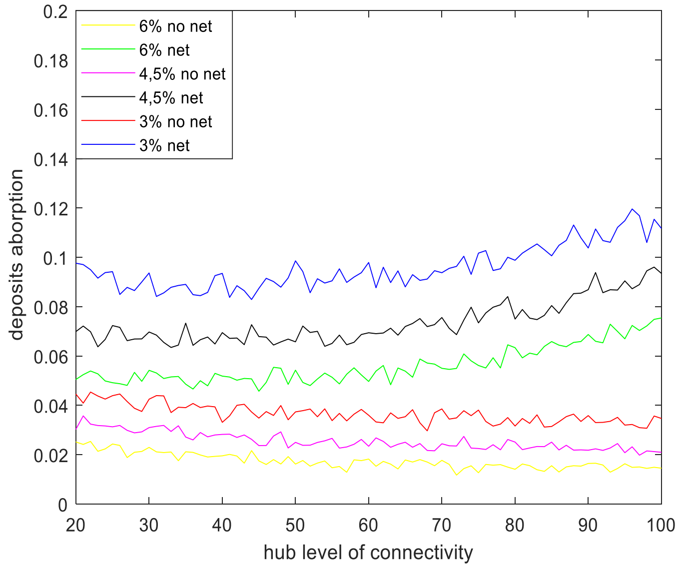


Figure A.2.38. Deposits absorption. Small shock (25%). Parameters: α = 0.4, n_b = 0.6.

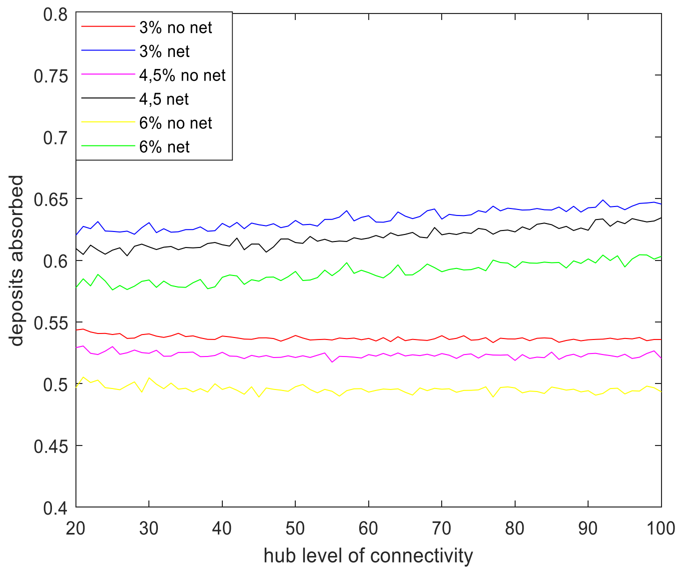


Figure A.2.39. Deposits absorption. Large shock (100%). Parameters: α = 0.3, n_b = 0.7.

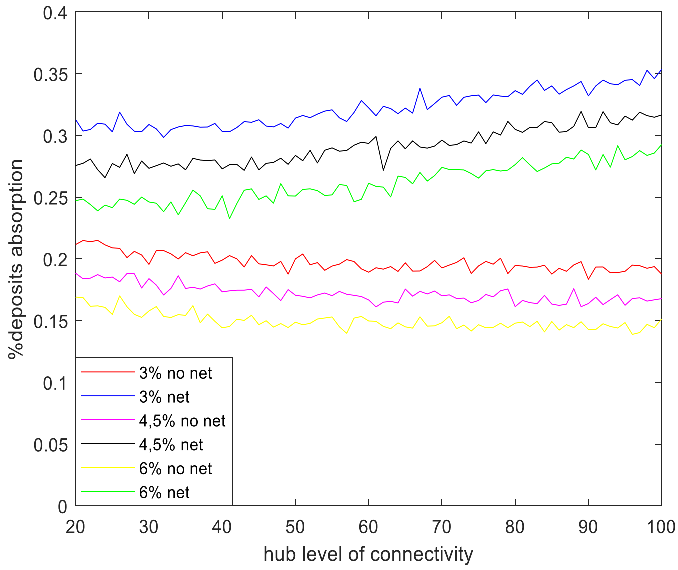


Figure A.2.40. Deposit absorption. Medium shock (50%). Parameters: α = 0.3, η = 0.7.

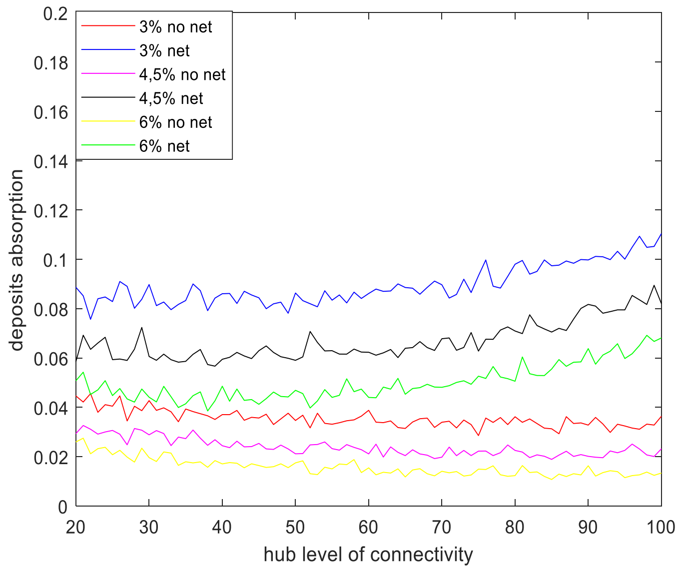


Figure A.2.41. Deposit absorption. Small shock (25%). Parameters: α=0.3, η = 0.7.

Endogenous clearing house formation in payment networks

Edoardo Gaffeo[†] Lucio Gobbi[†]

Abstract

This paper analyzes the endogenous formation of clearing houses within payments networks. More precisely, the process of formation of a clearing house as an act of collective coordination is investigated. This dynamic presents elements of analogy between real and financial environments. To this purpose, we developed an agent based model elucidating dynamics that lead the agents to regulate their transactions through gross settlement, bilateral netting or multilateral netting when these systems are in competition. In particular, the conditions for the formation of clearing houses within real and financial payment networks are investigated, focusing on two real cases. The first case concerns the creation of corporate barter circuits à la Wir, while the second investigates the development of clearing houses that managed interbank payments in the Free Banking America. The main difference between the analysis on real and financial markets concerns the type of frictions considered. In the first case we analyze a friction of spatial coordination, in the second case the presence of transaction costs. Our work shows how the formation of clearing houses that manage payments derives critically from the topology of the payment network, from the phases of the business cycle, from the transaction costs, and from the agents' ability to interact and coordinate with each other.

1. INTRODUCTION AND LITERATURE REVIEW

The payment system is one of the main institutions that define a modern economy, the goal of which is to allow the exchange of goods and services whilst avoiding the complications and the inefficiencies tied to barter. The principles regulating payment systems are similar to those ones which define payment agreements: the main differences are that the latter are concerned with the wealth to be transferred, whereas the former are concerned with the methods of the transfer upon which the parties have already agreed.

Payment systems are typically categorised according to the different procedures and the timeframe in which they allow the extinguishing of the obligations between two agents. We tend to distinguish between two types of systems: The Real Time Gross Settlement (RTGS) and the Net Settlement (NS). RTGS is a system in which credits and debts between agents are continuously regulated through payments established by the legislator, by private regulations and by customs. The main advantage of RTGS is the elimination of the insolvency risk, since contracts amongst the parties involved are settled through payment. The main disadvantage of said system is the cost of locating the necessary financial means, to which the operational risk during the execution of the payment is added. Whenever the system does not involve continuous payments, we talk about Gross Settlement (GS).

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Typically, the GS is used on real markets and the rationality in the use of this settlement method consists mainly in the fact of being a form of financing that the creditor grants to the debtor.
The Netting Settlement (NS) deviates from the RTGS both in timing and in regulatory procedure. In specific terms, the structures of NS allow to clear two or more obligations at the same time through a bilateral netting settlement (BNS) or a multilateral netting settlement (MNS). Netting procedures allow to lower the expository level between the agents, reducing the liquidity costs necessary at the time of execution. Thus, on the one hand we reduce both liquidity costs and operational risk; on the other hand, by postponing the execution time of the payment, a credit risk surfaces, which disincentivizes the use of the procedure altogether. In general, two settlement principles can be identified in the payments system as well as in the real market: the "liquidity" principle and the clearing principle. These "principles" are taken as ideal types and therefore serve as theoretical benchmarks. Specifically, the principle of liquidity requires the extinction of an obligation by means of legal tender, on the contrary the principle of clearing allows the extinction of a debt through the use of a credit[27]. Economic practice gives us a more complex reality where the concept of liquidity is rather elastic and not defined over time. Here we refer to these theoretical extremes that will be useful in the construction of the models, for a contextualization on the theme of liquidity and its creation is suggested without pretension of exhaustiveness Amato and Fantacci (2012), Biondi (2018), Graziani (2003), Hicks (1974), Keynes (1936,1950), Ricks (2015). The table summarizes the benchmarks and payment methods considered in this work.

Table 1.1 Principles and settlement modes

Principles			
Liquidity principle	Obligations between agents are extinguished at face value by means of central bank money.		
Clearing principle	The bonds between the agents are offset and only the net exposure is extinguished by central bank money. Compensation can take place between mutual obligations (Bilateral clearing) or between obligationss relating to more than 2 agents (Multilateral clearing)		
Settlement modes under liquidity principle			
Type of settlments	**Timing**	**Advantages/Disadvantages**	**Type of market**
Real time gross settlement	Continous	Null credit risk/ high liquidity costs and operational risk	Real and financial markets
Gross settlement	Deferred	Funding channel /Credit risk, liquidity costs and operational risk.	Real markets
Settlement modes under clearing principle			
Type of settlments	**Timinig**	**Advantages/disadvantages**	**Type of market**
Multilateral clearing	Deferred	Low liquidity costs and operational risk, funding channel/ High credit risk	Real and financial markets
Bilateral clearing	Deferred	Low liquidity costs and operational risk, funding channel/ High credit risk	Real and financial markets

[27]Indeed, even the legal tender is a credit towards the central bank. Here we consider all the credits without the legal tender.

With regard to the real market, the present study will attempt to scrutinize the dynamics according to which agents choose to regulate their obligations through MNS or GS systems whenever they coexist in such contexts. Specifically, starting from the experience of clearing houses such as Wix, Sardex and similar (Lucarelli and Gobbi 2016, Stodder 2009, Stodder and Lietaer 2016), we will investigate the reason why the aforementioned experiences are isolated and whether or not it is possible to draw innovative guidelines from these examples on how to manage payments between companies. The circuits that are discussed here are clearing houses for credits and debits amongst network members; these circuits can be seen as an advanced form of trade credit.

In the real market it is not uncommon for companies to give credit to each other in the form of a deferred payment; this option is usually referred to as trade credit and is quite widespread. Hence, a new source of financing emerges between suppliers and customers which departs from the traditional ones, namely the banking system and the capital market. Two main theoretical strands account for the emergence of trade credit: non-financial and financial. As for the non-financial strand, the most relevant theories identify the rationale of trade credit with the lowering of transaction costs (Ferris 1981), with trade credit being a de facto price discrimination (Schwartz and Whitcomb 1979, Brennan et al. 1988), with the granting to the buyer of the possibility of inspecting the purchased goods (Long et al. 1993), and with the fostering of a long-term relationship between the buyer and the seller (Summer and Wilson 2002). As for the financial strand, most theories highlight that suppliers can grant credit to their customers more efficiently than the banking system can. Such advantage would depend on the ability of the supplier to minimize losses in the case of the customer's default (for instance Petersen and Rajan 1997; Frank and Maksimovic 2005; Fabbri and Menichini 2010) other than to cheaply acquire information about the latter, thus managing to grant credit to those who could not obtain it from the baking system (Biais and Gollier 1997; Burkart and Ellingsen 2004 among the others). Finally, some works highlight that suppliers tend to gran trade credit to smaller and credit-constrained customers (for instance McMillan and Woodruff 1999; Marotta 2005) while others put emphasis on the market power of buyers and sellers (Goncalves et al 2018, Fabbri and Klapper 2016 among the others). For a literature review, see Seifert et al. (2013). Overall, the aforementioned contributions set out to investigate the bilateral relationship between supplier and customer; just few of them attempt at analysing the trade credit process in a multilateral perspective (Kim and Shin 2007, Fabbri and Klapper 2008). According to the latter, the companies which grant trade credit are often the same ones which benefit from it from their suppliers. The matching of incoming to outgoing financial flows is indicative of the companies' intention to balance assets and liabilities. This intention can be considered as an informal step towards multilateral clearing which is the core of the functioning of corporate barter circuits such as Wir and Sardex (Stodder 2009, Stodder and Lietaer 2016.)

The Swiss system Wir, in particular, is centred on the homonymous cooperative bank. Wir Bank was founded in 1934 by 16 members and, despite having implemented changes to its statute and its operating procedures over the decades, the keystone of its activity has not been substantially altered yet, and the number of participants in the system has reached 70.000 units. The core business of Wir Bank is the management of the clearing system, which is based on an internal account unit called Wir. Wir is tied to the Swiss franc through a fixed exchange rate, which serves principally for the accounting of the operators since the conversion between the two currencies is allowed only in exceptional cases and carried out exclusively by WirBank. All the members of the circuit have a checking account at Wir Bank where, whenever an exchange of goods or services takes place, the incoming and outgoing flows of transactions are recorded. In particular, a credit will be issued to the seller's account and a debt will be recorded on the buyer's account. At the end of the transaction the creditor, who is now a Wir credit holder, will be able to use his availability to acquire goods and services from other circuit members. The debtor, instead, having made the transaction through the credit guaranteed by the clearing house, will pay an interest rate on his overdrafts. Wir circulate only within the clearing house and cannot be converted into Swiss francs or any other currency except for specific reasons. For creditors, joining the system is advantageous because credits in Wir can circulate as currency and without discounts within the circuit and, therefore, the holder of a credit does not have to wait for the payment to be liquidated before using his own availabilities. For debtors, the main advantage is the possibility to access a credit at lower costs than loans at the official currency rate. This is because Wir is created endogenously by the bank, therefore eliminating the cost of obtaining the resource; thus, the bank imposes on the debtor only the cost necessary to cover the credit risk and the operating costs. This advantage will be particularly accentuated in the occurrence of business cycle fluctuations giving rise to the counter cyclical trend in the number and value of transactions carried out within the circuit (Stodder 2009). In general, it is convenient for a company to join the circuit if its activity is such as to allow it to pay its debits with credits generated by transactions with other members. Wir Bank’s functioning is based on the interest expense paid by the debtors of the clearing house and on the fees charged on circuit members. The disadvantages of the system mainly correspond to constraining the purchase capacity within the circuit, to the risk of not being able to spend the assets in Wir due to a low variety of offer, and to the possibility of not obtaining remuneration on current accounts at the clearing house.

Sardex system is a clearing system working with principles and rules similar to the ones that regulate Wir and it can be considered as an answer to the credit crunch that hit the Italian economy and the Sardinia region in 2010. Such system is not managed by a bank but by a company not authorized to banking activity that allows circuits members to offset their debits and credits denominated in Sardex. Similar to the previous case, Sardex is also tied to the official currency through a fixed exchange rate but it is not convertible. The exchange rate indentification is mainly used for accounting reasons. Each company joining the system is paired up with a broker specialized in the sector of classification of the company, with the responsibility of reporting the possibilities of spending and purchasing within the system. Therefore, active and passive positions of circuit members are being balanced. Client fees cover operating and service costs. The denomination of your own productive assets in a limited circulation currency is possibly the foremost deterrent to the activation of similar circuits, because potential new members may have doubts on the possibility of interfacing with a number of counterparties sufficient to cover their own financial exposures within the circuit. Operating costs during this stage can be considerable and, in most cases, they can be higher than the benefits of becoming a member. Wir Bank and Sardex operating principles lend themselves to analyses on monetary policy and the nature of currency (Amato and Fantacci 2012, 2013). In general, we can argue that systems of this kind lower the costs of financing the working capital of members, feed an internal demand to the circuit, and decrease the need for liquidity of the system.

As for the payment system, we discuss a case study of endogenous formation of a clearing house, namely the New York clearing house (1853) established in the times of the Free-Banking America as described by Gorton (1984). Theoretically speaking, free banking consists in a monetary system with no barriers to entry the banking sector and where all banks can issue their own currency, the value of which is determined by the market forces just like general goods. The most renowned cases of free banking occurred in Scotland (1716-1845) and in the United States (1837-1864). While the former is considered a success story, the latter is regarded as a case of high monetary instability which led to the formation of some key institutions of the contemporary monetary systems (Gorton 1984, 1985). Gorton illustrates how, during the first phase of the American Free Banking, the financial system was populated with small-size banks issuing checks to their account holders. Checks played a particularly prominent role in the large cities where most economic activities were concentrated, for people in business considered the check a more practical and secure means than cash to regulate their transactions. The checks clearing mechanism which was in operation among the banks is described by Gorton as follows:

"Before 1850 banks cleared checks with a daily exchange and settlement-each bank sent a porter to make the round of all the other banks. The porter carried a ledger book, check drawn on the other banks, and bags of gold. At each stop the porter turned over checks drawn of that bank and picked up check drawn on his bank. If the value of the checks he presented exceeded the value of those he picked up, he collected the difference in gold. If the balance netted out against his bank, he paid in gold" (Gorton 1984 pp.4).

In the light of the description, such payments system can be categorized as a bilateral netting system whereby the banks cleared the net value of their mutual exposures in cash (i.e. gold). Other than to the liquidity costs, the "porters" system was subject to the operational risk of incurring into a robbery during the settlement. According to Gorton, the number of banks and transactions involved in the system increased to the extent that the inefficiencies due to the parallel increase in transaction and liquidity costs, other than in operational risks, led the transactors to conduct the settlements multilaterally, in a specific place, once a week. Hence, in 1853 the first clearing house of America saw the light in New York, and since then many others came to life across the country. Therefore, in accordance with Gorton's argument, as soon as the payments network became complex enough, multilateral netting overtook bilateral netting. Here we seek to identify what conditions enable the endogenous formation of a clearing house when BNS and MNS are in competition with each other in contexts similar to the one described above.

The aim of the present paper is to reflect on whether the multilateral netting may have a spontaneous spreading once the netting circuit is managed by a clearing house. The originality of the work consists mainly in presenting a process of endogenous creation of the clearing houses vis-à vis the extant literature on netting rules, which usually considers the presence of clearing houses as exogenously determined (see Chu and Lai 2007 for a literature review).To this purpose, we developed an agent-based model suitable for analysing the economic dynamics which emerge in the real markets and in the payment systems whenever the agents can freely decide the type of settlement to resort to.

For what concerns the real markets, our model is characterized by the investigation of the agents' behaviour according to the variation in the topology of the payment network, to the presence of spatial frictions which might prevent the interaction amongst all members of the system, and to different contexts that may represent different phases of the business cycle. We tested the competitiveness between GS and MNS. Our findings point to the coordination problems between agents, to the different phases of the business cycle, and to the different topologies of the payment network as the main factors which obstruct the formation of a clearing house, turning corporate barter networks à la Wir into isolated cases. Following from this, we verified if, loosening the spatial friction of the model, a multilateral netting system was more likely to emerge. Our simulations suggest the existence of a strong potential for multilateral clearing in all the topologies and the phases of the business cycle that we tested. The suppression of such potential is mainly due to the aforementioned problems which, however, could be mitigated by technological innovation. The emergence of large internet platforms connecting thousands of companies, such as Amazon B2B, can ease the coordination problems between them, engendering new opportunities which are currently unavailable. Therefore, we reckon that the topic of multilateral clearing between companies will grow more and more important in the future, and that the present work can be the starting point of a wider research agenda.

For what concerns the payment systems, we adapted our agent-based model to the free-banking context described by Gorton through the addition of the transaction and operational costs associated with the type of settlement typical of that context. We tested the model for two competing system of settlement, namely the BNS and the MNS, and for different economic scenarios, payment networks, and transaction costs. In most cases, the results of our simulations are consistent with the argument developed by Gorton.

The paper will be divided into other five sections: in the second section, we will describe the technical aspects of the models and the underlying dynamics; in the third, we will present the results of the simulations of the spatial friction model for enterprise circuits; in the fourth section, we will analyse the potential of clearing for enterprise small circuits while in the fifth we test the free banking story. The last section is dedicated to the conclusions.

2. THE MODEL

2.1. The Network Analysis and Payment Network

The methodology used for this work is network analysis. This methodology is well-suited for the study of credit networks and is widely employed for the analysis of both interbank markets and real markets. As for the interbank markets, since Allen's and Gale's (2000) analysis of the process of contagion of a financial crisis in a small network composed of four banks, numerous theoretical and empirical contributions have added up to the extant literature exploring how financial crises develop within complex networks (for a non-exhaustive review, see Nier et al. 2007, Gai and Kapadia 2010, Battiston et al. 2012, Gaffeo and Molinari 2015). As for credit networks between companies, network analysis has been employed for studying how crises spread in supply chains where trade credits are in operation (Boissay 2006, Battiston et al. 2007, Delli Gatti et al. 2010, Lee et al. 2016). Furthermore, an implicit advice on the methodology to employ was provided by Gorton himself, who describes the functioning of the free banking system as follows:

> "The system had the simplicity of Indian camps in which each tepee had a path leading to every other tepee. But as the number of the banks grew, these paths became a tangled web" (Gorton 1984 pp.4).

The web of credit and debit relations among the agents is exactly what network analysis enable us to detect and is of paramount importance to identify what obligations can be cleared bilaterally, multilaterally, or regulated in GS. In the following section, we will present the network model and we will show some basic dynamics which will guide the simulations. A first segment will describe the shaping of the payment network and, subsequently, the agents' behavioural schemes. The aim of the research is to reveal which forces compel the agents either to join a multilateral netting circuit or to regulate their transactions through GS vs MNS or BNS vs MNS. To this end, we construct an artificial economic system composed of N nodes, where each node represents an agent (a firm or a bank) and where the nodes are linked to each other by debit credit obligations. The simplest way to model a network of this type is through an adjacency matrix X, that is a matrix composed of N rows and N columns in which each single value of $x_{i,j} \forall i, j \in N$ can assume the value of zero or one, respectively in the presence or the absence of a debt-credit relationship between agent *i* and *j*. In particular, the *i-th* row represents the credit agreements that the agent holds towards the other agents, while the *j-th* column represents the agent's debit contracts *j* towards the rest of the network. It follows that, by construction, the main diagonal of X is composed of zeros as shown in Figure 2.1. We set an illustrative matrix X*example* among all the possible adjacency matrix with 3 agents which can be generated by a random process.

Figure 2.1. Adjacency matrix for a network of N=3.

$$\text{X}=\begin{bmatrix} 0 & x_{1,2} & x_{1,3} \\ x_{2,1} & 0 & x_{2,3} \\ x_{3,1} & x_{3,2} & 0 \end{bmatrix} \qquad \text{X}example=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Necessary parameter for contracts attribution is the probability $p_{i,j}$ that the two agents be connected to each other, thus *p* is the main driver determining the topology of payments. In the present study, we will consider two topologies of payments: uniform and concentrated. For the uniform topology, commonly known as Erdos Renyi, the probability that two agents be connected to each other is the same. For the concentrated topology, the probabilities of among-agents connectivity are not homogenous; another probability of connectivity p_h characterizes few nodes and the rest by a lower p_s .Following the identification of the adjacency matrix, attributing value to the obligations between agents is necessary in order to define the payments network. We define therefore the weighted adjacency matrix by means of a procedure which is split in three phases: first phase is to identify the total value of the obligations *TV* in the system. Once *TV* is fixed, a credit quota of *TV/N* is attributed to each agent. In the last phase, for each agent the quota is divided by the equivalent number of credits. The number of credits is obtained by summing the values of the corresponding adjacency matrix row.

This procedure shows a network characterized by agents who have an equal total credit value divided into a number of different contracts and therefore heterogeneous. The net financial position of each agent, calculated as the sum of the receivables minus the sum of the debts of each agent, may be in debit, credit or balanced position. The strength of this procedure consists mainly in the fact of considering heterogeneous obligations allowing a well-founded investigation of the dynamics of clearing. The weakness could concern the fact that the heterogeneity between obligations can not assume high values, this limitation does not seem relevant to homogenous networks but could influence the analysis of concentrated topologies. The calibration of our model on the base of empirical data could correct this point of weakness. In order to facilitate the understanding of the procedure, we propose an example.

Consider the topology defined by the adjacency matrix *Xexample* and a *TV* value equal to 300. Following the aforementioned procedure, firstly, we identify the final quota of credits for each agent as equal to 100 (300/3), secondly, we divide said quota by the number of debtors for each agent, respectively 1 and 1, 2. The weighted adjacency matrix *X'* in Figure 2.2 represents the outcome of the process. As a result, Agent 1 is a net creditor of 50 to the system, as he has a credit of 100 to Agent 2 and a debit of 50 to Agent 3; Agent 2, on the other hand, has a negative net position equal to -50 given that the value of his credits (100) is less than the value of his debits (150); Agent 3 has a zero-position given that his total credits and debits have a value of 100.

Figure 2.2. The weighted adjacency matrix for the network described in *Xexample* and TV=300

$$X' = \begin{bmatrix} 0 & 100 & 0 \\ 0 & 0 & 100 \\ 50 & 50 & 0 \end{bmatrix}$$

2.2. Settlement methods

Once determined the payment matrix, we will proceed by describing possible settlement methods. Three systems will be taken into consideration: the GS, the BNS and the MNS. In the standard GS system, debits are extinguished by means of nominal payments without any type of clearing. In reference to the payment matrix *X'* described above, this indicates that Agent 1 will bear a monetary outlay of 50, Agent 2 will bear one equal to 150 whilst Agent 3 one equal to 100. The BNS presents with the same characteristics as the GS with the difference that the bilateral arrangements between the agents are offset. Technically, this payment method transforms the payment matrix *X '* into an *X' '* matrix where the bilateral positions are compensated according to the formula $x_{i,j} - x_{j,i}$ (Figure 2.3).

Figure 2.3. Payment matrix GS and BNS

$$X' = \begin{bmatrix} 0 & 100 & 0 \\ 0 & 0 & 100 \\ 50 & 50 & 0 \end{bmatrix} \qquad X'' = \begin{bmatrix} 0 & 100 & 0 \\ 0 & 0 & 50 \\ 50 & 0 & 0 \end{bmatrix}$$

In conclusion, we take into consideration the MNS which can be differentiated from the previous methods both by expiration and by execution. A feature of netting systems is the presence of a new agent within the payment system, the clearing house. Agents apply to clearing houses in order to compensate for their positions. Usually said procedure takes the form of a novation agreement, that is, a transformation of bilateral exposures between agents into a single net position towards the clearing house. The task of the clearing house is to offset debits and credits of the agents, to collect its own debits and to liquidate its own creditors. Figure 2.4 shows the "weighted adjacency matrix" of the clearing house for the payment system identified with *Xexample*. Each *i*-th row represents the account of the *i*-th agent to the clearing house; by construction the sum of the exposures to the Clearing House is zero. This is due to the fact that the agent's credit towards agent *j* is at the same time the agent agent's debt to agent *i*. Thus, when all exposures are added, agent's credit to agent *j* is eliminated by agent's debt to agent *i*.

Figure 2.4. Clearing House accounts

$$X_{cl} = \begin{bmatrix} +50 \\ -50 \\ 0 \end{bmatrix}$$

2.3 Model operability, Time frame and Variables

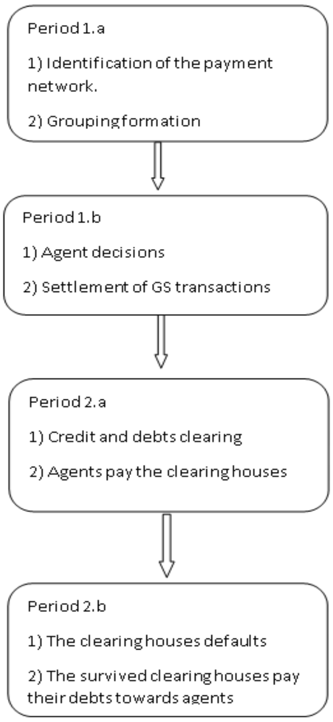
Having described payment network and settlement agreements, we proceed to describe the procedure according to which agents select their chosen payment option among the ones offered.

In general, the model presented has to be used only to investigate the dynamics of the activation of the clearing house and how this can be influenced by spatial frictions, transaction costs, payment topologies and business cycles. Upon activation of a clearing house there exists a minimum mass of agents who rationally decide to adhere to the room on the basis of their preferences, the economic context, the topology of credit network and without a prior story. For this reason, the choice of static networks and optimizing behavior of the agents can provide a good approximation of the phenomenon of coordination of a minimum mass of agents. Our model is not suitable for the analysis of the functioning and growth in size of the clearing houses once the institution has been set up. For this kind of analysis, it is necessary to use dynamic models and networks characterized by the strategic and repeated interaction among the agents.

Initially, agents' behaviour in the case of a binary choice between GS or MNS is analysed; at a later stage, BNS and MNS will be taken into account. The system is made up of two intervals and each of them is, in turn, divided into two more sub-intervals. Within the first half of the first interval, exchanges arise amongst agents and credit and debit reports are assigned to each agent. After identifying the payment network, agents are grouped exogenously.

The dimension of the group formation coincides with the dimension of the clearing house to which agents can have access to. Within the second half of the first interval, agents are called to decide whether to settle their positions by means of liquidity or whether to participate in a clearing house to compensate them. In the first half of the second interval, clearing houses compensate the positions amongst the agents who choose to become members, then they levy a commission from all the members for the service provided and eventually they recover credits from the agents. In the second half of the second interval, the clearing house can fail, thus avoiding liquidating net creditors. Otherwise, the house will liquidate its own creditors. Figure 2.5 depicts the graphic of the model's sequences. As for the case where the competing systems are the BNS and the MNS, the scenario is similar to the one presented above, except for the fact that the BNS replaces the GS and, under such circumstances, only one group comprising the totality of the agents is considered.

Figure 2.5. Model sequences for GS vs MNS



We establish that the utility function of the agents depends solely on the monetary variable in linear fashion, and that the value of a unit of currency in interval 1 is the same of a unit of currency in interval 2. For what concerns the GS vs the MNS case, agents decide which payment system to make use of, jointly analysing a set of variables such as interest rate, probability of default, commissions to the clearing house and net exposure.

Interest rate (i_t) represents the necessary cost to obtain liquidity that the agents are subject to. In our model, agents are not provided with initial liquidity; however, we hypothesize that they would be able to obtain any amount of liquid resources. The burden of such loan is defined by the interest rate in force within the interval in question. The borrowed currency can only be spent in the interval in which it is firstly requested. Agents are aware during interval 1 of the interest rates of interval 1 and 2.

The Default Probability (*pd*) is the variable that expresses the risk of default of the clearing house. Each clearing house is provided with a credit collection unit such as to always obtain its credits in the first half of interval 2. Nonetheless, due to technological or fraudulent reasons, the clearing house can fail before the liquidation of its own debts towards agents in the second half of the second interval. Agents are aware of the probability of failure of clearing houses. The default probability is equal for each clearing house.

In this way we are able to incorporate credit risk within our framework without incurring in computational problems that would inevitably arise if we consider the individual credit risk of each agent. Potential creditors of the clearing house must carefully assess the risk of bankruptcy of the clearing house as in the event of a manifestation of this risk the entire credit would become uncollectable. Extensions of our model could include margins or collaterals as insurance tools.

The fee is the commission rate to pay in order to benefit from clearing services. The fee is proportional to the volume of obligations cleared. The term fee encompasses also the cost of collecting the resource necessary for the payment of commission. We set a proportional fee as for many derivatives clearing houses and for some classes of members of the Wir circuit. An alternative solution would be to consider a flat fee. A proportional fee increases the weight of the commissions with the increase in the volume of compensated credits, which tends to penalize the membership of a large room. On the other hand, a flat fee tends to encourage the creation of a large room where the fixed cost of the operation is distributed over a larger volume of credit.

The direct exposition (Δ) indicates the agent net exposure within the group.

The size is the dimension of the clearing house, that is the number of agents necessary for the activation of a clearing house. The size corresponds also to the dimension of the group formation created. Within the group agents will have to decide whether to activate a clearing house with other members or whether to regulate said obligations with a GS approach, the same as the approach towards agents outside of the group formation.

As for the case where the competing systems are the BNS and the MNS, there are no significant differences on a conceptual level. With the aim of adapting the model to the different dynamics of the American Free Banking context, we add the relevant transaction and operational costs if the agents opt for the BNS and we leave the model unchanged if they opt for the MNS.

The transaction-operational costs (*trc*) is a variable taken into consideration only for the banking network. This variable comprises all the costs related to the activity carried out by the porters in the system described by Gorton, including the operational cost entailed by the procedure. The cost is proportional to the value of the payments that every bank has to make. This is due to three reasons. The first reason is that the operational risk is directly connected with the amount of gold carried by the porters, which in turn is directly connected with the value of the payments to be made. The second reason is that, in such system, the size of the banks was similar; in consequence, it is reasonable to assume that the variance of the payment amounts was quite low. This implies that the more the value of the payments to be made, the more the number of banks to which the porter has to go, thus increasing transaction costs. Finally, the third reason is that the bigger the value of the payments to be made in gold, the more expensive the transportation of the commodity.

2.4. Decision making process

The agents' decision-making process is similar for the two cases investigated. We will begin by describing the case for the circuits between companies (GS vs MNS) and, subsequently, the case concerning the system of interbank payments (BNS vs MNS).

For what concerns the real payment network, during interval 1, each agent needs to decide whether to take part to the clearing house according to conditions (1) and (2) valid respectively for the net creditor and the net debtor as compared to the group formation. Within the group we define each single credit from the agent *i* to agent *s* as $L_{i,s}$ and each single debt as $B_{i,s}$.

$$(1)\sum_{S=1}^{y} L_{i,s} - \sum_{S=1}^{y} B_{i,s} - \sum_{S=1}^{y} B_{i,s} * i_1 > (\sum_{S=1}^{y} L_{i,s} - \sum_{S=1}^{y} B_{i,s}) * (1 - pd) - \sum_{S=1}^{y} B_{i,s} * fee$$

$$i, s \in (1,2,3...size), i \neq s, y = size$$

$$(2)\sum_{S=1}^{y} L_{i,s} - \sum_{S=1}^{y} B_{i,s} - \sum_{S=1}^{y} B_{i,s} * i_1 > (\sum_{S=1}^{y} L_{i,s} - \sum_{S=1}^{y} B_{i,s}) - (-(\sum_{S=1}^{y} L_{i,s} - \sum_{S=1}^{y} B_{i,s})) * i_2 - \sum_{S=1}^{y} L_{i,s} * fee$$

for $(\sum_{S=1}^{y} L_{i,s} - \sum_{S=1}^{y} B_{i,s}) < 0$, $i, s \in (1,2,3...size), i \neq s, y = size$

Given that $(\sum_{s=1}^{y} L_{i,s} - \sum_{s=1}^{y} B_{i,s}) = \Delta_i \; i, s \in (1,2,3...size), i \neq s, y = size$, the condition (1) and (2) can be written as

$$(1)\quad (\Delta_i) - (\sum_{s=1}^{y} B_{i,s} * i_1) > (\Delta_i) - (\Delta_i * pd) - (\sum_{s=1}^{y} B_{i,s} * fee) \qquad i, s \in (1,2,3...size), i \neq s, y = size$$

$$(2)\quad (\Delta_i) - (\sum_{s=1}^{y} B_{i,s} * i_1) > (\Delta_i) - (-\Delta_i * i_2) - (\sum_{s=1}^{y} L_{i,s} * fee) \quad \text{for } \Delta_i < 0,\; i, s \in (1,2,3...size), i \neq s, y = size$$

The left side of the condition (1) includes the creditor's payoff in case he makes use of the GS to regulate the relations within the group. Said payoff consists of the monetary value deriving from credits minus the cost of obtaining the liquidity on nominally regulated debts. On the right, the expected payoff in the event of subscription to the clearing house. The value depends on the net position at the conclusion of the procedure minus the expected loss on net credits and the commission to the clearing house. Said commission is proportional to the offset values.

For the case of a net debtor ($\Delta_i < 0$), condition (2) has the same method of calculating payoffs in the GS case. In regard to multilateral netting, the settlement cost of this position is subtracted to the negative net position, in addition to the commission on the volume of the compensated obligations.

As for the case of free banking, we considered an instance where the competing systems are the BNS and the MNS, and the conditions of choice are defined by (3) and (4) for respectively the net creditor and the net debtor.

$$(3)\qquad \Delta_i - \sum_{S=1}^{y} BN_{i,s} * i_1 - \sum_{S=1}^{y} BN_{i,s} * trc > (\Delta_i) * (1 - pd) - \sum_{S=1}^{y} B_{i,s} * fee \quad i, s \in (1,2,3...size), i \neq s, y = size$$

$$(4)\quad \Delta_i - \sum_{S=1}^{y} BN_{i,s} * i_1 - \sum_{S=1}^{y} BN_{i,s} * trc > (\Delta_i) - (-\Delta_i * i_2) - \sum_{S=1}^{y} L_{i,s} * fee \text{ for } \Delta_i < 0,$$

$i, s \in (1,2,3...size), i \neq s, y = size$

It can be noted that two differences stand out in comparison with the previous case. The first difference lies in the term *BN* which, contrary to *B*, internalizes the bilateral netting, reducing the amount of payment means that the debtor must find if he does not participate in the clearing house. This increases the competitiveness of the BNS. The second difference lies in the term *trc* which represents the transaction and operational costs, being proportional to the payments due. In order to identify the leading dynamics of the agents' choices in regard to the system to be implemented, we propose an example of a uniform network composed of 25 agents united in a single group. The total value of the assets of the system is 100000. The size of the potential clearing house is 25. The systems taken into account by the agents are the GS-MNS and the BN-MNS. Only for this example, we set the transaction costs *trc=0* in order to be able to detect the differences between GS and BNS modes in a clearer way when connectivity varies. A default probability of 5% is taken into consideration, three different levels of flat curves of interest rates (1%, 5% and 10%).

Figure 2.6. Graph a) case GS-MNS b) case BNS-MNS.On the x axis the percentage level of connectivity (scale 1*10^{-1}) and on the y-axis the percentage of agents who apply for the clearing house. For y=1 the clearing house is activated. Three flat interest rate curves tested.

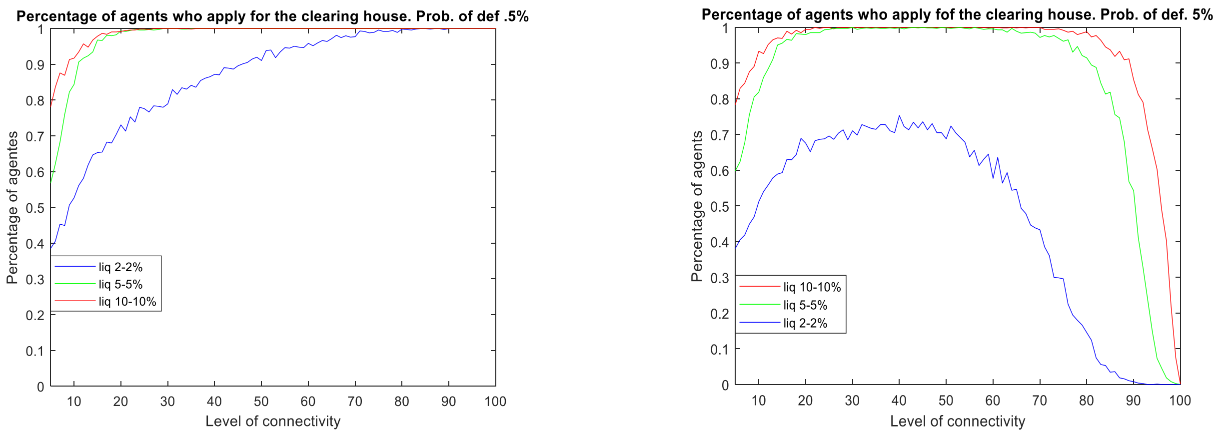


Figure 2.6.a shows the pecentage of agents asking for access to the clearing house as the interconnectedness amongst agents changes whenever the concurrent payment systems are GS and MNS. Y axis range is 0-1, the clearing house is activated when all the agents have the incentive to enter and therefore the percentage of agents is equal to 1. It can be seen how agents do not have any incentive to take part of the clearing house when the interest rates curve takes low values, with the exception of particularly high connectivity levels. This is because liquidity costs are as low as to encourage agents to avoid the risk of the clearing house failing as well as the weight of commissions. The agents will agree to become members only for particularly high connectivity levels, given that the net exposure towards the house lowers with the rise of the interconnectedness. As interest rates rise and as liquidity costs increase consequently, agents will prefer to apply for the clearing house already at low connectivity levels.

Figure 2.6.b shows agents' behaviour when the concurrent settlement systems are BNS and MNS. By contrast to the previous case, a non-linear relation can be seen between connectivity level and applications for the clearing house. The reason behind such phenomenon is that, whenever the probability of interconnectedness amongst agents rises, a concomitant rise in bilateral exposures is registered. This dynamic will increase the number of bilaterally netted positions, thus allowing for a cost reduction of the non-participation to the clearing house. In detail, with the growth of connectivity from the lower end of the domain to the upper one, the net debits to be welded first increase, until reaching a maximum point for an average connectivity level, then they fall almost symmetrically in the second half of the probability domain. This explains why the incentive to take part to the clearing house increases in the first half of the probability domain, and why it decreases in the second half. The same dynamic presents itself for each of the three rate curves observed. As for the example of GS-MNS treatments, rate curves have a positive relation to the number of agents applying for the clearing house. In contrast to the previous case, for the BNS the interval of connectivity during which

it is possible to activate the clearing house will be shorter and, in case of an interest rate low curve, null.

Figure 2.7. Graph a) case GS-MNS b) case BNS-MNS.On the x axe the percentage level of connectivity (scale $1*10^{-1}$) and on the y-axis the percentage of agents who apply for the clearing house. For y=1 the clearing house is activated. Three levels of default probability tested

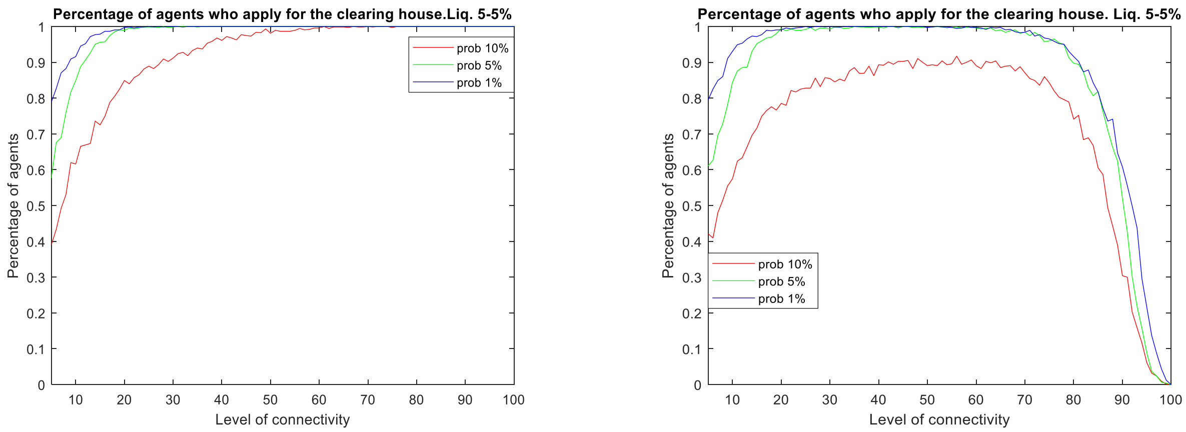


Figure 2.7a shows how the choice dynamic varies as the default probability of the clearing house fluctuates. We consider a flat interest rate curve at 5% and three default probability levels of the clearing house, fixed at 1%, 5% and 10% respectively. As it shown by the graphic, we can observe how, when the default probability is high, the house can be activated only for high connectivity levels. The linear rapport between interconnectedness probability and agents' inclination to apply to the clearing house is because the higher the level of *p* the lower the value of Δ_i for each agent. In the extreme case of *p*=100 and equal value of obligations amongst agents, the clearing house would be activated for whichever level of default probability, on the ground that all the positions would be offset and agents should only have to pay commission expenses. A negative relation between default probability of the clearing house and agents' admissions requests to it is observed. Intuitively, the higher the risk of do not recover the credit, the lower the number of members. Figure 2.7b takes into consideration the same dynamic for the BNS-MNS. For this process as well, the default probability has a negative relation to the number of agents willing to participate to the clearing house. However, in contrast to the previous instance, the probability domain for the activation of the clearing house can be null in case of a high default probability. This is because in the BNS treatment, agents will accept to become members of the clearing house only for low risk credits, given the savings inherent to bilateral netting.

3. SIMULATION OUTPUTS

3.1. Exogenous groups analysis

The first case study is structured accordingly to exogenous group analysis in a context where GS and MNS are in competition. After defining the payment matrix, we proceed to randomly group agents in groups of various dimensions. The dimension of the group is an indicator of the size of the clearing house that could be activated by agents belonging to each group. Exogenous grouping can be seen as a friction limiting the creation of clearing houses; in this framework, in fact, agents are not free to look for a counterparty with whom to offset their credits. Such instance reduces the domain of activation of the clearing houses.

In our case, the exogenous groups analyses carried out are two, respectively 5 and 12 of scope, to which is added the only possible 25 scope group (the only one possible given the correspondence with the total amount of agents of the system). The selection process of the payment options is up to the individual and its functioning is as previously presented in the above section of the paper. Four possible scenarios in which the interest rates curves are combined with the default probability curves of the clearing house are taken into consideration. We test a scenario in which the interest rate curve as well as the default probability curve are increasing; a scenario in which the interest rate curve as well as the default probability curve are decreasing; a scenario in which the interest rate curve is increasing, and the default rate curve is decreasing; a scenario in which the interest rate curve is decreasing, and the default rate curve is increasing. Three phases along the curves are considered. Each phase is identified by two interest rates, by the default probability of the clearing houses and by three agents' connectivity levels. We test two topologies, uniform and concentrated. The metric we take into consideration in this section is the number of clearing houses resulting from the model for agents' selection. Table 3.1 identifies the parameters based on which we will perform Monte Carlo simulations.

Table 3.1. Parameters values (GS-MNS)

Scenario	**Phase**	Pd	il	i2
1	**1**	1%	2%	2%
	2	5%	2%	5%
	3	10%	5%	7%
2	**1**	10%	7%	7%
	2	5%	7%	5%
	3	1%	5%	2%
3	**1**	10%	2%	2%
	2	5%	2%	5%
	3	1%	5%	7%
4	**1**	1%	7%	7%
	2	5%	7%	5%
	3	10%	5%	2%

Homogeneous Topology	P (0,3-0,5-0,7)
Concentrated Topology	Ph range (0,3-0,5-0,7)
	Ps fixed (0,1)
N	25
Nh	4
Ns	21
TV	100000
Fee	0,5 %

Table 3.2. Output for homogeneous topology (GS-MNS). The table shows the mean and the variance of the number of clearing houses activated.

Homogeneous Topology (GS-MNS)									
	Phase 1			Phase 2			Phase 3		
Connectivity	0,3	0,5	0,7	0,3	0,5	0,7	0,3	0,5	0,7
Size 5									
Scen.1 mean and (var.)	0,66 (0,65)	1,70 (1,39)	3,50 (1,10)	0,05 (0,05)	0,10 (0,09)	0,30 (0,22)	0,05 (0,05)	0,35 (0,34)	1,10 (0,07)
Scen.2 mean and (var.)	0,30 (0,25)	0,74 (0,70)	1,65 (0,55)	0,75 (0,55)	1,90 (0,83)	3,60 (1,35)	1,45 (0,89)	3,55 (1,44)	4,65 (0,23)
Scen.3 mean and (var.)	0,00 (0,00)	0,00 (0,00)	0,15 (0,14)	0,05 (0,05)	0,10 (0,09)	0,30 (0,22)	0,74 (0,70)	2,30 (0,64)	3,90 (0.93)
Scen.4 mean and (var.)	1,45 (0,89)	3,45 (1,41)	4,80 (0,16)	0,75 (0,55)	1,90 (0,83)	3,60 (1,35)	0,10 (0,09)	0,25 (0,19)	0,75 (0,80)
Size 12-13									
Scen.1 mean and (var.)	0,80 (0,40)	1,45 (0,36)	2,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,20 (0,13)	0,00 (0,00)	0,05 (0,05)	1,11 (0,51)
Scen.2 mean and (var.)	0,05 (0,05)	0,65 (0,45)	1,60 (0,30)	0,30 (0,22)	1,65 (0,23)	2,00 (0,00)	1,60 (0,35)	2,00 (0,00)	2,00 (0,00)
Scen.3 mean and (var.)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,20 (0,13)	0,45 (0,36)	1,70 (0,22)	2,00 (0,00)
Scen.4 mean and (var.)	1,80 (0,23)	1,95 (0,05)	2,00 (000)	0,30 (0,22)	1,65 (0,23)	2,00 (0,00)	0,00 (0,00)	0,65 (0,55)	0,94 (0,40)
Size 1									
Scen.1 mean and (var.)	1,00 (0,00)	1,00 (0,00)	1,00 (0,00)	0,00 (0,00)	0,01 (0,01)	0,55 (0,09)	0,10 (0,01)	0,65 (011)	0,90 (010)
Scen.2 mean and (var.)	0,65 (0,23)	0,95 (0,05)	1,00 (0,00)	1,00 (0,00)	1,00 (0,00)	1,00 (0,00)	1,00 (0,00)	1,00 (0,00)	1,00 (0,00)
Scen.3 mean and (var.)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,01 (0,01)	0,55 (0,09)	1,00 (0,00)	1,00 (0,00)	1,00 (0,00)
Scen.4 mean and (var.)	1,00 (0,00)	1,00 (0,00)	1,00 (0,00)	1,00 (0,00)	1,00 (0,00)	1,00 (0,00)	0,10 (0,09)	0,70 (0,22)	1,00 (0,00)

Table 3.2 illustrates the results of our simulations; the table indicates the average number of clearing houses activated and its variance for each scenario. For the sake of clarity the graphical analysis has been abandoned in favour of tables. Graphs would have been composed by too many overlapping lines, even candle graphs. We begin by analysing the results for the homogenous topology. In order to explain the outputs, we have identified four effects.

The reported effects derive from an analysis of the maximizing behavior of agents on the basis of their preference system. Once combined, these effects determine the results of simulations.

Effect 1. The first effect is due to the rise in interconnectedness amongst agents. A consequence of the first effect is a reduction trend of the net exposures towards the potential clearing house. *Ceteris paribus*, as the connectivity level increases, the number of agents applying for the clearing house increases.

Effect 2. The second effect takes into consideration the interest rate curve. *Ceteris paribus,* each time agents are confronted with a decreasing rates curve, they are also given an incentive to enter the clearing house because the liquidity cost in interval 2 is lower than that in interval 1. The opposite holds true in case of an increasing curve, for which the liquidity cost in interval 2 will be higher than that in interval 1. For a flat interest rate curve, effect 2 does not prompt action towards either of the two treatments.

Effect 3. The third effect pertains to the absolute value of the interest rate in interval 1. The relation between interest rate in interval 1 and willingness of agents to offset is positive. *Ceteris paribus*, the higher the interest rate, the higher the number of agents wanting to offset their credits and debits.

Effect 4. The fourth effect takes into consideration the default probability of clearing houses. In this instance, the relation to the willingness of agents to offset is negative. *Ceteris paribus*, the higher the default probability, the lower the number of agents willing to offset.

3.2. Small-size Clearing Houses, 5 scope group

In regard to the first scenario, Table 3.2 shows that the average number of activated chambers is, on average, low. In phase 1, effect 1 and 4 stimulate the development of chambers while effect 3 acts as a brake towards such choice. For high connectivity levels, less of 4 out of 5 possible rooms are activated. In phase 2 the number of houses activated is on average close to zero and this is mainly due to effect 2 and effect 4. In phase 3 the effects tend to balance, the number of rooms increases slightly and for maximum connectivity levels it reaches the unit permanently.

Analysing the second scenario, an increase in the number of chambers formed going from phase 1 into phase 3 can be seen; this is mainly due to effects 2 and 4. In phase 1, effect 3 and 4 tend to compensate each other flattening towards zero the number of houses of medium-low level; for high connectivity levels, effect 1 prevails on the others. In phase 2 and 3, effects 1, 3 and 4 stimulate the development of houses, which almost reach 5 units per high levels of connectivity on average.

In regard to the third scenario, in the first two phases the number of houses is close to zero, given the effects 3 and 4 in phase one and effect 2 in phase 2. In phase 3 with three effects, we are barely able to reach one single stable house for high connectivity levels. In regard to the fourth scenario, we observe a downward trend for the number of houses going from phase one to phase three. In phase one, for high connectivity levels, it is possible to reach the five houses with very high frequency. This is because almost all the effects move in the same direction. In phase two, we register an average fall of clearing houses due to effect 4, fall that persists in phase three, due mainly to effects 3 and 4.

3.3. Medium-size Clearing Houses, 12-13 scope groups

For medium-size houses agents are grouped into two formations, 12 and 13 agents respectively, thus widening the domain for compensation possibility but at the same time complicating the instance in which all the agents are given incentives to activate the clearing house of their reference-frame. In regard to the first scenario, during phase one, effects 1 and 4 dominate effect 3, pressing through high connectivity levels all the agents to participate to the clearing houses. In phase two, effects 2 and 4 reduce the number of houses to a value close to zero; phase three, in a similar manner, displays a few active clearing houses, with the exception of high levels of connectivity where, through a high number of simulations, it is possible to activate a house owing to the positive thrust of effect 3. The second scenario displays a growth in the number of houses going from phase 1 into phase 3. In phase 1 effects 1 and 3 are dominated by effect 4 through low levels of connectivity; in phase 2 effects 1 and 4 dominate over effect 2 pressing through high levels of connectivity all the agents to offset. A similar dynamic presents itself in phase 3. The thirds scenario is characterized in phase 1 by a null development of houses due to the strength of effect 4, whilst in phase 2 by the joint thrust of effects 2 and 4. In phase 3, effects 1, 3 and 4 dominate effect 2. In the fourth scenario, three effects out of four move towards offsetting in phase 1, therefore two clearing houses are always activated for medium-high connectivity levels. In phase 2, effects 1 and 2 tend to dominate effect 4, even though the two houses can be activated only for high connectivity levels.

3.4. Big size, 25 scope group

As a final point, we analyse the largest possible grouping, the 25-scope group. Such a large grouping formation allows agents to offset the all of their obligations within the house. For that reason, this fact occurs within many phases and we can consider the data shown by Table 3.2 as probability of occurrence. In the first scenario, phase 1 presents, once again, the development of the clearing house propelled by effect 4. In phase 2, the house is activated only for high levels of connectivity; nonetheless, the number of simulations where this occurs is slightly above average. In phase 3, due to the positive influence of effect 1, for high levels of connectivity, the house is active at 90% of probability while for a medium connectivity it reaches the average 65%. In the second scenario, the house is activated for almost all connectivity levels and rates, because in phase 1, effect 1 and 3 dominate effect 4. In regard to phases 2 and 3, the clearing house is always active, given that almost all the effects move towards clearing. In the third scenario, the house is structurally active only in phase 3, due to the domination of effects 1, 3 and 4 over effect 2 in this phase. In the fourth scenario, the house is usually active; only in phase 3 for medium-low levels of connectivity, effects 3 and 4 dominate effects 1 and 2.

We can claim that in the case where the agents are not spatially constrained the domain of activation of the clearing house is extremely higher than the other cases. It is a confirmation of our thesis which look at the coordination problem.

3.5. Concentrated topology

In this case, payments display a particular topology: only a few agents make numerous payments and numerous agents make only a few payments, mainly towards a highly connected minority. In such instances, clearing houses are usually never activated. As Table A1 in the Appendix shows, of the various scenarios tested only the case in which the largest grouping was present succeeds in obtaining the activation of the house structurally. In particular, for small sized groups, simulations at best can detect the activation of two houses but only with a much lower frequency compared to the number of simulations performed. The low connectivity level of the majority of agents within the network does not provide the incentives to offset; presumably, the critical mass of number payments for the majority of the agents is missing. For the case of two clearing houses, the houses themselves are usually never activated if not during some extremely favourable phases of the second and the fourth scenario. For the second scenario, in phase 3, effects 1, 2 and 4 move towards clearing, whilst only effect 3 is pushing towards the opposite direction, thus resulting in one active house for almost all the simulations. In the fourth scenario, in phase 1, it is possible to activate a house given that effect 1, 3 and 4 move towards clearing and, at the same time, effect 2 is neutral. Concerning the one-house

grouping, comparably to the two-group instance, there are only two phases of activation of the clearing house and this is due to the same effects described for the two-houses case.
In summary, for this topology of payment, the number of active houses is extremely low, and it is tied to specific phases of the proposed scenarios. The low level of interconnectedness among the majority of agents seems to be the main cause of the obtained results. In order to understand whether the 5-scope and the 12-scope grouping is a significant determinant, it is necessary to test the case in which agents choose partners in a context dominated by less urgent frictions.

3.6. Summary

In this section, the possibility of creation of clearing houses according to three types of grouping has been analysed. Tests have been carried out through two systems, GS-MNS and BNS-MNS, for two topologies of payments. Each topology has been studied through the lens of four different economic scenarios. The results can be summarized into the following core concepts:

1. Competition between GS and MNS systems leaves room for the coexistence of the two systems.
2. In the contest characterized by spatial friction credits and debits compensation is lower respect to the cases without this type of friction.
3. In homogeneous networks, more chambers are formed compared to concentrated topologies, regardless of the considered scenario.
4. The result of the tested topologies is that the economic cycle considerably influences the choice of agents in regard to the payment methods to be used.
5. The variable that indicates connectivity is significant for most of the phases and scenarios.

4. POTENTIAL OF CLEARING

The results expressed by the previous section suggest the difficulty in developing clearing houses along the lines of the Wir-Sardex model, when frictions limiting the possibility of organization among agents are present. In the absence of frictions, the issue of maximizing agents' payoff would become extremely costly from a computational perspective. Consider the hypothetical instance in which an agent evaluates whether to apply to a size 5 clearing house, where the possible combinations of a system populated by 25 agents become 25!/20!5!=53130 and each of them has to be confronted with all the possible combinations and sequences per different sizes. Therefore, the question we address in this section is not which the optimal configuration for clearing houses is, but, merely, whether the system might support a potential for multilateral netting. Due to the new aim of the research, the procedure presented in the previous section has been modified, in order to openly distinguish the effects of the elimination of "spatial friction". In detail, we hypothesize that, once the payment network had been identified, public authority, which we hypothesize would be able to map accurately the network and agents' preferences, would fix a size indicative of the dimensions of the group subsequently extract, one at a time, all the possible combinations associated with that size. Furthermore, we supposed that the public authority would have the power to compel agents to form a clearing house every time their feasibility conditions described in section 2.e are fulfilled.

Consequently, an optimal configuration is not achieved. It is achieved, however, a feasible incentive configuration which specifies the possible size of the multilateral netting. We take into consideration 6, 5, 4 and 3 size houses, starting with the extraction of the largest-size combinations. The same procedure is adopted for both topologies tested in section three. We consider the first and the second scenario of the previous section. Our reference metrics are the number of clearing houses constituted and the portions of the value of the credits handle by the clearing houses. Results shown by the Tables A2 and A3 in the Appendix concern the system in which GS-MNS are in competition.
For the homogeneous topology, an extremely high application rate towards larger-size houses is immediately evident. This is due to both the fact that the higher the size, the higher the number of potential clearing contracts, and to the order of the extraction. In the first scenario we notice a substantial number of houses in phase 1, reaching on average 31 units of 6 size for high levels of connectivity. Typically, larger size houses absorb the majority of debit-credit relationship among agents. Thus, only a few smaller size houses will be activated. In phase 2, in contrast with the results shown by the previous section, it is possible to achieve up to 27 houses of 6 size and, in a structural manner, smaller size houses. Similarly, in phase 3, despite effects 2 and 4 disincentivize the development of houses, for highly connected cases it is possible to develop on average as many as 28 houses of 6 size. Concerning the volume of offset credits in this scenario, phase 1 registers a range of 75-95%, a slight decrease is registered in phase 2 and 3, still granting, however, a 60-90% of credits handle by the clearing houses. The second scenario, more favourable to multilateral netting, allows to register a higher number of houses formed, with peaks of 37 developed houses of 6 size for highly connected networks. The value of offset credits in clearing houses nearly achieves 100% per highly connected networks in phase 2, if we consider the credits handle by the clearing houses of different sizes. Moreover, it achieves consistently more than 85% in phase 1 and 70% in phase 3 considering the entire system.
In regard to the concentrated topology, in contrast with the spatial friction cases of the previous section, the simulations we performed point to the activation of a relevant number of houses and offset credits. Analysing the first scenario, the least favourable scenario to clearing between the two, it emerges that in all phases a minimum average of 7 to a maximum average of 12 houses are activated. The total value of credits managed by the clearing houses goes from a minimum average of 50% to a maximum average of slightly more than 80%.
In regard to the second scenario, the most favourable to netting, a slight increase can be noticed in the number of active houses able to reach 14 units in phase 3 with high levels of connectivity and a netting range varying from 55% and 85% of the total of credits in the system.

5. FREE BANKING CASE

In this section we present the results of the scenario where the BNS and the MNS payment systems are in competition with each other as illustrated in section 2. In particular, we investigate the process of endogenous formation of a clearing house in the context of an interbank payment network whose features resemble the free banking system described by Gorton. To this purpose, we focused on homogeneous topologies that better represent the relations among banks of similar size, which are generally characterized by payment orders with a low variance. Furthermore, because clearing procedures occurred weekly, we assumed interest rate curves to be flat. We also took into account three levels of transaction costs and three default probabilities for the clearing house as shown in the Table below. As is the case with GS vs. MNS, we identified 4 effects which determine the results of our simulations.

Effect 1. The first effect concerns the connection probability. As described in section 2, when it comes to the BNS, the probability of connection between the agents engenders an incentive to the formation of the clearing house in the first half of the connectivity domain. In the second half, on the contrary, effect 1 will obstruct the formation of the clearing house due to the possibility of netting a high number of obligations in a bilateral way.

Effect 2. The second effect concerns the liquidity cost. We test three flat interest rate curves – low, medium, and high. With the increase of the interest rate from the low to the high curve, the liquidity costs increase too; as a consequence, the incentives to join the clearing house grow. Hence, effect 2 shows a positive association with the activation of the clearing house.

Effect 3. The third effect concerns the operational and transaction costs. As illustrated in the second section, with the increase of the transaction costs within the BNS, the incentives to the formation of the clearing house increase too. Therefore, effect 3 shows a positive association as well with the activation of the clearing house.

Effect 4. The fourth effect concerns the default probability of the clearing house. The higher the default probability, the lower the incentive to join the clearing house. Effect 4 thus shows a negative association with the formation of the clearing house.

According to Gorton, as the number of transactions between intermediaries increased, the increase in operational and transaction costs would not have made the bilateral netting system sustainable. From a theoretical point of view the result is not so obvious. The increase in the number of transactions certainly increases the cost of transactions, but also could increase the possibility of netting bilaterally. This trade off is the basis of our investigation. Table 5.1 shows the network and scenario parameters.

Table 5.1 Parameters values (BNS-MNS)

Scenario 1 i1=2%, i2=2%	trc=0,1%	Default probability=1,3 and 5%
	trc=0,5%	
	trc=1%	
Scenario 2 i1=3,5%, i2=3,5%	trc=0,1%	Default probability=1,3 and 5%
	trc=0,5%	
	trc=1%	
Scenario 3 i1=5%, i2=5%	trc=0,1%	Default probability=1,3 and 5%
	trc=0,5%	
	trc=1%	

Homogeneous Topology	P (0,3-0,5-0,7)
N	25
TV	100000
Fee	0,5 %

Tabella 5.2 Simulation outputs (BNS-MNS). The table shows the mean and the variance of the number of clearing houses activated.

Homogeneous topology BNS-MNS									
Transaction costs	**0, 1%**			**0,5%**			**1%**		
Connectivity	**0,3**	**0,5**	**0,7**	**0,3**	**0,5**	**0,7**	**0,3**	**0,5**	**0,7**
Prob 5									
Scen.1 mean and (var.)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)
Scen.2 mean and (var.)	0,15 (0,13)	0,25 (0,19)	0,10 (0,09)	0,35 (0,23)	0,50 (0,26)	0,15 (0,13)	0,75 (0,19)	0,77 (0,20)	0,30 (0,22)
Scen.3 mean and (var.)	0,88 (0,66)	0,95 (0,05)	0,70 (0,22)	0,90 (0,09)	0,95 (0,05)	0,75 (0,19)	1,00 (0,00)	1,00 (0,00)	0,85 (0,13)
Prob 3									
Scen.1 mean and (var.)	0,00 (0,00)	0,05 (0,05)	0,00 (0,00)	0,35 (0,24)	0,50 (0,26)	0,00 (0,00)	0,65 (0,23)	0,70 (0,22)	0,05 (0,05)
Scen.2 mean and (var.)	0,85 (0,09)	0,86 (0,12)	0,30 (0,22)	1,00 (0,00)	1,00 (0,00)	0,35 (0,22)	1,00 (0,00)	1,00 (0,00)	0,70 (0,25)
Scen.3 mean and (var.)	1,00 (0,00)	1,00 (0,00)	0,45 (0,26)	1,00 (0,00)	1,00 (0,00)	0,50 (0,26)	1,00 (0,00)	1,00 (0,00)	0,70 (0,22)
Prob 1									
Scen.1 mean and (var.)	0,75 (0,19)	0,45 (0,25)	0,00 (0,00)	0,95 (0,05)	0,60 (0,25)	0,05 (0,05)	1,00 (0,00)	0,75 (0,25)	0,15 (0,13)
Scen.2 mean and (var.)	0,90 (0,07)	0,93 (0,07)	0,10 (0,09)	0,95 (0,05)	0,95 (0,05)	0,45 (0,26)	1,00 (0,00)	1,00 (0,00)	0,80 (0,16)
Scen.3 mean and (var.)	0,90 (0,10)	0,95 (0,05)	0,75 (0,19)	0,95 (0,05)	0,95 (0,05)	0,90 (0,09)	1,00 (0,00)	1,00 (0,00)	1,00 (0,00)

The Table 5.2 presents the results for each scenario tested. They report the mean and variance of the number of activated clearing houses. Mean values can also be considered as activation probabilities insofar as they are determined through the number of positive cases divided by the number of possible cases. Below we analyse the results of each scenario in relation to the cases of a high, medium, and low default probability of the clearing house.

5.1 High default probability of the clearing house

The first case to be considered is the one whereby the clearing house is marked by a high default probability. When it comes to the first scenario, where liquidity costs are low, the activation of the clearing house does not occur at any level of the tested operational and transaction costs as well as at some levels of connection among the bank participating in the system. As follows, in this scenario effects 2 and 4 significantly prevail against effects 1 and 3.
In the second scenario, where liquidity costs are medium, the activation of the clearing house can be observed in several cases. In particular, when operational and transaction costs rise from 0,1% to 1%, it can be noted that the probability of activation of the clearing house triples and, on average, reaches

75% at medium connection levels – while being subject to a variation between 15% and 70% at low connection levels. For all the levels of operational and transaction costs, the maximum probability of activation occurs at medium connection levels: this is due to effect 1 which crucially contributes to limit the activation of the clearing house at high connection levels.
In the third scenario, the probability of activation of the clearing house is very high, leading to the structural activation of the latter both at medium and at low connection levels in combination with high operational and transaction costs. In general, it is effect 2 to determine the results of this scenario; the impact of effects 1 and 3, though still present, is significantly lower.

5.2 Medium default probability of the clearing house

Where the clearing house is marked by a medium default probability, the impact of effect 4 is generally less intense than in the previous case.
As for the first scenario with low operational and transaction costs, the clearing house is rarely activated, whereas with medium and high operational and transaction costs the activation probabilities can respectively hit 50% and 70% on average. In this scenario as well, effect 1 tends to prevail against the BNS at high connection levels.
The second scenario features the dominance of effects 1 and 2, structurally determining a high probability of activation of the clearing house at medium and low connection levels. At high connection levels, though, effect 1 tends to decrease the activation probability, an outcome considerably mitigated by effect 3 which push the average activation probability up to 70% for high transaction costs.
The dynamics of the third scenario are similar to the former: in this case, the structural activation of the clearing house occurs also in the presence of low operational and transaction costs, especially thanks to effects 1 and 2.

5.3 Low default probability of the clearing house

When the default probability of the clearing house is low, effect 4 tends to rise the average probability of activation. There are plenty of cases in which the activation of the clearing house is structural. With particular regard to the first scenario, the activation probability is high, by virtue of effect 1, at medium and low connection levels also for low operational and transaction costs. At high connection levels, the combination of effect 1 and 4 keeps down the probability of activation of the clearing house.
As for the second scenario, the probability of activation of the clearing house is high due to the combination of effects 2 and 4. In this case, effect 4 eventually prevails against effect 1, leading the probabilities of activation to hit 80%. Finally, in the third scenario, the activation of the clearing house proves to be structural because, except of effect 1 at high connection levels, all effects come out positive towards multilateral clearing.

5.4 Summarizing

The results of the present section enable us to confirm the thesis put forward by Gordon whereby the clearing houses for the management of interbank payments emerged with the purpose of mitigating liquidity and transaction costs. Indeed, in the presence of near zero operational and transaction costs, only for very high connection levels combined with low liquidity costs or high probability of failure of the clearing house the BNS system is cheaper than the MNS.

6. CONCLUDING REMARKS

In this work we sought to deepen the dynamics leading to the endogenous formation of clearing houses. In particular, we focused on two specific cases. The first one concerns corporate barter networks like Wir and Sardex. The second one concerns the emergence of clearing houses for interbank payments in the Free Banking American as described by Gorton (1984). When it comes to corporate networks, we developed an agent-based model encompassing the endogenous formation of clearing houses in contexts where multilateral clearing and gross settlement systems are in competition with each other. We integrated the model with a spatial friction which could represent the problems of coordination between the agents and, subsequently, we tested the model for different payment topologies as well as for economic scenarios exemplifying various phases of the business cycle.

The results of our analysis show how the presence of a spatial friction significantly obstructs the endogenous formation of clearing networks between companies. Besides, the activated networks could be unstable if the different phases of the business cycle are taken into account; as a consequence, the number of transactions handed by the clearing houses is variable, as demonstrated by Stooder's (2009, 2016) empirical analyses.

In section 4, we attempted to evaluate whether a potential for multilateral clearing exists by relaxing the spatial friction. The results of such evaluation are indicative of the fact that, even for a low number of groups tested (6, 5, 4 and 3 size groups) the value of cleared credits can reach extremely high rates. Such example occurs for various topologies as well as for the vast majority tested scenarios. If the aforementioned results are combined with the results from fixed and exogenous grouping, it is possible to notice, on one hand, the difficulty in the spontaneous development of clearing houses whenever frictions are present, on the other hand, the enormous potential for clearing per low size houses. If reducing the general need for liquidity by the banking system is deemed an important policy to pursue for the system efficiency, a possible solution to implement the offset could be for the banking system itself to mandate this service. A sufficiently large bank will be able to offer its clients the possibility of admission to clearing houses, at least for smaller sizes. In this respect, the path would be opposite to the one followed by Wir Bank, which made a start as a clearing house and grew to become a bank. If such hypothesis appeared to be precipitate, one should think about the conjoint effect that the new EU directive on the PSD2 payment systems and the spread of platforms for the exchange of goods between companies, such as Amazon B2B, might have on the supply of payment services. On the grounds of the new legal framework, the managers of these platforms could provide their customers with a system of credit clearing, being able to mitigate the coordination problems among the companies thanks to the significant number of actors participating in the system. Following from this, the competition might sway the banking system towards the same practice, revolutionizing the mode of financing commercial credits.

For what concerns the second case, we adjusted the agent-based model through the addition of the transaction costs which characterized the Free Banking system. Hence, we tested the model for an homogenous payments topology and for different economic scenario. The results of our simulation suggest that the strong incentive to the multilateral clearing of interbank payments is to be attributed to the inflated liquidity and transaction costs that featured in the original Free Banking system described by Gorton.

Our work leaves room to further theoretical and empirical interventions. As for the clearing between companies, from a theoretical perspective it would be worth investigating the pros and cons of multilateral clearing vis-à-vis other forms of trade credit commonly employed by the firms. From an empirical perspective, it should be tested what topologies of real payments would be convenient to finance through clearing. Lastly, as for the case of the Free Banking system, it would be interesting to calibrate the mode on the basis of historical data.

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APPENDIX

TABLE A.1. Number of clearing houses for concentrated topology (GS-MNS). The table shows the mean and the variance of the number of clearing houses activated.

Concentrated Topology (GS-MNS)									
	Phase 1			Phase 2			Phase 3		
Connectivity	**0,3**	**0,5**	**0,7**	**0,3**	**0,5**	**0,7**	**0,3**	**0,5**	**0,7**
Size 5									
Scen.1 mean and (var.)	0,30 (0,32)	0,55 (0,68)	0,85 (0,76)	0,35 (0,34)	0,50 (0,36)	0,65 (0,34)	0,45 (0,26)	0,34 (0,35)	0,45 (0,36)
Scen.2 mean and (var.)	0,45 (0,26)	0,69 (0,61)	0,70 (0,25)	0,60 (0,35)	0,75 (0,51)	0,85 (0,45)	0,90 (0,51)	1,35 (0,55)	1,60 (0,77)
Scen.3 mean and (var.)	0,40 (0,35)	0,65 (0,45)	0,75 (0,51)	0,35 (0,34)	0,50 (0,36)	0,65 (0,34)	0,75 (0,82)	0,60 (0,50)	0,85 (0,51)
Scen.4 mean and (var.)	0,95 (0,78)	1,50 (0,89)	1,55 (0,47)	0,60 (0,35)	0,75 (0,35)	0,85 (0,45)	0,40 (0,30)	0,41 (0,29)	0,45 (0,35)
Size 12-13									
Scen.1 mean and (var.)	0,10 (0,09)	0,05 (0,05)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)
Scen.2 mean and (var.)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,01 (0,01)	0,00 (0,00)	0,00 (0,00)	0,30 (0,21)	0,75 (0,19)	0,95 (0,15)
Scen.3 mean and (var.)	0,00 (0,00)	0,00 (0,00)	0,00 (000)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,05 (0,05)	0,00 (0,00)
Scen.4 mean and (var.)	0,35 (0,29)	0,75 (0,30)	1,05 (0,05)	0,01 (0,01)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)
Size 25									
Scen.1 mean and (var.)	0,45 (0,26)	0,60 (0,23)	0,60 (0,25)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)
Scen.2 mean and (var.)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,50 (0,26)	1,00 (0,00)	1,00 (0,00)	0,55 (0,26)	1,00 (0,00)	1,00 (0,00)
Scen.3 mean and (var.)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,45 (0,26)	0,80 (0,16)	1,00 (0,00)
Scen.4 mean and (var.)	0,80 (0,15)	0,90 (0,09)	1,00 (0,00)	0,50 (0,26)	1,00 (0,00)	1,00 (0,00)	0,00 (0,00)	0,00 (0,00)	0,00 (0,00)

TABLE A.2. Number of clearing houses, without spatial friction (GS-MNS). The table shows the mean of the number of clearing houses activated.

Number of clearing houses									
Homogeneous Topology (GS-MNS)									
	Phase 1			**Phase 2**			**Phase 3**		
Scenario 1									
Connectivity	**0,3**	**0,5**	**0,7**	**0,3**	**0,5**	**0,7**	**0,3**	**0,5**	**0,7**
Size 6 mean	15,3	24,3	31,7	13,5	19,7	27,2	13,2	20,8	28,8
Size 5 mean	0,3	0,1	0,2	0,9	2	0,3	0,6	1	0,5
Size 4 mean	0,2	0,2	0,4	1,2	1,2	1,1	0,6	0,9	0,6
Size 3 mean	0,1	0,2	0,1	0,6	1,3	1,8	0,7	0,5	0,9
Scenario 2									
Size 6 mean	30,1	22,1	29,9	18,7	27,9	36,6	14,2	24,1	33,6
Size 5 mean	0,3	0,2	0,2	0,2	0,3	0,2	0,4	0,5	0,2
Size 4 mean	0,2	0,3	0,4	0,2	0,2	0,2	0,6	0,5	0,6
Size 3 mean	0,2	0,4	0,4	0,1	0,2	0,3	0,4	0,1	0,2
Concentrated Topology (GS-MNS)									
Scenario 1									
Size 6 mean	6,9	9,6	11,7	5,6	7,7	9,2	5,8	7,6	9,7
Size 5 mean	0,3	0,2	0,2	0,9	0,8	1,2	0,6	0,6	0,7
Size 4 mean	0,1	0,2	0,2	0,9	0,1	0,8	0,5	0,6	0,7
Size 3 mean	0,1	0,1	0,2	0,4	0,6	0,9	0,6	0,5	0,4
Scenario 2									
Size 6 mean	6	8	10,6	8,4	11,6	13,1	8,7	12	13,9
Size 5 mean	0,7	0,5	0,4	0,2	0,1	0	0,2	0,2	0,1
Size 4 mean	0,3	0,4	0,3	0,1	0,3	0,1	0,2	0,1	0,9
Size 3 mean	0,3	0,3	0,3	0,1	0	0,2	0,1	0	0,2

TABLE A.3. Volume of clearing, without spatial friction (GS-MNS). The Table shows the percentage portion of credits handled by the clearing houses of size 6 in the first row. For the other sizes the table represents the percentage portion of the credits remained after the extraction procedure of the previous size managed by the clearing houses.

Volume of clearing									
Homogeneous Topology (GS-MNS)									
	Phase 1			**Phase 2**			**Phase 3**		
Scenario 1									
Connectivity	**0,3**	**0,5**	**0,7**	**0,3**	**0,5**	**0,7**	**0,3**	**0,5**	**0,7**
Size 6 mean	78,4	89,6	94,5	63,0	70,1	82,7	63,1	77,7	87,8
Size 5 mean	4,2	4,0	6,1	7,4	15,4	12,5	5,4	8,6	5,7
Size 4 mean	2,9	2,0	9,2	8,0	7,5	9,4	4,0	6,6	5,6
Size 3 mean	1,3	2,0	0,7	3,4	6,4	5,8	3,9	3,0	6,7
Scenario 2									
Size 6 mean	92,1	83,7	90,2	82,3	90,0	95,8	66,9	83,2	90,1
Size 5 mean	5,7	2,0	3,5	3,4	7,1	8,2	3,2	6,5	3,9
Size 4 mean	3,0	2,6	5,1	2,9	2,8	4,1	4,3	5,3	4,0
Size 3 mean	1,7	3,1	4,6	0,4	1,6	5,7	2,3	1,2	2,6
Concentrated Topology (GS-MNS)									
Scenario 1									
Size 6 mean	61,8	72,0	81,5	44,1	57,7	63,9	47,5	57,2	68,5
Size 5 mean	3,8	3,3	2,7	9,7	7,1	9,8	6,5	8,3	9,0
Size 4 mean	0,6	2,5	2,3	7,6	8,3	10,4	4,2	5,6	6,9
Size 3 mean	1,0	1,1	1,6	2,2	4,2	6,3	3,8	3,8	3,4
Scenario 2									
Size 6 mean	51,3	63,2	75,8	65,9	77,6	84,6	43,3	46,6	36,2
Size 5 mean	7,4	5,5	4,9	2,2	1,7	0,0	5,0	4,3	3,2
Size 4 mean	3,8	2,4	3,8	1,2	3,3	0,7	4,1	2,9	1,7
Size 3 mean	2,1	2,2	1,9	0,8	0	1,8	3,3	1,3	2,4