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A Dynamic Model for Optimal Covenants in Loan Contracts

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Acknowledgements

> There is probably no pleasure equal to the pleasure of climbing a dangerous Alp; but it is a pleasure which is confined strictly to people who can find pleasure in it
>
> *Mark Twain*

Pursuing a PhD degree in Trento turned out to be not an easy hike trail, but rather demanding and challenging one with quite an extreme elevation gain. It is like climbing a mountain, when after you reach a certain level you only see where to proceed.

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Introduction

Covenants are an important part of financial contracts, that are used for resolving the conflicts of interest between borrowers and lenders. In more formal way covenants can be determined as special provisions in loans that give lenders the possibility of putting certain actions in force (normally early repayment) when covenants are violated. For instance, a covenant may restrict the company in taking additional credit, or require a firm to maintain certain financial ratios, such as leverage, coverage, liquidity ratios, etc.

According to empirical studies, almost every loan in US market includes covenants (Chava and Roberts, 2008; Demiroglu and James, 2010). For example, Demiroglu and James (2010) report that 94% of private debt agreements from their database include at least one financial covenant. Starting from the 1970s, the substantial empirical work in this field has been done. However, there are few studies that elaborated theoretical models for covenant pricing.

There is an evidence in the empirical literature that covenants are usually set strict at the origination and as a consequence, the debt contracts are often renegotiated. It is reported that more that 90% of the contracts are renegotiated. In addition, the average number of covenant renegotiations per contract is stated to be 3.5. Normally, renegotiations are costly for the bank, but not every study (both empirical and theoretical) takes it under consideration. Taking into account the recent findings of frequent renegotiations and tightly set covenants, it is becoming more important to investigate the optimal covenant strength in loans.

Our study develops a theoretical framework that allows to determine the covenant strength index that should be included in a debt contract in a way that minimizes expected losses for a bank subject to the rising restructuring costs. This optimal covenant is found in order to better allocate control rights ex ante and to minimize the costs of renegotiations for both parties.

The approach that explores dynamic contingent claim models is applied to the problem. This approach was pioneered by Black and Scholes (1973) and Merton (1974), and extended by Black and Cox (1976). The dynamics of the optimal covenant strength with respect to various model parameters is investigated.

Different modifications to the initial model are considered which are important in exploring more realistic model setting. First of all, we introduce the concept of deadweight costs of distress or firesale price. The concept of deadweight losses imply that the debt holder gets some fraction of the asset value on default instead of the fundamental asset value (Das and Kim, 2015). Along with the concept of deadweight costs, the notion of firesale price is used, that represents the price at which the asset can be sold before the contract maturity. We explore how this extension of our baseline dynamic model influence the optimal level of covenant strictness in debt contracts.

We further develop a model of an optimal covenant in bank loans with information asymmetry. Asymmetric information as a source of agency problems is very important in studying control rights in financial contracting. The conclusions of the papers on information asymmetry regarding control rights allocation and covenant strictness are often ambiguous. Different papers demonstrate more or less control rights of lenders or greater or lesser strictness of covenants depending on the setting and model parameters. Our model is unique in a sense that it unites different implications of empirical and theoretical models with information asymmetry and reflects both perspectives.

We also introduce a framework for accessing the consequences of covenant violation in Monte Carlo simulation. Our simulation model allows us to measure different risk-parameters of a project, such as the probability of covenant violation and the probability of repayment of the loan. The measurements (average number of covenant violations per contract, frequency of covenant violation, frequency of loan repayment) can be used in implementation of different rules for a bank that extends the traditional risk-analysis of a project. Moreover, we implement a recursive technique for determining the level of covenant strength that allows the bank to maintain the performance of a specific risk-parameter. We employ a dynamic approach in the spirit of Borgonovo and Gatti (2013); Chang and Lee (2013); Liang et al. (2014) by simulating project value paths over time.

The dissertation proceeds as follows. Chapter 1 describes the baseline theoretical model for determining optimal covenant strength. In Appendix A we present a model for empirical analysis, in Appendix B we investigate deadweight costs of distress as an

extension of our baseline model. In Chapter 2 we explore our continuous time model under asymmetric information. In Chapter 3 we outline how the theoretical model can be turned into Monte Carlo simulation procedure. In Appendix C we provide code for our simulation model in Mathematica 10.

Keywords covenants; credit risk; dynamic contingent claim models; renegotiations; asymmetric information; Monte Carlo simulation

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Chapter 1

Continuous Time Model

Covenants are an important part of debt contracts and are a valid instrument for resolving the conflicts of interest between borrowers and lenders. Although substantial empirical work in this field has been done, few studies have elaborated a theoretical model for covenant pricing. Taking into account the recent findings of increased probability of covenant violations in repeated borrowing and frequent covenant renegotiations even outside of default, it is becoming more important to investigate the optimal covenant strength in loan contracts.

This study develops a model for covenant pricing in bank loans in order to explore how covenant strength affects the price. The model builds on the approach pioneered by Black and Scholes (1973) and Merton (1974), and extended by Black and Cox (1976). The approach that explores dynamic contingent claim models is applied to the problem. Numerical results demonstrate the impact of various parameter values on the optimal covenant strength.

The model can support the development of more effective internal risk-management procedures for banks to assess their expected loss rates in order to protect themselves against incurring significant losses in cases of possible insolvencies of firms. Furthermore, the model provides the basis for amending the global regulatory standards known as Basel II and Basel III to recommend procedures for banks that follows an advanced risk-management process.

1.1 Introduction

Covenants are special terms of debt contracts, which are used for resolving the conflicts of interest between the stockholders and debtholders. More formally, covenants are defined as specific clauses in debt contracts of firms that restrict business policy and give creditors the possibility of putting precise actions into force (normally early repayment) when the covenants are violated (Bazzana and Broccardo, 2013a). For example, a covenant may not allow the firms to issue additional debt, or not let leverage ratios get too high. According to empirical studies, almost every loan in US market includes covenants (Chava and Roberts, 2008; Demiroglu and James, 2010). For instance, Demiroglu and James (2010) report that 94% of private debt agreements from their database include at least one financial covenant.

The increasing frequency of the inclusion of covenants in debt contracts has inspired substantial empirical work in this field. Since the 1970s, a variety of empirical studies of covenants have been conducted covering a range of topics, including variations in performance loan pricing (e.g., how changes in the borrower's risk affect interest rates), frequent covenant violations, differences in debt design for private and public firms, and various practical observations (e.g., the observed tightness of covenants) (Ackert et al., 2007; Asquith et al., 2005; Mather and Peirson, 2006; Denis and Wang, 2014).

Although empirical studies may be helpful in eliciting important aspects of including covenants in debt contracts, theoretical loan pricing models need to be more elaborated to extend traditional risk-analysis in loan granting. Banks following an advanced risk-management process need more effective methods to assess their expected losses in accordance with the global regulatory standards known as Basel II and Basel III (Basel Committee, 2006, 2017). The above mentioned banks (upon approval) can choose their model for loan loss provision (as an estimate of bank's future loan losses) calculation. Those provisions as a part of a balance sheet are used in prudential coefficients that represent bank's stability. Furthermore, this methodology can help banks insure against incurring significant losses in cases of possible insolvencies of firms as well as frequent renegotiations. However, only a few studies have elaborated a theoretical model for loan pricing.

Moreover, the vast majority of theoretical research that has been conducted on covenants takes the covenant variable as exogenous as well as empirical studies operate with the available data on covenants. Since a substantial part of empirical literature is on covenant violations and renegotiation of loan contracts and there is an evidence that more than 90% of the contracts are renegotiated, the question on the covenant strength initially fixed in loan contracts become even more important.

In this paper a theoretical framework is developed to explore how covenant strength affects the expected losses of loans. To address this problem, the dynamic contingent claim model is build up. In this model the value of the firm's assets (or investment project) follows a stochastic process representing random shocks on the firm's assets value that corresponds to the covenant index. In order to explore the influence of the covenant strength on the expected losses, the formulated equation takes into account the probability of covenant violation as well as the probability of bank's waiver of the covenant violation. By studying the numerical examples, it is shown that for a given firm's assets value there exists a covenant strength that minimizes the expected losses for a bank subject to restructuring costs.

To summarize, the model can be used not only for explaining the differences described in the empirical studies, but also for computing the optimal covenant strength in order to minimize banks' risks and facilitate more effective monitoring.

1.2 Literature Review

Covenants have been described as one of the possible instruments for resolving the conflicts of interest between debt contract parties: bondholders and stockholders in public debt and creditors and borrowers in private debt. Jensen and Meckling (1976) and Smith Jr and Warner (1979) contributions were seminal in describing the conflict between stockholders and bondholders. The early theoretical literature of the 1970s involved reasoning mainly backed by the authors' expert assessments with only a few formalized models. Subsequently, a growing number of empirical studies have appeared which reflects the increasing frequency of the inclusion of covenants in debt contracts.

1.2.1 Theoretical Studies

Although the majority of theoretical works devoted to covenants were expert-based (meaning that the patterns of relations with covenants were defined by authors' expert judgment), there have been few studies formalizing findings on covenants. The earliest theoretical study is Black and Cox (1976), in which the authors proposed a continuous time model of valuating bonds with safety covenants based on an options valuating model. After more than thirty year period with a few theoretical contributions, several formal theoretical models began to appear.

Different approaches have been applied to the problem of formalizing findings on covenants. Some researchers have focused their attention on the distribution of property rights while resolving the conflicts of interest in the process of renegotiating debt covenants (Gârleanu and Zwiebel, 2009; Freudenberg et al., 2013). Others applied an accounting method approach (Lu et al., 2011) or game theoretic approach (Kahan and Tuckman, 1993; Bazzana and Broccardo, 2013b; Billett et al., 2013). In the present study the approach of evaluating the impact on expected losses will be used (Bazzana, 2009), which is common in the credit risk literature (Bouteille and Coogan-Pushner, 2012). Moreover, this approach will support the development of more effective methods for assessing banks' expected losses and hence reducing the amount of loan-loss reserves, which in turn will allow banks to save on internal capital (Basel Committee, 2010).

Apart from the earliest theoretical model of covenants in continuous time (Black and Cox, 1976), the majority of models available in the literature are either static (Matvos, 2013; Bazzana and Broccardo, 2013a) or two-period models (Bazzana, 2009; Lu et al., 2011). Since banks usually perform planning on a multiple time-periods base (e.g., quarterly or annually), multi-period models will be constructed.

In theoretical studies, the impact of inclusion covenants is analyzed from the perspective of different counterparts. For example, in the private lending context, the analysis can be provided either for a bank (Borgonovo and Gatti, 2013) or for a firm (Lu et al., 2011). The present study will examine the models from the perspective of the party that decides on the contract design — i.e., in private lending, not from the perspective of a firm, but rather from the perspective of a bank that offers the covenants according to a firm's risk-profile and performs monitoring. For example, Matvos (2013) considers the case when a firm is constrained to choose among a small number of boilerplate (i.e. standard form) covenants. Hence, it is not a firm that decides on the contract characteristics, but rather a bank that does.

Therefore, on the base of option valuation models we develop a theoretical model whose aim is to support bank's decision making on covenant strictness in order to minimize expected losses and renegotiation frequency. Moreover, our model will be dynamic in contrast to the majority static or two-period models existing in the literature.

1.2.2 Empirical Studies

The present study is based on the well diversified empirical studies that have examined different features of debt contracts. Early empirical studies mostly describe various aspects of using covenants: firms' costs in cases of covenant violation in debt contracts (Beneish and Press, 1993), differences in assessment of event-risk covenants by two rating agencies – Moody's and Standard and Poor's (Bae and Klein, 1997). Over time, increasingly complex patterns have been identified in the empirical stream of the literature; Rauh and Sufi (2010), for example, connected debt structure with capital structure and Vanderschraaf (2007) studied behavioural aspects of covenant violation. One of the well know empirical facts about covenants is the strictness of covenants that restricts the borrower behaviour a lot (Bradley and Roberts, 2003; Dichev and Skinner, 2002; Chava and Roberts, 2008). The present study will allow to loosen these constraints by better allocation the control rights ex ante.

There is a stream of empirical literature that investigates the pricing of debt covenants. Since the present study is devoted to theoretical covenant pricing models, these results are of particular interest. A lot of findings are consistent with a Costly Contracting Hypothesis developed by Smith Jr and Warner (1979). Bradley and Roberts demonstrate empirically the negative relation between the presence of covenants and the promised yield of a debt agreement (Bradley and Roberts, 2003, 2004). It was also showed by Chava et al. (2004) that the presence of covenants reduces the cost of debt. In addition, Wei (2005) reports the decreasing credit spreads of bonds in the strength of covenant protection. The same results are also demonstrated by Goyal (2005), Asquith et al. (2005), Chan and Chen (2007). Evidence to support the Costly Contracting Hypothesis are also given for UK (Moir and Sudarsanam, 2007) and Finland (Niskanen and Niskanen, 2004).

There is also a substantial part of empirical literature on covenant violations and renegotiation of loans. It is important to emphasize this stream of literature, since our study contributes to it al well. The process of renegotiation (how it is organized, by whom it is initiated, etc.) is described in detail in the literature. The aspect of our particular interest for the present research work is the frequency of renegotiations. The presence of frequent contract renegotiations has been proven by recent empirical studies. Moreover, the frequency of loan renegotiations has been demonstrated both for US and for European countries.

In US the statistics on loan renegotiations is very pronounced and proven by different independent empirical studies. The researchers report that over 90% of long-term (with a maturity of three to five years) private credit agreements are renegotiated prior to their maturity (Roberts and Sufi, 2009; Nikolaev, 2015). In addition, it is important to emphasize the increasingly frequent loan renegotiations. Whereas Denis and Wang (2014) report that there are in average 2.7 renegotiations per each contract, Roberts (2015) with his more recent database states that the average bank loan is renegotiated five times (or every nine months) according to US data. Moreover, in both studies it is argued that the majority of renegotiations occur outside of default. European statistics in general confirms the trends of US data, it is not so impressive though. According to Godlewski (2015), the average number of renegotiations by borrower is 1.5. It has been also demonstrated that covenants in public debt contracts are highly renegotiated as well (Nikolaev, 2010, 2015).

In practice, the borrower begins the process of loan renegotiation by contacting the lender and the main reason for this is the restrictiveness of the initial contract (Roberts and Sufi, 2009; Godlewski, 2015). Apart from the other loan parameters, covenants are modified significantly during each renegotiation. This fact is also well known and proven to be true both for US and European loans data (Roberts, 2015; Godlewski, 2015). In Europe one of the major changes during renegotiations concern covenants (10%) (Godlewski, 2015). In US covenant violation (or anticipation of covenant violation) is a reason of less than 28% of renegotiations (Roberts, 2015). In addition, more than 75% of all debt contract renegotiations modify at least one of the restrictive or financial covenants. Moreover, the bulk of renegotiations (46%) modify only the covenant package (Roberts, 2015). Hence, renegotiation of covenants is the most significant amendment type. These finding support the importance of investigation of the optimal covenant strength.

In general, the relation of covenants on the likelihood and the frequency of the renegotiation can be viewed from different points of view. Nikolaev (2015) systematizes them in 3 groups. According to the first one, covenants determine the bargaining power of lenders and borrowers that influence the division of renegotiation surplus (Roberts and Sufi, 2009; Li et al., 2012). This, however, do not imply any impact on the renegotiation frequency. The second point of view implies covenants as state-contingent control rights that should reduce the need and scope of future renegotiations (Asquith et al., 2005). According the third (so called strategic) view covenants set ex ante help allocate control rights efficiently ex post (Berlin and Mester, 1992; Gârleanu and Zwiebel, 2009). It worth mentioning that only according to the second view point the presence of covenant should be negatively related to renegotiations, the last two view points does not imply any dependency of this kind.

There exist different models in support of above mentioned points of view on the problem. Moreover, different aspects of all approaches are proven empirically. In general, there are no one proven direction of thinking. One of the important aspects here is the type of covenant under consideration. For example, Nikolaev (2015) demonstrates negative relation between covenants and frequency of renegotiations for the performance pricing type of contingency.

Although the number of contingencies increase in the contracts, this fact indicates the fundamental incompleteness of loan contracts (Roberts, 2015). According to one of the approaches mentioned above covenant renegotiations are positioned as the dynamic way to the complete contracts and the way of allocation of the control rights ex post (Gârleanu and Zwiebel, 2009; Roberts, 2015; Godlewski, 2015). Although according to Godlewski (2015), "renegotiation leads to more efficient or more complete contracts as it translates new information into an updated contract", the results of his paper are controversial and prove this statement to be true only partially. Moreover, as stated by Godlewski (2015), the probability of renegotiation is missing in his model. In the present work the probability of renegotiation is one of the important component of the model.

We provide one more empirical evidence of the approach chosen for the current study regarding the impact of covenant tightness on the frequency of renegotiations. This evidence is derived from the effect of covenant violation on subsequent borrower behaviour. This topic has been investigated by Freudenberg et al. (2013), who have shown that the probability of covenants violation in a subsequent new loan increases by 30% for borrowers who have violated covenants in prior contracts. At the same time, the authors report increased loan spreads and tightened covenants in newly issued loans. Consequently, this correspondence between covenant strictness and renegotiations in repeated borrowing supports the approach chosen for our theoretical model, namely, the negative relation between covenant strictness and renegotiation frequency.

In the present study we consider frequent renegotiations as a negative phenomenon because of the following reasons. First of all, renegotiations are costly. Researchers agree on this point and provide a lot of examples of both direct and indirect restructuring costs in their studies. Costs vary with the size of loans and their complexity. For more information on renegotiation costs, please, refer to Section 1.3. Secondly, the changes in the initial conditions of a loan leads to increased risks for a bank. Indeed, according to the empirical studies, renegotiations usually result in loosening of loan conditions, which in turn changes the risk-profile of a firm. For example, in case of increase in credit amount (or limit of the overdraft line of credit), it becomes more difficult for a firm to return debt because of increased leverage, hence, bank bears more risk as a result.

The next reason is stated by Godlewski (2015), who argues that negative reactions of stock market can be a consequence of late and frequent renegotiations. Finally, credit constraints (and further renegotiations) has been proven to have the effect on real economy. For example, the recent paper by Ersahin and Irani (2018) proves the increase in employment expenditures when the value of firms' collateral increases. We argue that the optimal covenant concept can help lenders to improve their contracting behaviour that leads to less frequent renegotiations, less costs and help to avoid negative consequences of frequent renegotiations.

One important and very recent paper on covenants is one by Griffin et al. (2018). The authors investigate evolution of loan contracts over past 20 years. Their focus is on covenant strictness and they report the declining trend in covenant strictness over this period. The reason of loosening of covenants is suggested to lie not in firm composition or credit supply but in “fundamental changes in the costs and benefits of tight covenants”.

Summing up, there are empirical studies that act as advocates of covenant renegotiations stating that frequent renegotiations lead to dynamic contract completeness. For example, in the recent paper Roberts (2015) states imperfect correlation between ex ante covenant strength measures and ex post renegotiations and transfers of control rights. Although Roberts (2015) is suspicious about the relation of the initial strictness of covenants and subsequent renegotiation process, at the end of his paper he suggests to make further research on the implications of the ex post restructuring processes to the ex ante contract design. The present study is aimed to provide theoretical framework for the suggested study.

Empirical studies are important not only for developing coherent theoretical models constructed in accordance with the main empirical findings, but also for understanding the contradictions and differences of these findings. For example, covenants in private debt contracts are more restrictive than these in public debt contracts (Mather and Peirson, 2006). These kinds of distinctions have become the building blocks of the theory in this field.

Thus, our theoretical model is build based on the well known empirical facts of strict covenants and frequent renegotiations both for US and European credit market. The approach of covenants as state-contingent control rights that should reduce future renegotiations is chosen for our model. Therefore, the aim of our model is to find an optimal covenant strictness (often less strict) that reduces renegotiation frequency. The reduction in covenant strictness has been proven by Griffin et al. (2018) in their very recent paper, that supports results of our model.

1.2.3 Approach Rationale

In describing the approach chosen for the problem, it is worth providing the terminology in order to link together and provide a review of diverse sets of models. The approach that was chosen for the formulated problem explores dynamic contingent claim models. In the literature, however, these models are often called structural credit risk models or structural dynamic models. Adhering to the viewpoint of Strebulaev and Whited (2012b) who argue that these models are called in this way "confusingly" and "mostly for historical reasons rather than for any connection to other uses of "structural" in economics". Therefore, we would stick to dynamic contingent claim models definition.

There is a recent debate in the literature on the usefulness of dynamic models in corporate finance in general. The sides of this debate are represented by I. Welch on the one hand and by I. A. Strebulaev and T. M. Whited on the other hand. The dispute has begun with the critique made by Welch (2011) in one of his working papers on the two articles: one by Hennessy and Whited (2005) and the other by Strebulaev (2007). While the first paper develops the dynamic model of investment and financing under uncertainty, the second one builds the dynamic capital structure model.

Welch (2011) criticizes the structural modelling trend in corporate finance in general by illustrating the following shortcomings on two above mentioned papers. First of all, Welch (2011) states that many plausible forces are ignored based on the authors' priors. Secondly, he doubts the reliability of the explanations based on the reduced-form findings. Thirdly, he points out some important econometric issues. And finally, Welch (2011) argues that these models do not meet high test standards.

Strebulaev and Whited publish the response to Welch (Strebulaev and Whited, 2013), in which they literally say that Welch's criticisms are incorrect and are based on the inaccurate literature interpretation and conclude that there is no logical reason to dismiss the whole stream of research methodology. According to Longstaff and Schwartz (1995), "traditional Black and Scholes (1973) and Merton (1974) contingent-claims-based approach to valuing corporate debt has become an integral part of the theory of corporate finance."

Apart from the arguments provided by Strebulaev and Whited (2013), the eligibility of use of dynamic contingent claim model for the present research problem is proven by two more reasons. Welch by himself has mentioned the structural models as "award-winning" papers in finance. Furthermore, this methodology has been widely used for the whole spectrum of problems in corporate finance, some of them are described below.

Moreover, structural approach is often opposed to reduced-form approach. According to Gündüz and Uhrig-Homburg (2013), providing an empirical comparison of these two approaches in CDS pricing, the reduced-form approach is not in general a superior one to the structural approach. At the same time, both of them can perform better depending on the (sub)investment grade rates and the maturity. Thus, we put aside this debate.

The comprehensive review of dynamic models in corporate finance has been performed by Strebulaev and Whited (2012b) based on the research during the last two decades. In their fundamental working paper, the authors cover three broad areas: dynamic contingent claim models, discrete-time investment models and structural estimation. In each of these areas the authors develop a typical model frame with the basic elements, it makes the interpretation of a lot of papers simpler and more intuitive.

According to Strebulaev and Whited, there are two main advantages of contingent claim models with respect to discrete-time investment models. Firstly, the pricing of claims can be based on techniques developed in the derivatives pricing literature. The second advantage is the allowance of the unbounded firm growth. In dynamic contingent claim models any claim can be represented as a derivative on underlying state variable (firm value, firm cash flow, etc.). Among the elements there are also the objective function, the set of instruments and constraints and the optimal set of controls.

The one of the first applications of the option pricing theory was to the area of real options. In this area the underlying state variable is the value of the investment project. The manager's control variable is the decision of the time of investment. The fundamental result is that the option to wait before committing to a financial or investment policy is valuable (and therefore the hurdle rates are higher). It underscores the dramatic difference between essentially static and fully dynamic decision-making models. Specific economic ingredients are: firm's flexibility, fixed investment cost, irreversible investment.

In order to further justify and support the approach chosen for the problem, several examples of its application in order to resolve various objectives are provided. Zhou (1997) develops a dynamic approach for valuing risky debt by modelling the firm value as a jump-diffusion process. The author has proven his approach in order to better explain empirical literature in the term structure of credit spreads and to be consistent with many other stylized empirical facts in the literature. Based on the standard assumptions, Zhou develops the valuation formula for the price of any derivative security with payoff at maturity and contingent on the market value of the firm's assets relative to the default threshold. The bond valuation formulas are provided for both frameworks where the default can occur either at maturity or at any time prior to maturity.

There are also some examples of applying the dynamic approach to develop bank run models. Papers by He and Xiong (2012) and Liang et al. (2014) provide some of them. In the first paper He and Xiong (2012) explore the dynamic coordination problem among firm creditors. In the second article Liang et al. (2014) decompose the total default probability as the sum of the insolvency and illiquidity default probabilities. Due to impossibility to compute the total default probability analytically, the authors implement their model in a binomial tree framework.

Summarizing, the thorough investigation of the informational sources including literature reviews as well as discourses allowed us to make a choice in favour of dynamic contingent claim models. Moreover, the study of applications of these models to various areas in corporate finance supported the choice and contributed with important elements and details.

1.3 Methodology and Results

The continuous time model developed in this paper is built on the framework inherited from the Bazzana's two-period model (Bazzana, 2009). Whereas in Bazzana's model there is only one rollover date, in the present model the rollover decision can be made at any time. The baseline model is described in the paper by Bazzana (2009), where the two-period loan pricing model is presented to investigate the optimal covenant strength.

The author considers a firm investing in a 2-period project, which is financed through a bank loan. The value of a project can be represented as a two-step binomial model (see Figure 1.1).

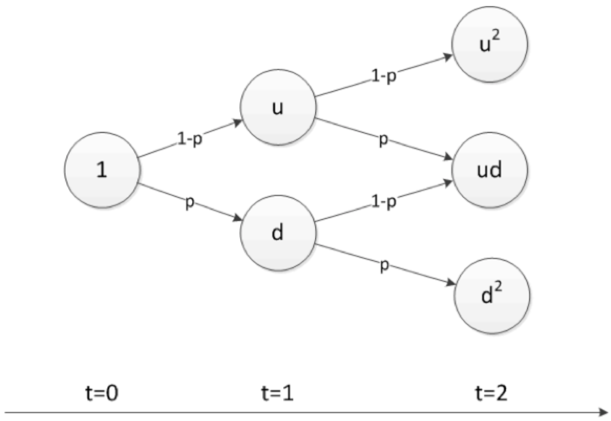

Figure 1.1: 2-Period baseline model

In this model the probability p of a decrease of project value in each time period is introduced. The value of the project can either go up (u) or down (d). Stock and option prices in a two-step binomial tree can be represented in an analogous form (see Figure 1.2) (Hull, 2009).

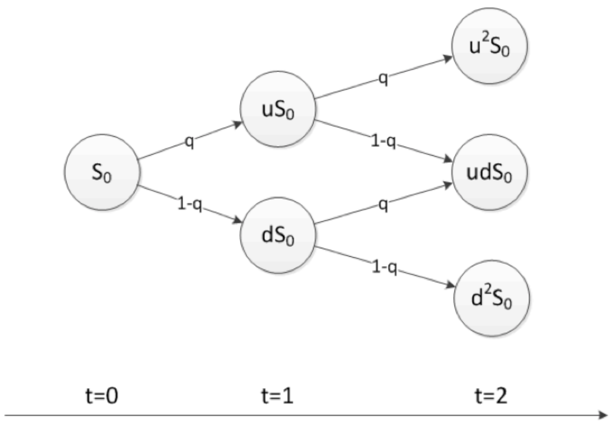

Figure 1.2: 2-Period derivative pricing model

In option pricing theory the value of the option is examined in a similar way with respect to the probability of up and down movements of asset returns. By analogy with option price, the equation for expected loss rate (elr) of a bank is provided for the model of covenant pricing (Bazzana, 2009). For the loan with a restrictive covenant on the investment project elr becomes

$$elr_{cp} = PD \times LGD = PD \times \left(\frac{EAD - R}{EAD} \right) \tag{1.1}$$

where PD is the probability of default and LGD is the loss given default. The latter can be represented by the ratio between the loss upon default and exposure at default (EAD), R is the value of recovery upon default.

Figure 1.3 illustrates the finding of Bazzana's paper (Bazzana, 2009), showing the expected loss rate as a function of covenant strength s when a bank sells the collateral upon covenant violation ($elr_{ca,s}$) and when the bank waives the covenant violation ($elr_{ca,w}$).

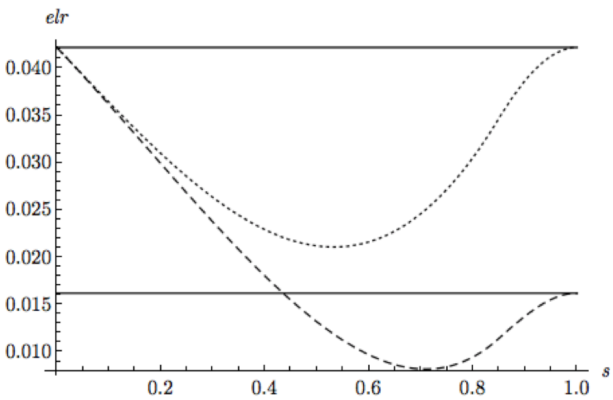

Figure 1.3: The expected loss rate as a function of covenant strength in a two-period model

The formulation of the problem in dynamics setting is similar to one in Bazzana's paper (Bazzana, 2009): a firm investing in a project, which is financed through a bank loan that contains covenants. The investment project itself acts as a collateral for a bank.

Our continuous time model extends the Bazzana's approach of a two-period model with only one rollover decision toward real-life applications.

In terminology of Strebulaev and Whited (2012b) the value of the firm's assets (or investment project) $X_t = X(t)$ in our model is a state variable and it follows a stochastic process

$$dX_t = \mu X_t dt + \sigma X_t dW_t \tag{1.2}$$

with drift μ and volatility σ, where W_t is a standard Brownian motion that represents random shocks on the firm's assets value.

On the liabilities side, S_0 is the amount of debt provided to the firm until the time T, r_S is the promised (continuously compound) rate of return. Hence, if there is no covenant violation the amount of debt at maturity will be $S_T = e^{r_S T} S_0$. Discounting takes place at the risk-free rate r. The difference between the final rate of return to the firm r_S and the market interest rate r is a credit spread.

The following form of the covenant is considered: if the value of the firm assets (or the value of the investment project) falls to a specified level, then the bank can incur losses. More precisely, covenant violation is triggered if at any time $t \in [0, T]$ the firm value reaches this specified level $\alpha(t)$, which may change over time. We will call this level as the covenant threshold value (or it is also called reorganization value). This threshold is defined as it is commonly used in the related literature (for example, as in classical paper by Black and Cox (1976) or one by Liang et al. (2014)) as follows:

$$\alpha(t) = e^{-r(T-t)} S_0 e^{r_S T} \rho, \tag{1.3}$$

where ρ is the covenant strength index. In other words, covenant threshold value is specified as a constant fraction (determined by the covenant strength index) of the present value of the promised final debt payment. Note here that the covenant threshold changes over time and depends not only on the covenant strength index ρ, but also on the amount of debt provided to the firm. The more debt the firm takes, the higher the covenant threshold is.

In practice, there is a number of covenants of different types in a contract. However, it is possible to unify them in one index and use as one variable in theoretical modeling. The simplest way is to consider this variable to be the number of covenants included in a contract. It is also possible to construct a unified index of covenant strictness. As it was mentioned before, Bazzana (2009) uses the relative distance between covenant threshold and default value of an investment project. Freudenberg et al. (2013) in their paper develop so called "Distance to Covenant Violation" – measure of covenants tightness. Hoberg and Maksimovic (2015) perform text-based analysis of 10K reports in order to obtain a measure of financial constraints. For example, in debt market they call it covenant violation score.

As stated above, at any time the bank checks whether the covenant is violated. Given the symmetric information between the bank and the firm, the outputs for the bank are summarized on Figure 1.4, which we now describe in detail.

By examining these scenarios, we can compute the expected losses for the bank. When the covenant is not violated, there is no potential losses for the bank. On the other hand, in case of covenant violation there is a possibility for the bank to incur losses. Along with this, when the information is symmetric and there are no uncertainties between the

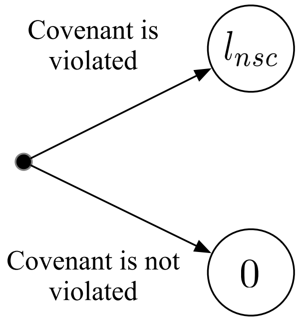

Figure 1.4: Decision tree in each point in time

parties, the bank has no opportunity to sell the collateral. Bank losses and expected losses are defined by the Equation (1.4) and Equation (1.5) respectively.

$$l_{nsc}(t) = S_T - X_t e^{rs(T-t)} = e^{rsT} S_0 - X_t e^{rs(T-t)}, \tag{1.4}$$

$$\begin{aligned} el_{nsc}(t) =& PCV(t)\left(S_T - X_t e^{rs(T-t)}\right) = \\ =& PCV(t)\left(e^{rsT} S_0 - X_t e^{rs(T-t)}\right), \end{aligned} \tag{1.5}$$

where $PCV(t) = P(X_t < \alpha(t))$ is the probability for a firm to violate the covenant from the time t up to the maturity T and it is another important component of the model. The expected losses defined in this way is in line with empirical studies. Indeed, according to Roberts (2015), there are three factors determining the timing of loan renegotiation: the renegotiation outcome, the financial health of the parties, and the uncertainty regarding borrowers' future profitability (which is in our model represented by the probability of covenant violation).

The probability of covenant violation is defined as the distribution function of a log-normal process with an absorbing barrier at the reorganization boundary (i.e. covenant threshold) $\alpha(t)$. For example, it is defined in the similar way in the papers by Black and Cox (1976), Zhou (1997) or Liang et al. (2014). In this model the following definition of the probability of covenant violation will be used

$$PCV(t) = P(X_t < \alpha(t)) = \Phi\left(\frac{ln\frac{\alpha(T)}{X(t)} - (\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right) + \\ + \left(\frac{X(t)}{\alpha(t)}\right)^{1-2\frac{\mu-r}{\sigma^2}} \Phi\left(\frac{ln\frac{\alpha^2(t)}{X(t)\alpha(T)} - (\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right). \tag{1.6}$$

It's natural to note that the stricter the covenant is, the greater the probability of covenant violation is. This positive relation with respect to covenant strength ρ is illustrated in the Section 1.4.

The expected losses when bank does not sell the collateral elr_{nsc} can be either positive or negative. This sign represents the profitability of not selling the collateral and is obviously determined by the value of the initial debt amount and the project value at each period of time. Given the assumption that all agents behave rationally, we will further consider the positive values of el_{nsc} meaning that non selling the collateral is profitable for a bank. In this case we observe positive relation of the expected losses with respect to covenant strength ρ that is illustrated in Section 1.4.

Further comes the analytic investigation of behaviour of the bank losses function from Equation (1.4) with respect to the key parameters of our model. Primarily let's explore how bank losses change with respect to the credit amount S_0. The analysis shows that bank losses increase in S_0:

$$\frac{\partial l_{nsc}(t)}{\partial S_0} = e^{r_S T} > 0 \tag{1.7}$$

and they increase linearly in S_0:

$$\frac{\partial^2 l_{nsc}(t)}{\partial S_0^2} = 0 \tag{1.8}$$

Consequently, the greater the initial amount of credit S_0 is, the greater the amount of debt at maturity. Therefore, the greater becomes the amount of final payoff the bank can get deducting the expected at any pointy in time t project value at maturity if the firm will not be able to meet loan requirements and return debt in full. In other words, from one hand, if the bank issues the greater amount of credit, it has greater expected payoff at maturity. From the other hand, if the distance between the current project value at time t and initial debt amount becomes larger, the situation is riskier because it's more probable the firm will not return debt in full at maturity.

Let's now explore how the bank losses change with respect to the current project value X_t. The loss function decreases linearly in X_t since:

$$\frac{\partial l_{nsc}(t)}{\partial X_t} = -e^{r_S(T-t)} < 0 \tag{1.9}$$

and:

$$\frac{\partial^2 l_{nsc}(t)}{\partial X_t^2} = 0 \tag{1.10}$$

Hence, although the expected payoff becomes less when the current project value gets bigger in any period of time, however the project becomes less risky.

At the same time, the bank losses when not selling the collateral grow when the time to maturity t shortens. Indeed,

$$\frac{\partial l_{nsc}(t)}{\partial t} = r_S X_t e^{r_S(T-t)} > 0 \tag{1.11}$$

and:

$$\frac{\partial^2 l_{nsc}(t)}{\partial t^2} = -r_S^2 X_t e^{r_S(T-t)} < 0. \tag{1.12}$$

In other words, when the maturity becomes closer the expected payoff for the bank grows.

All these dependencies are demonstrated numerically in the Section 1.4 in this Chapter.

Cost function and optimal covenant strictness.

Our objective is to determine the covenant strength index ρ at which debt will be written down in a way that minimizes expected losses for a bank subject to the rising restructuring costs. Renegotiation process is in general assumed to be costly (Mather and Peirson, 2006). Although some model settings allow costless renegotiation, it is usually taken as baseline, that is later compared to the costly renegotiation case (Li, 2013).

The costs of renegotiation is one of the determinants of the frequency and nature of renegotiations as an implication of incomplete contract theory to debt contracts (Denis and Wang, 2014). For example, as pointed out by Gârleanu and Zwiebel (2009), the cost of renegotiation facilitates the optimal contract to be written in the way that minimizes the probability of renegotiation. In other words, the renegotiation cost gives the two parties an incentive to write such an optimal contract.

In practice, the costs of renegotiation depend on the size of the loan, complexity of the amendment, time and effort spent by both parties (Roberts and Sufi, 2009; Godlewski, 2015). In the literature there is plenty of examples of these costs, both direct and indirect. One can think of a payment in favour of accountants or lawyers as an example of direct renegotiation costs (Gârleanu and Zwiebel, 2009). Indirect restructuring costs may be in the form of free-rider or externality costs in case of involvement of multiple creditors or opportunity cost of time (Gârleanu and Zwiebel, 2009; Li, 2013).

The present approach that involves a comparison between the expected losses and restructuring costs is common in other important tradeoffs in the literature. As an example we can provide the models developed to discover the optimal capital structure. In these papers capital structure adjustments are opposed to transaction costs (An et al., 2015) or tax advantages of debt are opposed to the risk of costly default (Gale and Gottardi, 2015).

Regarding the functional form of cost functions, there exist different approaches in the literature. The simplest way to include restructuring costs in the model is to assume them to be constant. For example, Li (2013) in the model to investigate the accounting conservatism on the efficiency of debt contract assumes that the renegotiation process has a fixed amount of cost and then considers a large, moderate and small renegotiation cost cases.

Now let's have a look at the relation of restructuring costs on covenant strictness. One of the implications of the model developed by Gârleanu and Zwiebel (2009) is that the dependency between renegotiation costs and covenant strictness is opposite. In other words, restructuring costs should decrease when covenant strictness increases. This inverse relation contrasts with the concept of the covenants as alternative mechanism of debtholder control to renegotiations. However, the implications of the model, proposed by Gârleanu and Zwiebel (2009), are confirmed by well known empirical facts. For example, it confirms the empirical fact that the covenants for private debt are stricter than those for public debt. Moreover, the model confirms also the empirical fact that the renegotiation process typically imply the loosening of covenants. In addition to that, in their recent paper Das and Kim (2015) also consider the decreasing form of cost function with respect to covenant strictness.

Regarding the precise functional form, let us refer to the literature on cost efficiency of producing some output bundle for a bank. For example, Hughes and Mester (1993) consider the exponential form of cost of production in their multiproduct cost function model. Although our approach is contract-oriented and it differs from the approach to investigate the cost of production for a bank, the exponential form of the cost function will be used. The same form of the cost function has been used in the dynamic debt model by Das and Kim (2015).

Considering all of the above, let's introduce the cost function of the following form:

$$c(\rho) = c_0 e^{-c_1 \rho}, \tag{1.13}$$

where $c_0, c_1 > 0$ (Das and Kim, 2015). Let's explore the behaviour of this function with respect to ρ. It's easy to see that, since

$$\frac{dc(\rho)}{d\rho} = -c_0 c_1 e^{-c_1 \rho} < 0 \tag{1.14}$$

and

$$\frac{d^2 c(\rho)}{d\rho^2} = c_0 c_1^2 e^{-c_1 \rho} > 0 \tag{1.15}$$

the cost function is a decreasing and convex function of ρ. It means that with the stricter covenant it becomes less costly to renegotiate the contract. Again, as has been discussed before, this behaviour is in accordance with such well known empirically proven facts as stricter covenants for private debt or loosened covenants as a result of restructuring process (Godlewski, 2015). Moreover, as stated by (Gârleanu and Zwiebel, 2009), stronger rights are granted to the lender in the initial contract (meaning stronger covenants) when the renegotiation process is less costly.

Although the functional form of renegotiation costs are highly dependent on the parameter values (c_0 and c_1), it is feasible to find the estimators in the empirical literature.

For example, Beneish and Press (1993) provide estimates for costs of the violation of accounting-based covenants in debt agreements. By examining the changes in loan terms as well as changes in investing and financing decisions, the authors provide evidence not only of refinancing costs, but also of the restructuring costs. Hence, there are no restrictions in verifying the present model empirically.

As we have seen above, the relation of the expected losses when not selling the collateral on covenant index ρ is positive (that implies increasing function in ρ). The optimal covenant strength index may be chosen when the value of costs and the value of expected losses are equal with this index in any rollover decision time. By equating right hand sides of the Equation (1.5) and the Equation (1.13) (expected losses and costs), we can obtain the optimal level of covenant strength that should be included in the contract in a way that minimizes the expected losses for a bank subject to costs of renegotiation:

$$PCV(t)\left(S_T - X_t e^{r_S(T-t)}\right) = c_0 e^{-c_1\rho} \tag{1.16}$$

or

$$PCV(t)\left(e^{r_S T} S_0 - X_t e^{r_S(T-t)}\right) = c_0 e^{-c_1\rho}. \tag{1.17}$$

Since it has been shown that the function of expected losses increases in ρ and the function of renegotiation costs decreases in ρ, there exists a tradeoff (an intersection point) between increasing the expected losses and decreasing the costs. Therefore, this intersection point is the optimal level of covenant strength for the debt contract.

1.4 Analysis and Numerical Results

1.4.1 Data Description

In order to perform ensuing analysis of the model, parameter values are now introduced. Like in Das and Kim (2015), Liang et al. (2014) the model can be analyzed on one numerical example. In order to empirically test the present model parameters of the paper by Liang et al. (2014) are chosen. In this paper example of Merrill Lynch has been used.

Let's assume drift $\mu = 1.56\%$ and the annual volatility $\sigma = 11.25\%$ for the value of the firm's assets (or investment project) X in the model given by the Equation (1.2). For the risk-free rate, $r = 1.56\%$ will be used. The asset value at time t is supposed to be $X_t = 726.0$.

The amount of debt $S_0 = 755.7$ is provided for the firm until the time $T = 3$ years (maturity) under the promised (continuously compound) rate of return $r_S = 5.72\%$.

Using this set of parameters, we obtain that the value of covenant threshold given by the Equation (1.3) varies from $\alpha(t = 0) = 771.5$ to $\alpha(T = 3) = 807.5$ (the covenant strength index $\rho = 0.9$ is chosen here just for illustrative purposes).

Some examples of the firm's asset dynamics together with the covenant threshold are represented on the graphs below (Figure 1.5).

Whereas on the first graph the dynamics of the firm's assets does not lead to covenant violation during the lifespan of the loan, on the second graph covenant is violated frequently during the contract period.

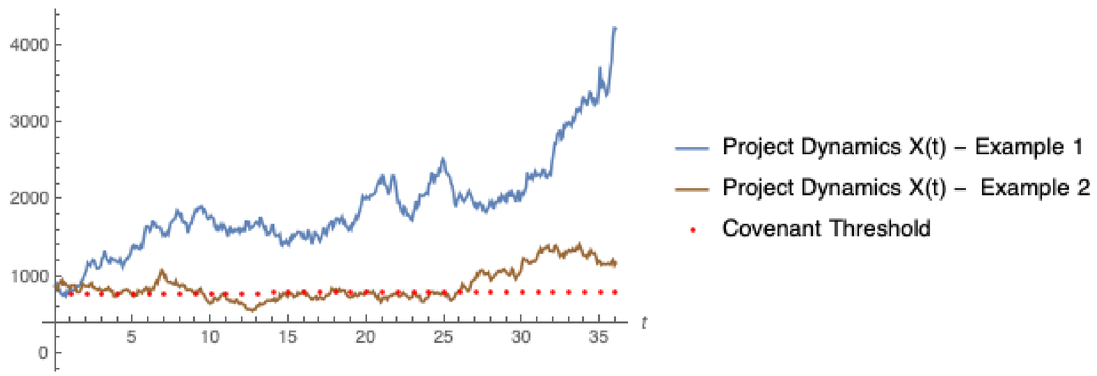

Figure 1.5: Examples of possible dynamics of the project value together with the restrictive covenant boundary.

1.4.2 Model Analysis

As we discussed in the Section 1.3, the losses l_{nsc} can be either positive or negative and different in magnitude. Depending on the correspondence between the project value X_t and the initial credit amount S_0 in any time, Figure 1.6 demonstrates the increasing and linear dependency of the bank losses with respect to the credit amount S_0, that was provided in the Section 1.3.

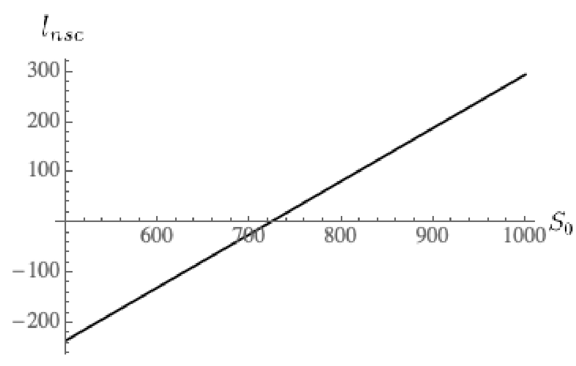

Figure 1.6: The dependency of bank losses on the credit amount

Moreover, according to the Figure 1.6 it's possible to say that with the credit amount less than 721.9 makes the loan agreement unprofitable at any point in time t. When the amount of credit goes below this value, the potential profit of investment after selling the project exceeds the value of debt at maturity.

As it was shown in Section 1.3, the bank losses when selling the collateral grow in case the time to maturity t shortens. Figure 1.7 demonstrates this positive relation.

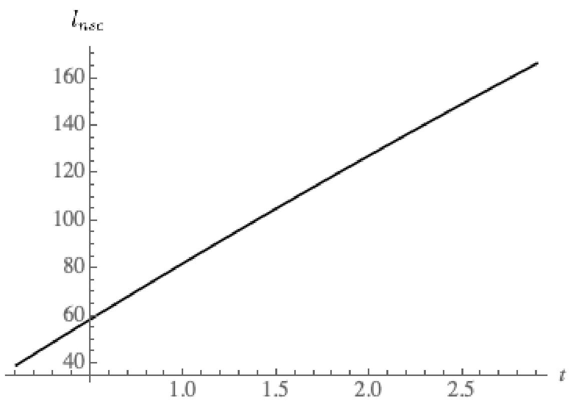

Figure 1.7: The dependency of bank losses on the time to maturity

According to the graph, the potential outcome for the bank is larger when it's close to maturity, however, at the same time it's a riskier situation because the project value becomes much lower than the promised debt value at maturity. The more safe time for the bank in terms of final payoff is closer to the loan issuance time.

Let's investigate the behaviour of the probability of covenant violation. This probability is given by the Equation (1.6) and is the important part of the expected losses for a bank when non selling the collateral given by the Equation (1.5).

Figure 1.8 demonstrates the straightforward fact that the stricter the covenant is, the greater the probability of covenant violation is.

As was discussed in Section 1.3, only positive values of the expected losses for a bank

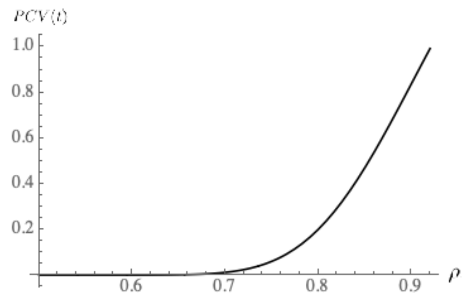

Figure 1.8: The dependency of the probability of covenant violation on covenant strength

when not selling the collateral are under consideration in the present model. This means that selling the collateral is profitable for a bank. In this case we observe positive relation of the expected losses with respect to covenant strength ρ, as illustrated on the Figure 1.9.

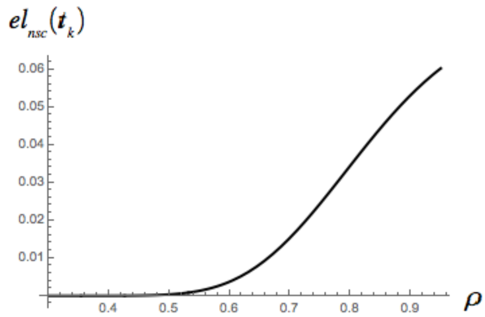

Figure 1.9: The dependency of the expected losses when non selling the collateral on covenant strength

The cost function was introduced in Section 1.3 of the form given by the Equation (1.13). This form ensures the negative relation of the cost function with respect to the covenant index ρ. To show that let's assume the variable values to be $c_0 = 50, c_1 = 1.5$ (Das and Kim, 2015). We obtain the following graph (Figure 1.10) for the cost function:

Thus, the cost function is the decreasing function of ρ.

When depicting both graphs on the same plot (expected losses and costs), we can observe the optimal level of covenant strength that should be included in the contract in

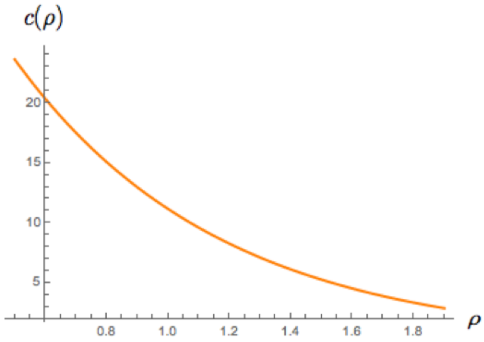

Figure 1.10: Cost function

a way that minimizes the expected losses for a bank subject to costs of renegotiation. As we can see on Figure 1.11, in this case the optimal level is 0.718.

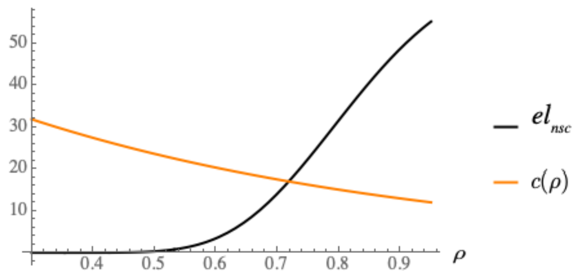

Figure 1.11: The optimal level of covenant strength as a tradeoff between increasing the expected losses and decreasing the costs

It is empirically demonstrated that the presence and strictness of covenants is a channel to discipline bank risk-taking (Goyal, 2005). It becomes extremely important in financial distress. We now demonstrate how a bank can choose the optimal covenant index when changing the debt value and also when the market volatility changes.

1.4.3 Sensitivity Analysis

In this section we investigate the model dynamics with respect to key parameters under financial distress. Along with other parameters of the model, key variables under investigation in this section will be the following: drift and volatility of the continuous time dynamic equation, the amount of credit, credit and market interest rates and the parameters of the renegotiation cost function.

The optimal covenant strength dynamics with respect to the drift.

Drift is a parameter that represents the direction of the project value movements in time. In financial distress the drift value decreases which in turn leads to difficulties for a firm in paying off its financial obligations or in maintenance of covenant values on the agreed levels. Hence, the bank may have problems with liquidity or expected losses in this case. We show that when the drift value decreases, the optimal covenant value decreases as well in order to meet the increased renegotiations costs. Figure 1.12 demonstrates that by decreasing the drift value from 1.56% to 0.20%, the optimal covenant strength index becomes less strict (decreases from 0.718 to 0.703). The same relationship holds for the majority of covenant index values. However, when the value of covenant strength index is greater than 0.85 the situation changes and we observe inverse relationship between the covenant strength index and the drift. In case the estimates of the cost function parameters will differ significantly from the given values, the conclusions and empirical predictions will be different.

One possible explanation of the above mentioned change in relationship is the following. In the context of drift reduction, when the estimates of the cost function are high enough,

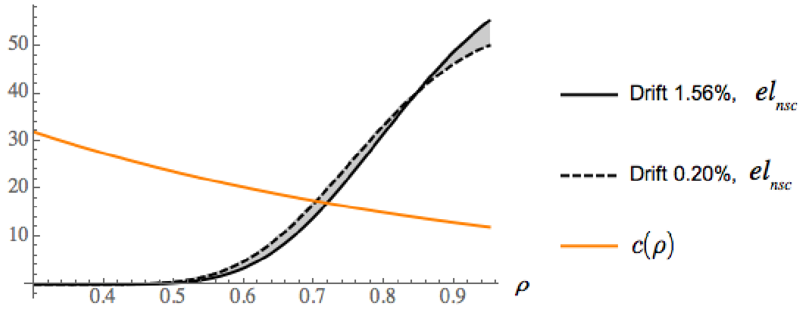

Figure 1.12: The change in optimal level of covenant strength with respect to the drift value

it becomes optimal for a bank to increase the covenant strength index and thereby trigger the possible default (i.e. covenant violation). It is worth mentioning that the turning point remains the same regardless of the drift value. It means that with any value of μ the turning point of describing interrelationship is always 0.85.

The optimal covenant strength dynamics with respect to the volatility.

As stated above, financial distress is characterized by changes in volatility in the model which influence the optimal covenant strength value. The greater the volatility value, the greater the uncertainty, the probability of covenant violation, expected losses and renegotiation costs for a bank. In order to compensate this influence, bank chooses a less strict covenant index as an optimal value that has to be included in a debt contract. As shown on Figure 1.13, if we consider more volatile situation in the market (increase of σ from 11.25% to 15.00%), the level of optimal covenant strength decreases from 0.718 to 0.681. The above mentioned relationship is true almost for entire spectrum of ρ values. However, when the covenant strength index passes the value of 0.85, the relationship changes and becomes positive. Interestingly, the turning point is the same as in case with the drift value - 0.85, and it remains the same independently of the volatility value.

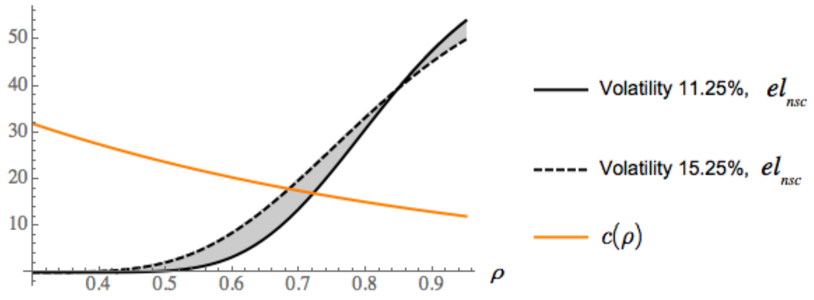

Figure 1.13: The change in optimal level of covenant strength with respect to the volatility value

The possible explanation of the above mentioned inverse relationship is the same as in drift case. When the estimates of the cost function are high and volatility increases, it becomes optimal for a bank to trigger the possible default (or covenant violation) by increasing the covenant strength index.

The optimal covenant strength dynamics with respect to the amount of debt issued.

It may sound natural that a bank when issuing greater amount of debt protects itself by setting the stricter value of covenant. However, taking into account the renegotiation costs, that are obviously increase when the amount of debt rises up, it is optimal for a bank to lower the level of covenant protection when issuing the greater debt value. Figure 1.14 demonstrates that by increasing the amount of debt from 755.7 to 800.0, the optimal covenant strength index becomes less strict (decreases from 0.718 to 0.621).

The optimal covenant strength dynamics with respect to the credit interest rate r_S.

On the one hand, with the increased value of the interest rate r_S the loan becomes more profitable for a bank. On the other hand, it becomes more problematic for a firm to meet

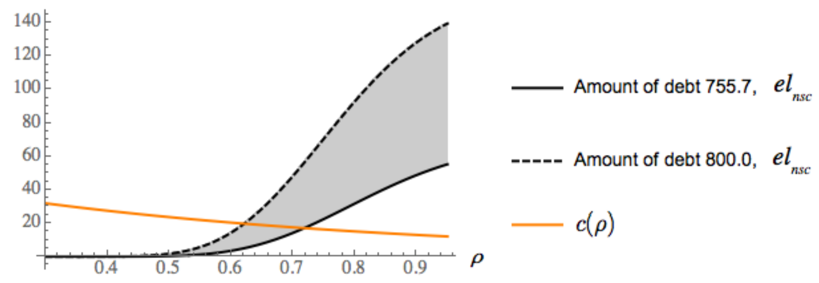

Figure 1.14: The change in optimal level of covenant strength with respect to the amount of debt issued

the covenant requirements that increases the bank's risks. In optimum, renegotiation costs are higher as well. Hence, in order to meet these increased costs, it is optimal for a bank to choose lower value of the covenant strength index. Indeed, Figure 1.15 demonstrates that by increasing the credit interest rate from 5.72% to 7.00%, the optimal covenant strength index becomes less strict (decreases from 0.718 to 0.689).

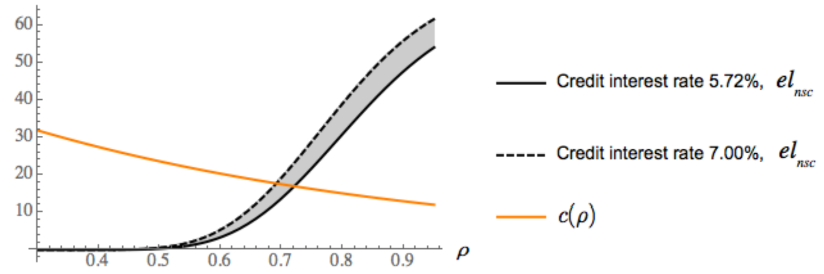

Figure 1.15: The change in optimal level of covenant strength with respect to the credit interest rate r_S

The optimal covenant strength dynamics with respect to the market interest rate r.

Financial distress can be characterized by the decreasing market interest rate. Indeed, it becomes less profitable for a bank to sell the collateral in case of covenant violation. Moreover the renegotiation costs increase in optimum as well. Consequently, in order to meet these increased losses a bank should choose the less strict value of the covenant strength index. Indeed, Figure 1.16 demonstrates that by decreasing the market interest rate from 1.56% to 0.20%, the optimal covenant strength index becomes a bit less strict (slightly decreases from 0.718 to 0.708).

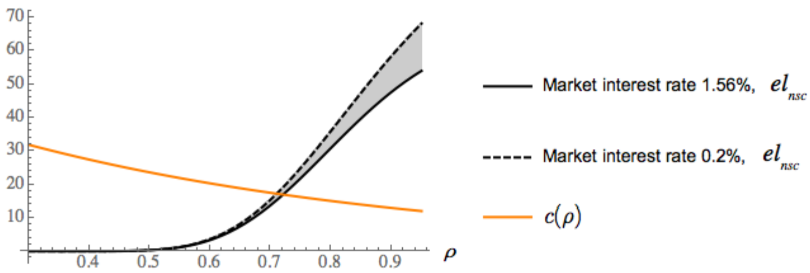

Figure 1.16: The change in optimal level of covenant strength with respect to the credit interest rate r

The optimal covenant strength dynamics with respect to the cost function parameters c_0 and c_1.

As it was discussed above, the exponential functional form of the cost function is quite common in the related literature. Moreover, there have been provided the estimates of renegotiation costs. The parameter c_0 acts as a multiplier in the exponential function. Increase in this parameter entails the jump up of the cost function, wherein the magnitude of the jump is greater with small values of the covenant strength index (due to the convexity of the cost function with respect to the parameter ρ). In other words, the greater value of the parameter c_0 leads to the increasing renegotiation costs for a bank and, consequently, the stricter optimal value of the covenant strength index. Figure 1.17 demonstrates that by increasing the parameter c_0 of the cost function from 50 to 70, the optimal covenant strength index become more strict (increases from 0.718 to 0.751).

On the other hand, parameter c_1 appears as an argument of the exponential cost function with a negative sign. Increase in this parameter entails the down shift of the

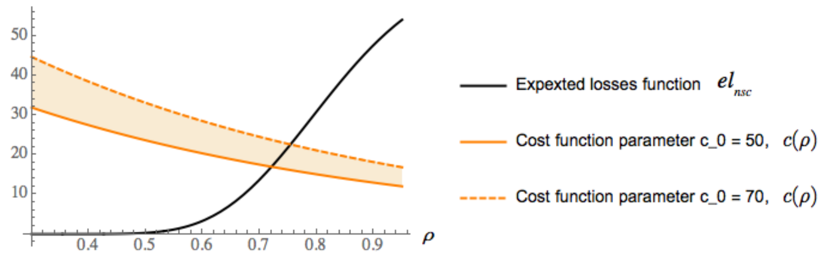

Figure 1.17: The change in optimal level of covenant strength with respect to the cost function parameter c_0

renegotiation cost function with a slight curvature transformation (more steep functional form). In other words, with greater values of the parameter c_1 covenant renegotiation costs for a bank decrease and, consequently, the optimal covenant strength index becomes lower. Figure 1.18 demonstrates that by increasing the cost function parameter c_1 from 1.5 to 1.7, the optimal covenant strength index become less strict (decreases from 0.718 to 0.706).

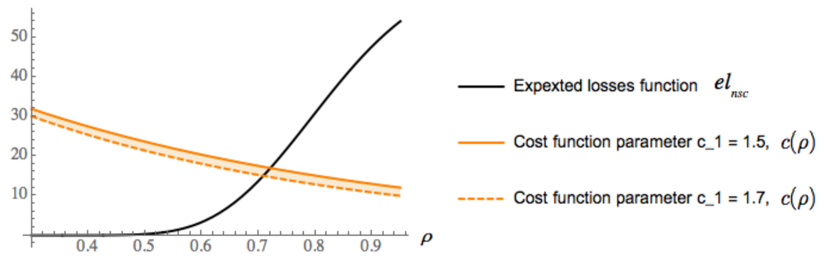

Figure 1.18: The change in optimal level of covenant strength with respect to the cost function parameter c_1

1.5 Baseline Model Extension — Deadweight Costs of Distress

The concept of deadweight losses imply that the debt holder gets some fraction of the asset value on default instead of the fundamental asset value (Das and Kim, 2015). The notion of deadweight costs is very common in the literature and is used in different contexts, including but not limited to investment, deposit insurance, modelling of different types of financial contracts.

Along with the concept of deadweight costs, the notion of firesale price is used. The firesale price (determined by the firesale rate) represents the price at which the asset can be sold before the contract maturity. These two notions are very similar in formulation. Therefore, the literature based on both of them is included in our analysis.

The concepts of deadweight losses or firesale price are included in the literature not only in order to build more accurate and realistic models, but also become their very important and critical factors and even foundational elements (or basic notions) in various streams of literature.

The concept of deadweight costs of distress is very common in the literature and is a building block in theoretical modelling. In general, the literature on the optimal capital structure is based on the trade-off between the costs and the benefits. The classic trade-off is the one between deadweight costs of bankruptcy and the tax saving benefits of debt. For example, in the contract literature, Viswanath and Eastman (2003) try to resolve the bondholder-stockholder conflict by finding the optimal contract as a trade-off between contract implementation costs and deadweight losses.

When being included as a parameter in the model, the notion of deadweight costs of default sometimes gets critical weight in the model. For example, in the paper by Matutes and Vives (1996) where they investigate deposit insurance, deadweight costs of default play an important role. The authors consider the trade-off between its positive impact (for example, it helps to avoid systemic confidence crises) and deadweight loss in the economy. Based on the model simulations the authors conclude the importance of market structure. They argue that the deadweight loss in the whole economy will preponderate all the advantages of deposit insurance in case when banks have direct competitors and does not have local monopolies.

Authors include this notion in order to build their models in a more accurate and realistic way. For example, Chava and Roberts (2008) investigate the impact of covenant violation on corporate investment. The authors consider three ways to influence investment. One of them is through deadweight loss or transfers to investors.

The concept of deadweight losses is widespread and is used in different streams of the literature: deposit insurance (Matutes and Vives, 1996), investment (Chava and Roberts, 2008), labor market model (bankers and regulators) (Bond and Glode, 2014). The notion of deadweight costs of distress is also used in different contract modelling settings: mandatory convertibles (Chemmanur et al., 2014), asset-backed commercial papers (Schroth et al., 2014), incomplete contracts (Tirole, 2009). Asquith et al. (2005) when investigating the performance pricing in bank debt contracts determine its importance by that it helps to alleviate the deadweight losses.

In order to broaden our analysis, it is worth including the literature based on different terminology. Together with the concept of deadweight losses of bankruptcy, the notion of firesale rate is used.

The term firesale means that the sale is performed during financial distress. Firesale rate is an important variable in many theoretical models and empirical works in banking and contract theory area. It represents "the rate by which the risky asset can be sold prematurely" (Liang et al., 2014). In their multiperiod bank run model for liquidity risk, Liang et al. (2014) use firesale rates as endogenous variables determined by leverage data. Therefore, the authors say that the firesale rates can be used as monetary policy tools along with interest rates and leverage.

Since the concepts of firesale rate and deadweight losses are very similar in formulation, we'll include the literature of both these streams. However, as stated by Gale and Gottardi (2015), unlike the deadweight losses, assets sold at firesale prices represent a transfer of value from creditors to buyers. Since we are considering the open model and the redistribution of values is not the focus of our study (as it would be in case of a close model), we will include both streams of literature in our analysis.

The model extension under our consideration in this chapter is the introduction of deadweight costs of distress or the notion of firesale price. Liang et al. (2014) in their paper state that the definition of firesale rate is determined by the concept of leverage. The authors develop the dynamic bank run model for liquidity risk. One of the two important variables in their model is the firesale rate. This variable is endogenous in their model and is determined empirically using the leverage data. Based on the relationship provided by Liang et al. (2014) we will use the following correspondence:

$$\psi X_t = (1-\beta)X_t = (1-H_t)X_t = \left(1-\frac{1}{L_t}\right)X_t, \tag{1.18}$$

where ψ is the firesale rate (which is also called recovery rate), β is the deadweight costs (or deadweight loss on default), L_t is the leverage ratio for the date t, H_t is the haircut rate for the date t (defined as the reciprocal of leverage). It is worth providing this relationship not only because it's beneficial in putting together all different terminology we use, but also in understanding the resulting inferences we obtain.

This relationship is supported by Strebulaev and Whited (2012b), who have also included the deadweight costs in their dynamic investment model and have stated the inverse relation between leverage and the deadweight costs.

Regarding our model, let's assume the deadweight costs α for the project (or the firesale rate ψ) with the value X_t at any date before the maturity $t < T$. The expected losses for a bank are defined in the following form in this case:

$$el_{nsc}(t) = PCV(t)\left(S_T - (1-\beta)X_t e^{rs(T-t)}\right) = PCV(t)\left(S_T - \psi X_t e^{rs(T-t)}\right), \tag{1.19}$$

where β is the deadweight losses, ψ is the firesale rate, $PCV(t) = P(X_t < \alpha(t))$ is the probability for a firm to violate the covenant from the time t up to the maturity T. According to Liang et al. (2014), the firesale rate has been proven to be time varying. For simplicity, we operate with the exogenously given and constant over time deadweight losses α and the firesale rate ψ. Due to the demonstrated interdependence of these notions, we'll include reasoning based on the firesale rate ψ in ensuing analysis.

Let's investigate the behaviour of expected losses with respect to the firesale rate ψ. The function of expected losses decreases linearly in ψ since

$$\frac{\partial el_{nsc}(t)}{\partial \psi} = -PCV(t)X_t e^{rs(T-t)} < 0 \tag{1.20}$$

and:

$$\frac{\partial^2 el_{nsc}(t)}{\partial \psi^2} = 0 \tag{1.21}$$

Hence, the greater the firesale rate is, the smaller the expected losses become and, obviously, the more profitable the project is. In other words, the smaller deadweight costs of distress lead to the smaller expected losses of a bank.

For our further analysis we use the value of firesale rate 0.7. As it was described above, Liang et al. (2014) have computed firesale rates for their model for liquidity risk using the leverage data. Based on the quarterly data for the high-yield market provided, the values of the firesale rate vary around 0.7. The authors also use the constant value of 0.7 for their further analysis in the paper. Moreover, Das and Kim (2015) use for their dynamic debt model the value of deadweight losses on default of 30% (meaning that the firesale rate $\psi = 0.7$).

Figure 1.19 demonstrates the linear dependency mentioned earlier: the greater the firesale rate is, the smaller expected losses are and the more profitable the project is.

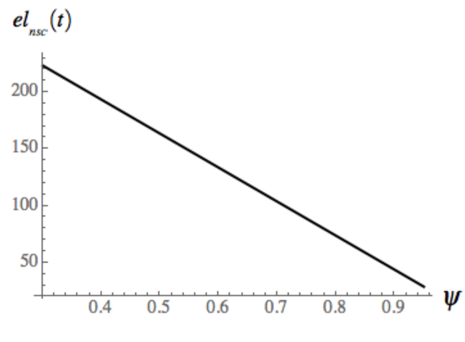

Figure 1.19: The dependency of the firesale rate on bank's expected losses

The presence of the firesale rate in the model increases the value of the expected loss rate. Indeed, it becomes less profitable for a bank to sell the collateral in case of covenant violation. In addition to that, the renegotiation costs increase in the optimal point as well. Therefore, when comparing results with the baseline model we observe the less strict optimal value of covenant strength index in the extended model.

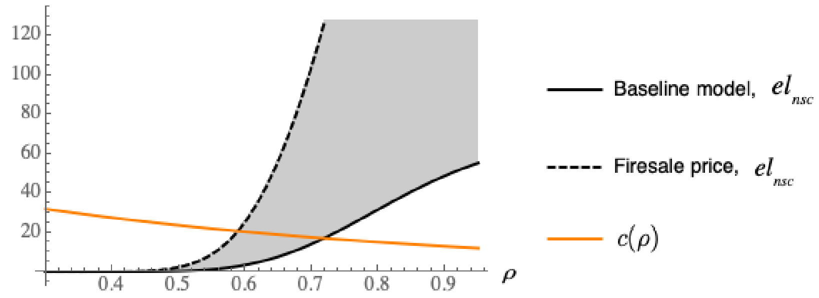

Figure 1.20: The change in optimal level of covenant strength with respect to the firesale rate

Indeed, Figure 1.20 demonstrates that by introducing the firesale rate of 0.7 value (or deadweight losses of 30%) in the model, the optimal covenant strength index becomes less strict (decreases from 0.718 to 0.589).

The smaller the value of firesale rate is (meaning that the situation on the market is more unfavourable for a bank i.e. selling the collateral is less profitable), the greater the value of expected losses are and the less strict the optimal covenant has to be in order to meet the increased costs of renegotiation in the optimal point.

This result is inline with the finding of Liang et al. (2014), who show empirically the inverse relation between the firesale rates and default probability. These findings mean that when firesale rates are decreasing, the total default probability is increasing. Since the probability of default is included into the expected losses formulation in our model, it means in turn that the expected losses for a bank should increase when firesale rates are decreasing. Hence, our results fully support these empirical findings.

The result is also supported by findings of Das and Kim (2015), who demonstrate an increase in spreads when deadweight losses decrease both for high leverage and medium leverage ratios. According to the Equation (1.19) the expected losses in our model also increase with spreads. In the present analysis technically we introduce decrease in deadweight losses from $\phi = 1.0$ to $\phi = 0.7$. Therefore, we observe increased losses when introducing the deadweight costs in our model.

1.6 Conclusion

In this study a solid theoretical framework is developed in order to investigate the problem of inclusion covenants in debt contracts. More precisely, this framework can allow us to determine an optimal level of covenant strength in a loan contract. The approach that utilizes dynamic contingent claim models is applied to this problem and is based on techniques developed in the derivatives pricing literature.

The optimal covenant strength index is chosen as a tradeoff between the expected losses and renegotiation costs. Whereas the bank expected losses are shown to be an increasing function of the covenant strength index, the renegotiation costs function decrease with respect to this index. Therefore, the intersection point is an optimal covenant strength index that should be included in the debt contract.

In order to explore the model, numerical analysis is performed using the example of Merrill Lynch. All functions in the model are analyzed in detail with respect to key parameters. Moreover, the sensitivity analysis is performed in order to investigate the model dynamics and the behaviour of the optimal covenant strength index in financial distress. The key parameter destructions during financial distress are explored, including the drift and volatility values, the credit amount, the market and credit interest rates, as well as the parameters of the renegotiation costs function.

There are potential policy implications of these findings. First, the theoretical models will support the development of more effective risk-management procedures for banks to assess their expected losses when granting debt contracts with covenants. Second, it is possible to suggest an amendment to global regulatory standards known as Basel II and Basel III to include the above mentioned risk-management procedures as recommendatory for banks that follows an advanced risk-management process.

Appendices

Appendix A

Model for Empirical Analysis

In order to verify the proposed model empirically, we introduce the following approach. Based on the theoretical analysis and numerical result demonstrated above we construct a regression model that is aimed to verify the basic derivations that were obtained. We propose to test on panel data the regression model of the following form:

$$\rho_{it} = \alpha_0 + \alpha_1 \mu_{it} + \alpha_2 \sigma_{it} + \alpha_3 S_{0,it} + \alpha_4 r_{S,it} + \alpha_5 r_{it} + \alpha_6 c_{0,it} + \alpha_7 c_{1,it} + \beta' Y_{it} + \epsilon_{it}, \quad \text{(A.1)}$$

where Y_{it} is a vector of control variables.

According to the conclusions derived in the Section 1.4.3, the predicted values of the regression coefficients are the following:

$$\alpha_1 > 0,\ \alpha_2 < 0,\ \alpha_3 < 0,\ \alpha_4 < 0,\ \alpha_5 > 0,\ \alpha_6 > 0,\ \alpha_7 < 0. \quad \text{(A.2)}$$

It is worth recalling once again the discussion about interrelationship of the optimal covenant strength with respect to the drift and the volatility values (the first two lines of the Equation (A.2)). As it was mentioned in the previous section, the signs of these coefficients remain as stated in the equation for the majority of the index ρ values. However, they may differ then the covenant strength index value becomes stricter.

Overall, the present model of the optimal covenant strength is a mechanism for a bank of choosing the covenant level as a tradeoff between the expected losses and renegotiation costs. The examined model dynamics demonstrates how the covenant strength should be chosen and adjusted in financial distress.

Chapter 2

Continuous Time Model Under Asymmetric Information

Asymmetric information as a source of agency problems is very important not only in studying control rights in financial contracting, but also in the fields of venture capitalists investment as well as control rights within an organization. Moreover, allocation of control rights is an important distinction between models with symmetric versus asymmetric information. The importance of information asymmetry is supported both by theoretical and empirical studies.

In this chapter we develop a model of optimal covenants in bank loans with information asymmetry. We compare it with our baseline model with symmetric information. In addition, we numerically investigate model behaviour with respect to key parameters.

There exist numerous settings and various formulations of information asymmetry in contract modelling. Moreover, the conclusions regarding control rights allocation and covenant strictness are often ambiguous. Our model unites different implications of empirical and theoretical models with information asymmetry.

2.1 Introduction

Asymmetric information as a source of various agency problems is very important in financial contracting environment. Information asymmetry is considered to be one of the sources of contractual incompleteness. The latter is indicated by the presence of frequent renegotiations in bank lending (Roberts, 2015). Covenants are considered to be an instrument for resolving those conflicts of interest between borrowers and lenders. Moreover, in the present theoretical model setting, optimal covenants are aimed to reduce costly renegotiations.

There exist a lot of formulations of asymmetric information in both theoretical and empirical papers. Models may or may not consider costly information acquisition, may explore an impact on outcomes of investment or potential transfers. There are also various trade-offs under consideration when investigating the impact of asymmetric information. Kaplan and Strömberg (2004) classify the risks into different categories. Although their paper is in the field of venture capitalists investment, some of them may be related to our topic. For example, asymmetry about the entrepreneur's quality or effort, or disagreements between the parties after the investment.

Although there are various formulations of information asymmetry, the importance of it was demonstrated both empirically and theoretically. It was empirically shown that there is an impact of different sources of agency problems on contractual terms. For example, Roberts (2015) in his empirical paper shows that information asymmetry in the model accelerates the renegotiation. Moreover, there are theoretical studies that emphasize the importance of information asymmetry in contractual models. For example, as was pointed out by Roberts (2015), "Gârleanu and Zwiebel (2009) show that the allocation of control rights is an important distinction between theories predicated on symmetric versus asymmetric information". Influence on covenants and renegotiation process are the ones of our particular interest.

Information asymmetry is an important element not only in private debt modelling, but also in other related fields of study. The significance of inclusion of asymmetric information was demonstrated when studying the allocation of control rights in financial contracts. In addition, information asymmetry is also investigated within an organization between a principal and an agent, when the problem of delegation is explored as an alternative to communication. Asymmetric information is also studied in venture capitalists investment. Unlike in bank-firm relationship, the venture capitalists usually more connected to the entrepreneur, actively involved in different project operations and hence, more informed about what's going on with the project in general (Schmidt, 2003). In combining together different streams of literature we discuss in this section, it is important to mention that according to Schmidt (2003) the covenants are used to allocate control rights while convertible securities are used to allocate cash-flow rights.

Summarizing, the current study unites different terminology and relates to different streams of literature, discussed in detail in Section 2.2. The conclusions of the papers on information asymmetry are often ambiguous, leading to more or less control rights of lenders or greater or lesser strictness of covenants depending on the setting and model parameters. Our model unites all the inferences of different models and reflects both perspectives.

2.2 Literature Review

The importance of information asymmetry in contract modelling has been demonstrated by the number of studies. Kaplan and Strömberg (2004) in their paper classify the risks of investments of venture capital firms. However, it's possible to generalize them to conflicts of interest between the parties and information asymmetry caused by them in financial contracts or at least relate this classification to our model setting and highlight the relevant agency problems. The authors identify four main agency problems that venture capitalists face in the investment process.

1. The effort of the entrepreneur is unobservable to the venture capitalist. The moral hazard approach in this case predicts the dependence of the entrepreneur's compensation on performance. In our interpretation it means stricter covenants.

2. Asymmetry exists in the knowledge about the entrepreneur's quality. Similar to 1, the solution is also greater pay-for-performance in this case. This means stricter covenants in the context of our model setting.

3. The venture capitalist wants the decision right in case of disagreement with the entrepreneur after the investment. Control theories suggest that the conclusion is state dependent. Namely, control goes to the venture capitalist in some states and to the entrepreneur in the others.

4. "Hold-up" problem, when the entrepreneur threatens to leave the venture in case the human capital is very important. The solution of this problem is in vesting the shares to the entrepreneur.

In general, the above mentioned classification indicates that the solutions of different agency problems may be different in terms of influence on the entrepreneur: they can be more favorable to the entrepreneur (shares vesting) or less favorable (greater tie to performance). Moreover, they can depend on the state of the world. In our model we consider two cases of asymmetric information that reflect first three cases in Kaplan and Strömberg (2004) classification. However, the conclusions of our model are closer to the ones of control theories, meaning that they are state dependent.

As it was discussed in Section 2.1, we integrate different streams of literature when developing our model with asymmetric information. Let's discuss those streams in more detail. Firstly, we consider asymmetric information and allocation of control rights in financial contracts. It's a more broad topic. There are papers on control rights allocation where covenants are mentioned as one of examples. In other cases we can relate more general conclusions about control rights allocation on our model setting with covenants. Different approaches are invented to explore the control rights patterns. The contingencies and covenants are used in financial contracting literature as an instrument to show control rights motion (Dessein, 2005; Gârleanu and Zwiebel, 2009; Denis and Wang, 2014; Roberts, 2015).

Secondly, we consider information asymmetry in lending relationships (Gertler, 1992; Sridhar and Magee, 1997; Quadrini, 2004; Clementi and Hopenhayn, 2006; DeMarzo and

Sannikov, 2006; Bazzana, 2009; Biais et al., 2010; Bazzana and Broccardo, 2013a; Lin et al., 2018). We also consider information asymmetry in venture capitalists' investment (Kaplan and Strömberg, 2004; Dessein, 2005). Finally, we explore asymmetric information within an organization and the problem of delegation as an alternative to communication (Aghion and Tirole, 1997; Dessein, 2005). Further we will examine these literature streams in more detail.

2.2.1 Asymmetric Information and Allocation of Control Rights in Financial Contracts

One of the recent papers on the allocation of control rights is a paper by Gârleanu and Zwiebel (2009). The authors develop a theoretical 2-period model in order to investigate how costly information acquisition influence the assignment of control rights in a financial contract. Primarily, they show the difference in the predictions of asymmetric information models compared to symmetric information models. This difference is namely in the allocation of control rights and the strictness of debt covenants. Gârleanu and Zwiebel (2009) distinguish two types of information asymmetry: one regarding outcomes of investment and another one regarding transfers between the parties. Although the majority of financial models focus on outcomes of investment (including our model), the authors investigate transfers and demonstrate contradicting results with respect to the traditional incomplete contracts literature. The main implication of the model is the stricter covenant with information asymmetry. Although our model in contrast shows less strict optimal covenant strength with information asymmetry, other conclusions and especially model setting in Gârleanu and Zwiebel (2009) are of our particular interest, since they support a lot of our findings. Moreover, as noted by the authors, it's difficult to test both types independently since they both may be present together.

Denis and Wang (2014) in their comparative review of different papers on control rights and covenants mention greater control rights of lenders and tighter covenant as stated in Gârleanu and Zwiebel (2009) paper. The authors also mention that empirically covenant thresholds are usually set close to the current values of variables, that altogether forms the setting for frequent (and costly) renegotiations. This judgment also justifies our result and recommendation on optimal covenant to be less strict in information asymmetry setting.

As in our model, the main results of the paper by Gârleanu and Zwiebel (2009) depend on the probability of a good or a bad state of the world as well as the costs. Namely, the interplay between renegotiation costs and costs of acquiring information influence the timing of information acquisition. According to Gârleanu and Zwiebel (2009), if the probability of a good state of the world p goes down, more control goes to the entrepreneur. In other words, if the probability of a good state of the world is small then the control rights transfer to entrepreneur, in our model it means that the renegotiation costs increase and the covenant becomes less strict. If the probability of a good state of the world p goes up, the result is twofold. On the one hand, entrepreneurs give up rights in renegotiations more frequently when information asymmetry increases. It means more control to the entrepreneur and less strict covenants in this case, which is supported by our model as well. On the other hand, lender renegotiates covenants more frequently when renegotiation costs and information acquisition costs increase. This finding is also supported by our model, meaning that if the increase in renegotiation costs is big enough compared to the increase in information asymmetry, the lender receives more rights and the covenant becomes stricter. Again, all of the above depends on the interplay between the degree of the asymmetric info and costs. Proposition 10 in Gârleanu and Zwiebel (2009) lists conditions under which the lender gives up rights, for example, if costs (renegotiation and information acquisition) are small compared to the degree of asymmetric information. In our model it means that the bank should lower the covenant strictness if the costs increase, but increase not greater than the level of info asymmetry increases (depends on the probability of selling or non selling the collateral in our model).

All in all, the dependency of the results on the probability of a state of the world and costs (both renegotiation and information acquisition) is demonstrated in the propositions in Gârleanu and Zwiebel (2009) paper. This paper demonstrates basically how the control rights vary depending on the probability of a state of the world and how it's changed based on relative values of the degree of asymmetric information and costs.

However, as stated by Gârleanu and Zwiebel (2009), the paper doesn't answer the question "Exactly how strict should one expect covenants to be?". Our model not only supports the deep and compound analysis described above, but also gives an answer on the posed question.

Both Dessein (2005) and Roberts (2015) show that creditors are granted strong control rights by borrowers. Moreover, the persistence of this finding throughout the lending relationship indicate the persistence of information asymmetry. In addition, Roberts (2015) demonstrates that the greater information asymmetry accelerates renegotiations process. Dessein (2005) shows that the borrower control is increasing both with respect to the level of asymmetric information and to the level of ex post uncertainty. However, it's worth mentioning that none of these papers take into account that renegotiation process is costly and don't include cost variables in their models.

Roberts (2015) in his empirical paper studies the allocation of control rights, renegotiations and covenant modifications in asymmetric information framework. He underlines the importance of information asymmetry in financial contracting. Roberts (2015) states that creditors have strong control rights not only at contract origination, but also throughout the lending relationship. The author shows that the renegotiations occur sooner if the borrower's uncertainty is larger. The uncertainty is determined by the borrower's volatility of the stock return. In order to study information asymmetry, Roberts (2015) builds the regression model of the dependent variable (which equals to 0 or 1 if the covenant were modified) on other model parameters. He bases the fact of the importance of information asymmetry on the following reason. The author states it is based on the observation that the covenants are modified frequently and mostly as a response to changing borrowers' demands given that "the joint distribution of loan term modifications is fairly stable across renegotiation rounds".

Roberts (2015) tries to determine factors that influence the allocation of control rights between the parties in financial contracting. He states that the ex ante measure of distance from a covenant threshold to the firm state (or covenant slack) could be misleading for ex post control rights transfers, so the asymmetric information is better to be assessed not only with this ratio. The factors that influence covenant modification in renegotiations are both macroeconomic and representing the borrower's financial strength. In addition, the author shows the significance of duration of lending relationship (both since origination of loan or since last renegotiation) in covenant modification. The author underlines that firm-specific uncertainty (not macroeconomic) influences the control rights allocation between the parties in lending relationships. In our model we differentiate firm-specific uncertainty and macroeconomic factors and show how they individually and interplay between them influence optimal covenant allocation.

According to Roberts (2015), there are certain factors that identify the number of renegotiation rounds. Firstly, macroeconomic factors are among them. Secondly, contract characteristics rather that borrower characteristics determine the number of renegotiation round. Thirdly, the interest rate is also positively related with this variable. Hence, the author predicts the positive relation between the information asymmetry and the intensity of renegotiations.

One contradictory result in Roberts (2015) paper concerns renegotiation timing. From the one hand, he shows that “the renegotiation round is unrelated to covenant modifications”. From the other hand, “covenants are less likely to be modified as time passes”. The author says this discrepancy may take place because of the omitted variables, however, it’s unlikely the reason because of the model construction (control for the fixed effects). Another interpretation can be the following: there is an asymmetry in accessing covenant slack. Although it stays the same from ex ante perspective, it becomes greater from ex post perspective. Roberts (2015) says that “this asymmetry is consistent with banks learning about borrowers during the relationship and the efficiency of the contract improving over time by avoiding costly amendments of covenant restrictions”. All in all, this asymmetry demonstrates the importance of the control right transfers throughout the contract during the lending relationship and asymmetric information in the financial contract modelling.

2.2.2 Information Asymmetry in Lending Relationships

Interestingly, both Dessein (2005) and Roberts (2015) show lenders' stronger control rights allocation in lending relationship. Moreover, the main point in the importance of information asymmetry underlined in both papers is that it's present because of frequent renegotiations. And renegotiations are said to be important as a mechanism of allocating control right ex post and completing the contract. However, Roberts (2015) says at the same time that it's natural and efficient for the parties to avoid costly covenant amendments and this is how a contract improves over time. The aim of our paper is to show that covenants not necessarily should be set that tight at origination and during renegotiations and loosen in certain scenarios in order to avoid costly renegotiations.

DeMarzo and Fishman (2007) develop a model for an agency problem in financial contracting. The information asymmetry as the source of an agency problem in this paper is in the cash flow. Whereas the investor does not observe the realization of the cash flow, the agent, or entrepreneur, observes it and can underreport the cash flow with some private benefit. As an optimal contract the authors search for a combination of standard securities, namely: long-term debt, equity and a line of credit. The basic trade-off considered in the paper is the agent's trade-off between taking an immediate compensation or expecting higher future payoffs.

Both Gertler (1992) and Clementi and Hopenhayn (2006) study the optimal contract and consider the trade-off of current and future financial position under the constraints in the setting of multi-period lending relationship with asymmetric information. Although the basic model of Gertler (1992) is 3-period, Clementi and Hopenhayn (2006) build a multi-period dynamic model. The tightness of covenants in Gertler (1992) model is determined not only by the present cash flow but also by the expectation about future profits. The information asymmetry in the model is defined in the following way: the project output is not publicly available, the lender and third parties can only observe firm investment and capacity utilization. One of the important elements of the model is allowance for hard times (analogue of macroeconomic dependence in our model), which is determined by the underemployment of capital in the bad productivity states. In the model setting macroeconomic conditions play an important role, since they may cause large fluctuations in the profits and covenant strictness. The optimal contract is found by solving the trade-off between the expected returns for the entrepreneur and opportunity costs. As a result the author obtains a simple time 0 cross-like model of an optimal contract. The author concludes that in bad states the entrepreneur is required to reduce production, which in our terminology means stringer the covenants. Since the setting of his model analyses the influence of bad states of the world, the main conclusion of the model proposed by Gertler (1992) is coherent with the part of our model dealing with bad states of the world and resulting in tighter covenants, although in most cases covenant strength should be relaxed because of the renegotiation costs. It is worth mentioning that renegotiation costs are not present in the Gertler (1992) model.

The model developed by Clementi and Hopenhayn (2006) is a dynamic extension of the model presented by Gertler (1992). Similar to our model, in this continuous time model the project can be liquidated every period. However, the revenue outcome is not a common knowledge but rather the private information for the entrepreneur. Hence, the source of asymmetric information in this model is in the inability to observe the project dynamics and the use of funds. One of the model variables is the probability of the project liquidation (analogue of our probability in asymmetric information setting). It is determined as being dependant of the history of reports provided by the entrepreneur in previous periods up to date of decision making. Clementi and Hopenhayn (2006) in their paper also use the recursive representation for the project value over time and across the states. Moreover, the probability of selling the collateral is determined by the distance to covenant. The trade-off in their model basically lies in the concavity of the value function in a certain range of project values. It means that the unlimited increasing in working capital meets the decreasing effect on firm value.

Overall, Clementi and Hopenhayn (2006) derive a model for an optimal capital adjustment policy for the firm. In an asymmetric information setting they observe the following: "in the case of our experiment success is always followed by an increase in the capital invested, while failure always triggers a decline". They separate the cases of high and low equity values and show different dynamics of optimal capital adjustments. It is similar to our model, where we take into account good and bad states of the world, the magnitude of renegotiation costs in these states and obtain different implications regarding the optimal covenant strength depending of these conditions. Regarding the liquidation value, Clementi and Hopenhayn (2006) argue that "a larger collateral S makes this risk less costly, resulting in a higher capital advancement" that means that the dependencies in our model are constructed in line with their model, meaning the decreasing costs of renegotiation (liquidation in their terminology) and increasing expected losses (a higher capital advancement in their terminology) when the covenant strength increases (larger collateral or the liquidation value S). What concerns the firm survival, their analysis is inconclusive, probably because they do not consider bad and good states of the world separately. Our analysis is more illustrative since it takes into account states of the world and builds conclusions based on that.

DeMarzo and Sannikov (2006) develop a dynamic continuous time model for an optimal contract. Their approach also supports our vision of information asymmetry: project value distribution is known to everyone, but the firm may undertake certain actions that cause the information asymmetry because of the possible private benefit for the firm as a result of these actions. Hence, from the bank's perspective, "there is the concern that a low cash flow realization may be a result of the agent's actions, rather than the project's fundamentals". The principal doesn't know the realization of the project cash flow and receives only the reports from the agent. The difference between the real and the reported cash flows and as a result a possibility of the agents private benefit forms the source of the agency problem and underlies the asymmetric information problem. In DeMarzo and Sannikov (2006) paper, the investor controls the firm's wage and its financial support and basically has an opportunity to make a termination decision which is one of the key features of their optimal contract implementation (in our model the bank makes a decision on whether to waive the covenant violation). The authors build the optimal contract model regarding the firm's capital structure choice. In DeMarzo and Sannikov (2006), the optimal contract is modeled in a setting when the access of credit is traded off with the operational losses. The optimal contract is found as a trade-off between maximization of the principal's profit and the agent's total expected payoff. After making some simplifying assumptions first, the authors eventually come up to an optimal contract for a general problem formulation. The optimal contract for the agent is truth-telling even in the case when stealing the cash flows is allowed. Using the martingale approach the authors are able to analytically derive the impact of the model parameters on the optimal contract (the same as we demonstrated in the part of analytic analysis of our model). Regarding the capital structure, it is also shown in the paper that the optimal contract can be build using capital structure based on the following securities: equity, long-term debt and credit line.

The optimal covenant with asymmetric information setting is also studied in the papers of Bazzana (2009) and Bazzana and Broccardo (2013a). Although in the first paper the author explores the optimal covenant allocation in two-period model in both symmetric and asymmetric information cases, the second paper is devoted to the comparison of public and private debt. The important modification of the model that is considered in Bazzana (2009) is an introduction of asymmetric information. In this framework the objective is to minimize the expected loss rate with respect to the covenant strength. Covenant strength index is determined as a relative distance between covenant threshold q and underlying value of the investment project d. In Bazzana (2009) the expected loss rate for a bank depends now on the firm behaviour, mainly on whether the firm will increase the capital in order not to violate the covenant. This behaviour is characterized by the variable, representing the level of relationship between a bank and a firm. The source of the asymmetric information is non-awareness of the bank regarding the behaviour of the firm in bad state, namely: whether the firm will increase the capital in order to avoid covenant violation. The probability of such an event is characterized by the density function which is assumed to be known to the bank. The information asymmetry in Bazzana and Broccardo (2013a) paper is defined through the different estimation of the probability of covenant violation between the borrower and the lender. However, then the authors assume that this assumption can be mitigated in such a way that the firm and the bank both have the same information about the probability of covenant violation. Hence, the information asymmetry and the agency problems associated with it can be mitigated because of the established relationships between the parties.

Biais et al. (2010) study continuous time moral hazard problem. In the introduction of the paper an example is provided that the explosion at the BP Texas refinery in March 2005 was happened because of the lack of the "effective process safety leadership" by their management. Similarly, insufficient risk control was named as one of the sources of the recent financial crisis. Consequently, the authors consider these aspects as well as their timing as the source of information asymmetry. In addition, information asymmetry itself (that the principal does not observe the agent's effort decisions, based on the key parameters $\Delta\lambda$ and B, described later) is a source of a model hazard problem in the model presented in their paper. According to their model, the entrepreneur can make an effort that reduces the probability of losses. This effort as well as the project value are unobservable by the principal. However, the principal (the bank) can observe the entrepreneur's continuation utility process, which is a mapping of the project value process. The bank has two ways to influence the project value and the relationship with the entrepreneur: payments in favour of the entrepreneur as well as project size adjustment (analogue of waiving or not the covenant violation in our model).

The optimal contract in Biais et al. (2010) is derived by maximizing the principal's (the bank's) expected profits subject to costs in the model (project expansion costs as well as adjustment costs). There is no covenant by itself in their model formulation, however there is a certain threshold set for one of the state variables of the model — the continuation utility of the agent (the entrepreneur). It is stated that the investment by the principal can take place only if the continuation utility of the agent is above a certain threshold. The optimal contract in Biais et al. (2010) is derived by equating the increasing costs of payments in favour of the agent and the marginal increase in the agent's continuation utility. The authors show that it only happens when the continuation utility is above the certain threshold. In general an optimal contract in their paper is characterized by the four regions on the figure of the position of the agents size-adjusted continuation utility relative to the certain thresholds. Based on this disposition, downsizing, investment, or transfers can take place as an optimum for a given contract.

One of the conclusion of the Biais et al. (2010) paper is one that demonstrates how the changes in asymmetric information $\Delta\lambda$ affect the banks requirements towards the firm. Here $\Delta\lambda$ and B (the key parameters of a moral hazard problem) are defined as follows: $\Delta\lambda$ is an increase in the probability of losses when no effort is made by the agent (in other words, the agent shrinks) and B models the cost of effort and represents the agent's private benefit when shrinking. Assuming that there is no investment and keeping variable b (the ratio of these key parameters) constant the optimal contract is in maximum risk prevention which means always make an effort for the agent.

In the recent paper Lin et al. (2018) investigates another type of agency problems in loans and its influence of financing costs. In this paper an agency problem is defined as private benefits of control of firm managers or large shareholders of the firm. Together with this, agency costs are also dependent on the bank's control over the firm, for example through renegotiation process. Since the private benefit of control is associated with the reduced creditworthiness, banks tend to influence the debt terms. Along with the increased price of debt, banks usually influence through the covenants. The intuition behind the study is the following: with more private benefits banks impose greater loan prices as well as stricter covenants. Connecting this work to our model we can assume that the greater private benefits as one of the forms of greater information asymmetry and the conclusion is in the stricter covenants in this case. When relying on other empirical work, the authors argue that the loan interest rates increase when more benefits of control are in place, which means increase in the expected losses in terminology of our model and we show the same. However, the final conclusion depends on the interdependence of these increased expected losses with the renegotiation costs. If the costs of renegotiation also increase (that is very natural) with the increase in the private benefits of control, that leads to increased heterogeneity among firm shareholders which make it more difficult to renegotiate.

Empirical results based on various regression models support the main conclusion of Lin et al. (2018), namely the positive and significant coefficients in regressions of the loan spreads on private benefits. The authors check different sets of controlling variables including firm and loan specific parameters and macroeconomic factors. However, at the same time they state the negative correlation between Z-scores and loan spreads. This means that the macroeconomic factors have the opposite influence with respect to the main focus of the model. In other words, even if the conclusion of the model remains the same controlling on macro factors, with increasing of influence of the latter ones (crisis situations, major economic problems) the conclusions may be the opposite. In sum, although the authors emphasize the main results of positive relation between loan scores and private benefits, the results of the paper support the duality of our model.

Another paper that considers influence of information asymmetry on covenants in loans is Sridhar and Magee (1997). In this paper the authors investigate how the opportunistic behaviour of the firm affects the design of debt covenants. Sridhar and Magee (1997) consider 2-period investment model and 2 different conflicts of interest that may arise in this setting. The first one is whether the firm will invest in another project at period 1 and the second one is whether the bank with waive the covenant violation and influence the firm's investment decision. The combination of these two conflicts of interest as well as possible outcomes for all parties form a setting for transfer of rights of the firm and the debtholder at period 1 based on the realization of both contractible and non-contractible terms.

Sridhar and Magee (1997) consider two types of public signals at period 2: contractible and incontractible. Although variables characterizing incontractible signals are not easily verifiable and rather subjective, the authors state they “play a critical role in the design and implementation of contracts”. They can be interpreted both as asymmetric information per se or renegotiation costs (“management skills”) or a state of the world (the market’s perception, an opinion of the overall economic environment). Hence, the result of the study can incorporate both effects of our model that can cause different results in terms of the covenant strength. Since they are not differentiated as in more recent papers, the conclusion may be misleading in some sense. The main result of the paper is the following: when the contractible variable becomes less informative (can be interpreted as increase in information asymmetry), covenants become more strict (the interest rate increases as well).

Quadrini (2004) investigates the long term optimal contract with repeated moral hazard problem. The source of the moral hazard (or information asymmetry) in his model is defined by unobservable for the lender resource allocation by the entrepreneur. The author defines information asymmetry as being dependent on the firm size, saying that for bigger firms investment decisions depend not on moral hazard but on future profitability. Another important element of his model is liquidation possibility in the presence of renegotiation in long-term contracts. One of the important conclusions of the model is the possibility for the firm to be liquidated under certain circumstances even if the liquidation is not free from the renegotiation. Quadrini (2004) also shows that the probability of liquidation is in positive relation with the relative value of the firm’s liability to its total value and in negative relation with the firm’s size and age. The aim of his paper is to find an optimal contract maximizing firm's revenue subject to the opportunity cost of capital. In some sense the author investigates inverse relation between the covenant strength and renegotiation process together with liquidation. First of all, he assumes the greater "lower bound to the entrepreneur's promised value" (which is the covenant strength in our terminology) and then concludes that the surplus decreases and the firm becomes smaller, but still can be liquidated. In our terminology he says that the covenant strictness is negatively related with the surplus and firm size which is true in our model. Quadrini (2004) also states that the smaller firms experience higher volatility which is also in line with the conclusions of our model regarding the covenant strictness and the project volatility. However, the author does not compare the optimal covenant strictness of the model with information asymmetry with one of the benchmark model.

2.2.3 Information Asymmetry in Venture Capitalists' Investment

Let's consider the literature stream on the allocation of control rights and information asymmetry in ventures (Schmidt, 2003; Kaplan and Strömberg, 2004; Dessein, 2005).

Schmidt (2003) investigates venture capital investment, mostly in the form of convertible securities. He studies the allocation of cash-flow rights depending on the effort of the entrepreneur, the state of the world and renegotiations. Although the setting is quite similar, it is important to emphasize the difference of the allocation of cash-flow right and control rights. According to Schmidt (2003) the covenants are used to allocate control rights while convertible securities are used to allocate cash-flow rights. Moreover, unlike in bank-firm relationship, the venture capitalists are usually more connected to the entrepreneur, actively involved in different project operations.

Dessein (2005) in his theoretical paper demonstrates that control allocation doesn't belong either to the investor or to the entrepreneur, but has a probabilistic nature. This probabilistic control allocation, according to Dessein (2005), depends on a performance measure of the entrepreneur. For example, if the entrepreneur shows bad performance, the control goes to the investor. The author provides two approaches in explaining why control rights matter, formulated by Hart (2001). The first one — "for influencing relation-specific investments", the second one — for signaling. In the first strand there are studies investigating different aspects of the allocation of control rights. One work is of our particular interest — the one by Aghion and Bolton (1988) who demonstrate the optimality of the contingent control rights allocation. In other words, whereas in good states of the world control goes to shareholders, in bad states of the world control goes to debt-holders. Dessein (2005) also states that even if there is empirical research supporting the influence of financial constraints on control right allocation, the theories mentioned above are not able to explain the influence of asymmetric information and uncertainty.

Moreover, Dessein (2005) in his theoretical paper differentiates between formal investor control and real investor control. The difference between those two is in the presence of actual inference by the investor in the second one. This differentiation was presented in the earlier paper by Aghion and Tirole (1997) and mean "the right to decide" as a formal authority and the actual influence in decision making as a formal authority. Dessein (2005) shows that the formal investor control increases when the venture uncertainty increases. At the same time, real investor control decreases with the presence of information asymmetry.

More formally, Dessein (2005) denotes by α and $\alpha_g > \alpha$ the probability of success and the probability of success when the entrepreneur receives no bad signal correspondingly, meaning that the entrepreneur has some private benefits and is better informed about the probability of success. In this case the signaling game comes into play, which is known by its multiple equilibria, that implies not the only one optimal solution but several solutions that is in tune with our model. The measure of information asymmetry between the parties is defined as $\alpha_g - \alpha$. Control allocation is introduces with the variable $\gamma \in [0, 1]$ — the probability of receiving control by the investor. This probability is contingent on the measure of performance of the entrepreneur and actually represents the formal investor control in the model. The real investor control in turn is denoted as the probability that the investor actually intervenes in the venture. One of the main results of the paper is formulated in Section II and sounds as follows — whereas under symmetric information we observe full entrepreneur control, with even small level of asymmetry in entrepreneur's awareness in project success some control passes to the investor. Section III provides the result that contrasts existing agency theory results. Although the latter argues that the control should be allocated to the better informed party, Dessein (2005) states that if the entrepreneur becomes better informed about the project profitability, he relinquishes more formal control over the contract to the investor. Meaning that although the concept of information asymmetry is defined in different way, with more information asymmetry we observe less strict covenants as in our model with asymmetric information.

Dessein (2005) also argues that more rights should go to the investor if the volatility of information increases or the entrepreneur is performing badly. This result together with probabilistic control allocation support the results of our model. However, the Dessein (2005) model does not account for the state of the world, macroeconomic condition and the renegotiation costs. In addition, the author does not make a difference between firm-specific and macroeconomic uncertainty, assuming their interdependence. The conclusions made in the Dessein (2005) paper are supported by empirical studies from different fields, for example, venture capital investment and technology alliances.

Kaplan and Strömberg (2004) not only classify and generalize the risks as it was shown earlier in this chapter, but test different approaches in their empirical analysis as well. The authors also differentiate internal and external risks and ascertain the ambiguity in the results with respect to external risks. The risks are considered to be external when they are equal for both parties. The authors also demonstrate empirically the correlation between external and internal risks. When they study the influence of risk on the control rights allocation, the results are ambiguous. On the one hand, Kaplan and Strömberg (2004) report significant positive relation of external and internal risks on the venture capitalist's control, meaning that greater risks are associated with more venture capitalist's board control. On the other hand, significant negative relation with respect to the execution risks is reported.

2.2.4 Asymmetric Information Within an Organization

Another stream of literature related to the information asymmetry in lending relationships covers a topic of information asymmetry and allocation of control rights inside an organization (Aghion and Tirole, 1997; Dessein, 2005).

As in Dessein (2005), Aghion and Tirole (1997) also differentiate formal and real control. Although the authors consider control inside an organization, the structure of the model is still similar. The subordinate proposes the project to the principle, and the principle decides whether to accept the project. The trade-off that is under consideration of this paper is between benefits of delegation in the form of increased initiative and participation of the subordinate versus the principal's costly loss of control. The authors conclude that the more information asymmetry there is between the principal and the subordinate, the less control the principal has over the project choice.

Another paper on delegation and control in an organization is one by Dessein (2005). Delegation is considered to be an alternative to communication. Given that the objectives of the principle and the agent are different, the author considers the following trade-off — "a loss of control under delegation and a loss of information under communication". The general conclusion of the paper is that the principle prefers to delegate control to the agent (meaning less control with information asymmetry in our terminology) as long as the incentive conflict is not extreme. The latter note supports the change of behaviour of the optimal contract of our model. If the situation is extreme, or in other words the uncertainty about the environment is large (which in our case implies a bad state of the world with increased costs of renegotiation), it is no longer optimal to cede the control and as in our model the covenant strictness increases with information asymmetry.

Summarizing, there are different models and different approaches in order to explore how information asymmetry affects control rights of the parties and how these shifts in control rights are reflected in the covenant strictness. We see that different models investigate different phenomena. For example, Gârleanu and Zwiebel (2009) deal with transfers in contrast to other financial contract papers who study investment outcomes. Moreover, some of them consider models with costly information acquisition (Gârleanu and Zwiebel, 2009), others differentiate between real and formal control (Dessein, 2005). Some of the authors take into account state of the world and renegotiation costs (Dessein, 2005; Gârleanu and Zwiebel, 2009), some of them - do not (e.g. Roberts (2015) in his empirical study). Hence, the conclusions are different and sometimes opposing. Whether Kaplan and Strömberg (2004) show less control with asymmetric information, other papers show more control "in order to avoid information distortion (Dessein (2002)) or to provide better incentives for information acquisition (Aghion and Tirole (1997))". Our model unites all the inferences of different models and allow for different conclusions based on the parameters configuration. It shows that with fixed state of the world and renegotiations costs the covenant becomes less strict with information asymmetry, meaning that entrepreneur gain more control rights. In turn, when the renegotiation costs increase it may become optimal to increase the covenant strictness.

2.3 Methodology and Results

Unlike the model with symmetric information that we considered in Chapter 1, in current model we admit information asymmetry between the firm and the bank. We assume that the bank is less informed about the project status and firm behaviour and strategy. Therefore, in case of covenant violation the bank has a possibility to decide whether to waive the covenant violation or to sell the collateral.

Consequently, in this chapter we will consider the following form of covenant: if the value of the firm assets (or the value of the investment project) falls to a specified level, then the bank can sell the collateral to refund the loan early. In the case with asymmetric information between the bank and the firm, the outputs for the bank are summarized on Figure 2.1, which we now describe in detail.

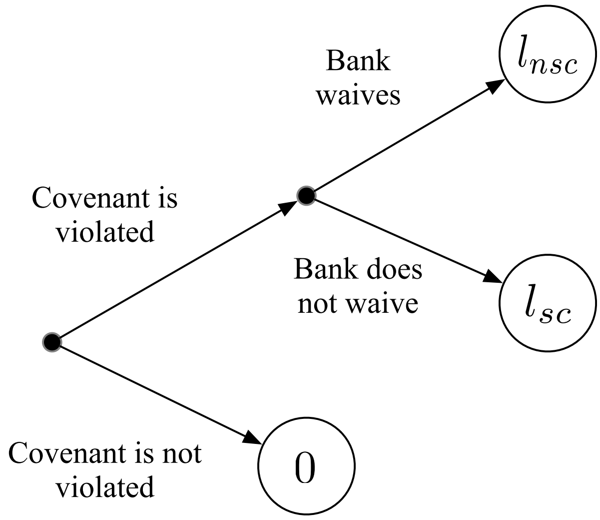

Figure 2.1: Decision tree in each point in time

In any time the bank checks whether the covenant is violated. If the covenant is not violated, there is no possibility for the bank to incur losses. Once the covenant is violated, the bank decides whether to waive the covenant violation or not.

2.3.1 The Bank Waives the Covenant Violation

In case the bank waives the covenant violation and decides not to sell the collateral, it incur potential losses, given by the Equation (2.1) and, multiplied by the probability of covenant violation, expected losses, given by the Equation (2.2).

$$l_{nsc}(t) = S_T - X_t e^{rs(T-t)} = e^{rsT} S_0 - X_t e^{rs(T-t)}, \tag{2.1}$$

$$el_{nsc}(t) = PCV(t)\left(S_T - X_t e^{rs(T-t)}\right) = PCV(t)\left(e^{r_sT}S_0 - X_t e^{rs(T-t)}\right), \tag{2.2}$$

where $PCV(t) = P(X_t < \alpha(t))$ is the probability for the firm to violate the covenant from the time t up to the maturity T. As in Chapter 1, this probability is defined as the distribution function of a log-normal process with an absorbing barrier at the reorganization boundary (i.e. covenant threshold) $\alpha(t)$

$$\begin{aligned} PCV(t) = P(X_t < \alpha(t)) = \Phi\left(\frac{ln\frac{\alpha(T)}{X(t)} - (\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right) + \\ + \left(\frac{X(t)}{\alpha(t)}\right)^{1-2\frac{\mu-r}{\sigma^2}} \Phi\left(\frac{ln\frac{\alpha^2(t)}{X(t)\alpha(T)} - (\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right), \end{aligned} \tag{2.3}$$

where the covenant threshold $\alpha(t)$ is determined exactly the same way as in Chapter 1 by the following equation:

$$\alpha(t) = e^{-r(T-t)} S_0 e^{r_sT} \rho. \tag{2.4}$$

The analysis of model behaviour when the covenant is violated and the bank doesn't sell the collateral is performed in Section 1.3 and Section 1.4.2 of Chapter 1. And we refer to this analysis to be correct and relevant to the Chapter 2 and our asymmetric information model as well.

2.3.2 The Bank Sells the Collateral

In case the bank decides not to waive the covenant violation, it sells the collateral (since the whole investment project value acts as a collateral). Provided the bank decides not to waive the covenant violation, it faces the losses and the expected losses determined as follows

$$l_{sc}(t) = S_T - X_t e^{r(T-t)} = e^{r_sT}S_0 - X_t e^{r(T-t)}, \tag{2.5}$$

$$el_{sc}(t) = PCV(t)\left(S_T - X_t e^{r(T-t)}\right) = PCV(t)\left(e^{r_S T} S_0 - X_t e^{r(T-t)}\right), \tag{2.6}$$

meaning that at date t the bank sells the collateral (the risky asset) at price X_t and get the profit of investing it under the risk-free rate with relation to the value of the debt at maturity that it would have to get. Hence, the difference with respect to Equation (2.1) and Equation (2.2) is in the interest rate used: either debt interest rate when non selling the collateral, or risk-free rate when selling the collateral.

Depending on the amount of credit S_0 and the project value X_t in any time of the credit lifespan, the expected losses when selling the collateral can be different in the magnitude and the sign (either positive or negative). The positive value of the expected losses when selling the collateral indicates on the unprofitability of selling the collateral, whereas the negative value indicates that the profit of investment after selling the collateral will exceed the value of debt at maturity.

Let's explore how the bank losses when the bank selling the collateral changes with respect to the credit amount S_0. First of all, it increases in S_0:

$$\frac{\partial l_{sc}(t)}{\partial S_0} = e^{r_S T} > 0. \tag{2.7}$$

Moreover, they increases linearly in S_0:

$$\frac{\partial^2 l_{sc}(t)}{\partial S_0^2} = 0 \tag{2.8}$$

Consequently, the greater the amount of credit S_0 is, the greater the bank losses are and, hence, the less profitable the selling of the collateral is. In other words, if a bank issues the greater amount of credit, it becomes less profitable for a bank to sell the collateral during the lifespan of the loan since the profit of selling doesn't exceed the value of debt bank gets back at maturity.

Let's now investigate the behaviour of the losses when the bank selling the collateral with respect to X_t. It decreases linearly in X_t since:

$$\frac{\partial l_{sc}(t)}{\partial X_t} = -e^{r(T-t)} < 0 \tag{2.9}$$

and:

$$\frac{\partial^2 l_{sc}(t)}{\partial X_t^2} = 0 \tag{2.10}$$

Hence, the greater the current project value in any period of time is, the smaller the expected losses become and, obviously, the less risky the project is.

At the same time, the bank losses when selling the collateral grow when the time to maturity t shortens. Indeed,

$$\frac{\partial l_{sc}(t)}{\partial t} = rX_t e^{r(T-t)} > 0 \tag{2.11}$$

and:

$$\frac{\partial^2 l_{sc}(t)}{\partial t^2} = -r^2 X_t e^{r(T-t)} < 0. \tag{2.12}$$

In other words, it becomes less profitable for a bank to sell the collateral when the maturity becomes closer.

All these dependencies are demonstrated numerically in the Section 2.4 in this Chapter.

2.3.3 Total Expected Losses for the Bank

In order to calculate the optimal covenant strength in asymmetric information model, let's define the total expected loss rate for the bank taking into account the probability of selling the collateral. This probability depends on the beliefs whether the bank will get the face value of the debt back. Intuitively, the ratio between the asset value at time

t and the covenant threshold α_t

$$\lambda_t = \frac{X_t}{\alpha_t} \tag{2.13}$$

represents this probability. The higher the value of Equation (2.13), the more assets the firm has at time t, and the more likely the bank is to get its debt back. Hence, the inverse value of λ_t (i.e. $1/\lambda_t$) can be interpreted as the probability for a bank to sell the collateral at time t.

However, it was shown in the literature that the distance to covenant threshold (our coefficients are based on it), is not empirically informative. It was also shown that the firm-specific uncertainty plays important role in allocating control rights in models with asymmetric information. As Roberts (2015) demonstrates, "it is interesting to note that firm-specific, as opposed to macroeconomic, uncertainty is what matters for the allocation of control rights between borrower and lender". Indeed, the distance to covenant could be informative only if all the firms were equal. Since different firms develop with their own pace and the changes of balance sheet ratios have different velocities for different firms as well, volatility σ_b can capture this difference.

Moreover, the importance of firm-specific parameters for covenant strictness in the asymmetric information setting is underlined in other studies (Bradley and Roberts, 2004; Gârleanu and Zwiebel, 2009). For example, such parameters as the size of potential transfers, operational complexity, lack of transparency within a firm to be positively related to the covenant strictness. In addition, the size of the borrower, it's growth opportunities, leverage are also shown to be related to asymmetric information and to the covenant strictness in loans.

Therefore, the bank's total expected loss rate can be represented by the following equation:

$$el_{ai}(t) = \left(1 - \frac{1}{\lambda_t/\sigma}\right) el_{sc}(t) + \frac{1}{\lambda_t/\sigma} el_{nsc}(t), \tag{2.14}$$

where $el_{sc}(t)$ and $el_{nsc}(t)$ are defined by Equation (2.6) and Equation (2.2) respectively. Although this expression written in full would be rather cumbersome, it contains nothing more complicated than standard normal distribution functions.

Multiplying the covenant threshold by σ_b implies the following: the more risky the firm is, the more the bank will be inclined to sell the collateral in case of covenant violation. In the same way, the less risky the firm is, the more probable will be the situation when the bank waives the covenant violation.

In the paper by Clementi and Hopenhayn (2006) the information asymmetry was defined as the private information on revenues for the firm and the possibility to waive the covenant for the bank. Revenue is a common knowledge in our model, but based on other uncertainties for the bank (managers behaviour, investment strategy etc.), the bank can waive or not the covenant violation depending not on the reporting, but rather on the distance to covenant together with project-specific volatility parameter. Thus, in our model in the setting with symmetric info everyone is risk-neutral and observe the realization of project outcomes. The bank has no right to waive the covenant and receives some profit if the covenant is not violated (depending on the probability of this event) and receives nothing otherwise. Thus, the expected losses for a bank are determined by the formula Equation (2.2). In case of asymmetric information, the bank has the right to waive the covenant violation and renegotiate the covenant based on the fact that there is an uncertainty for the bank in firm's management strategy and investment strategy or in the project behaviour. The expected losses in this case are determined by the weighted sum of expected losses when selling the collateral and when non-selling it. The weight coefficient (the probability) depends on the distance to covenant together with the firm-specific (in other words, project-specific) variation σ.

2.3.4 Two Types of Information Asymmetry

In our model we distinguish the following two types of information asymmetry: asymmetric information based on the behaviour of the project (possibility of selling the collateral and weighted expected losses for the bank) and asymmetric information based on the behaviour of the manager (the same form of expected losses for the bank, but different volatility σ for the bank because of the uncertainty in firm managers behaviour, investment strategy, the uncertainty about the manager's work effort etc.).

Consequently, total expected losses for the bank in the first type of information asymmetry are given by the Equation (2.14).

When we consider the second case of information asymmetry we have two different volatility variables in the model. One - σ_f - is project-specific and is used to calculate expected losses for the bank (both when selling the collateral $el_{sc,f}(t)$ and not selling the collateral $el_{nsc,f}(t)$). Another one - σ_b - based on the behaviour of the manager. Due to the uncertainty in firm managers behaviour, investment strategy or the uncertainty about the manager's work effort, the bank can have different assessment of the volatility for weighting coefficients.

The form of total expected losses for the bank in this case is given by the following equation:

$$el_{ai,bf}(t) = \left(1 - \frac{1}{\lambda_t/\sigma_b}\right) el_{sc,f}(t) + \frac{1}{\lambda_t/\sigma_b} el_{nsc,f}(t), \tag{2.15}$$

where expected losses $el_{sc,f}(t)$ and $el_{nsc,f}(t)$ are defined in the following way:

$$el_{sc,f}(t) = PCV_f(t)\left(S_T - X_t e^{r(T-t)}\right) = PCV_f(t)\left(e^{r_S T} S_0 - X_t e^{r(T-t)}\right), \tag{2.16}$$

$$el_{nsc,f}(t) = PCV_f(t)\left(S_T - X_t e^{r_S(T-t)}\right) = PCV_f(t)\left(e^{r_S T} S_0 - X_t e^{r_S(T-t)}\right). \tag{2.17}$$

In other words, expected losses when selling and non selling the collateral in Equation (2.15) are defined using the same functional forms as in Equation (2.6) and Equation (2.2), but the project specific volatility σ_f is used in the calculation of the probability of covenant violation $PCV_f(t)$:

$$\begin{aligned} PCV_f(t) = \Phi\left(\frac{ln\frac{\alpha(T)}{X(t)} - (\mu - \frac{1}{2}\sigma_f^2)(T-t)}{\sigma_f\sqrt{T-t}}\right) + \\ + \left(\frac{X(t)}{\alpha(t)}\right)^{1-2\frac{\mu-r}{\sigma_f^2}} \Phi\left(\frac{ln\frac{\alpha^2(t)}{X(t)\alpha(T)} - (\mu - \frac{1}{2}\sigma_f^2)(T-t)}{\sigma_f\sqrt{T-t}}\right). \end{aligned} \tag{2.18}$$

2.3.5 The Cost Function and the Optimal Covenant

As in Chapter 1, the cost function is given as follows:

$$c(\rho) = c_0 e^{-c_1\rho}. \tag{2.19}$$

The optimal covenant strength index is chosen in any rollover decision time when the value of costs and the value of total expected losses are equal for the bank. By equating right hand side of the Equation (2.14) (or Equation (2.15)) and the Equation (2.19), we obtain the optimal level of covenant strength that should be included in the contract:

$$\left(1 - \frac{1}{\lambda_t/\sigma}\right) el_{sc}(t) + \frac{1}{\lambda_t/\sigma} el_{nsc}(t) = c_0 e^{-c_1\rho} \tag{2.20}$$

the the fist type of information asymmetry and

$$\left(1 - \frac{1}{\lambda_t/\sigma_b}\right) el_{sc,f}(t) + \frac{1}{\lambda_t/\sigma_b} el_{nsc,f}(t) = c_0 e^{-c_1 \rho} \tag{2.21}$$

for the second type of information asymmetry.

2.4 Analysis and Numerical Results

We perform our analysis using the parameter values from Section 1.4.1.

2.4.1 Losses When Selling the Collateral

As in the Section 2.3, the case of selling the collateral is considered first. Depending on the correspondence between the project value X_t and the amount of credit S_0 in any time, the losses l_{sc} when selling the collateral can be different in the magnitude and the sign (either positive or negative). Figure 2.2 demonstrates the increasing and linear dependency of the bank losses when selling the collateral with respect to the credit amount S_0, that was provided in the Section 2.3.

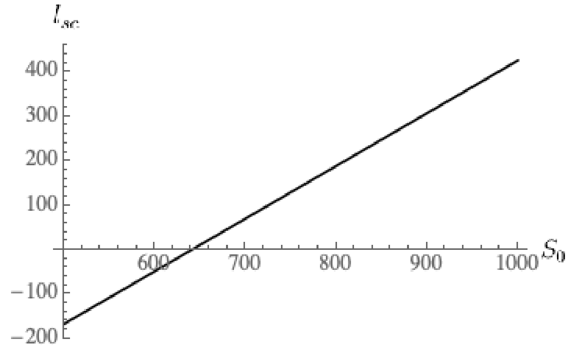

Figure 2.2: The dependency of bank losses when selling the collateral on the credit amount

Moreover, according to the Figure 2.2 it's possible to say that with the credit amount greater than 639.8 it's unprofitable for a bank to sell the collateral. When the amount of credit goes below this value, the profit of investment after selling the collateral exceeds the value of debt at maturity and hence selling the collateral becomes profitable.

As it was shown in Section 2.3, the bank losses when selling the collateral grow when the time to maturity t shortens. Figure 2.3 demonstrates this positive relation.

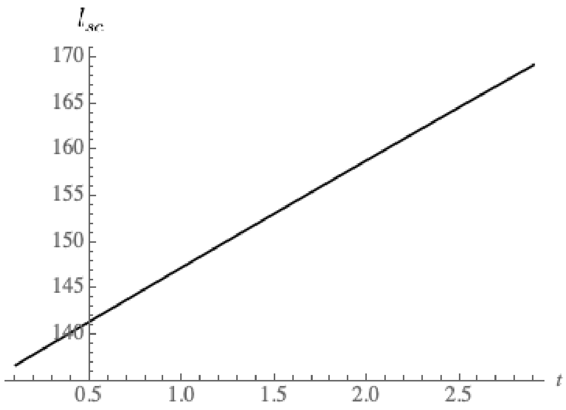

Figure 2.3: The dependency of bank losses when selling the collateral on the time to maturity

According to the graph, the most profitable time for a bank to sell the collateral is closer to the loan issuance time. This fact is supported by the empirical studies on loan renegotiations saying that the majority of the renegotiations happen close to the issuance time. Moreover, when loan approaching the maturity, it becomes less profitable for a bank to sell the collateral.

2.4.2 Information Asymmetry - Behaviour of the Project

The Figure 2.4 represents the first type of asymmetric information: asymmetry because of the behaviour of the project with respect to the baseline case with symmetric information between the parties. Compared to the baseline model, covenant strictness index in the model with information asymmetry decreases from 0.718 to 0.701.

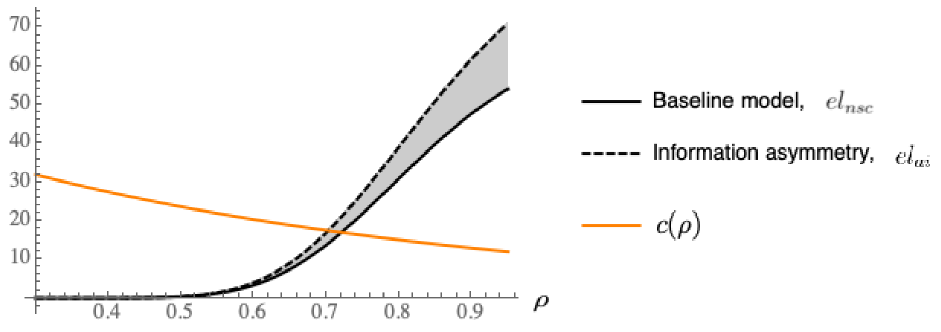

Figure 2.4: The optimal level of covenant strength with information asymmetry type 1 - behaviour of the project, compared to the baseline model

Below is the sensitivity analysis of the first type of information asymmetry (behaviour of the project) with respect to changes in volatility σ. The following figure (Figure 2.5) shows the uplift of expected losses function (and hence less covenant strength) when σ increases from 0.15 to 0.25.

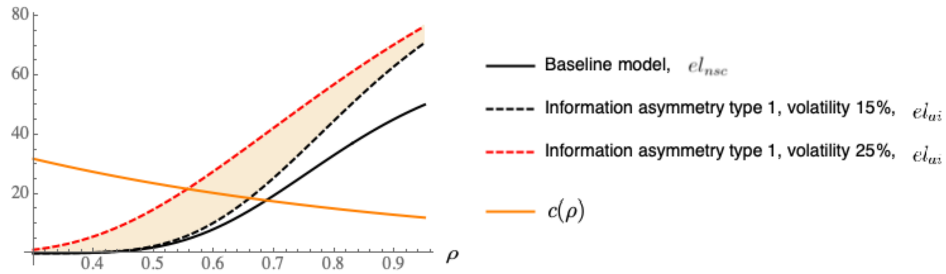

Figure 2.5: The change in optimal level of covenant strength with information asymmetry type 1 - behaviour of the project, when volatility increases from 15% to 25%

However, the change in volatility σ is not that straightforward and we observe slightly different behaviour with changes in smaller sigmas. The following figure (Figure 2.6) shows the uplift of expected losses function and change in behaviour when the covenant strength is high when σ increases from 0.05 and to 0.15. Hence, depending on the cost function, covenant strength can become smaller or bigger.

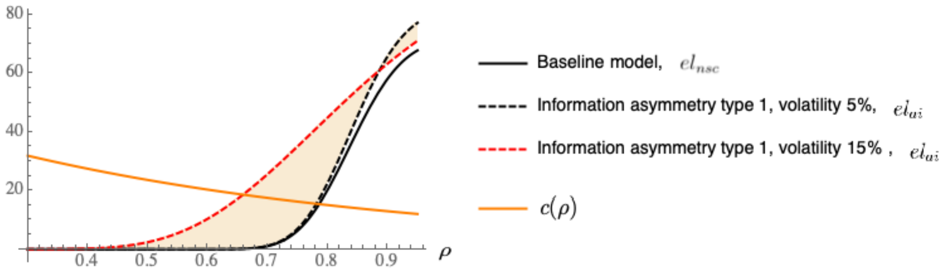

Figure 2.6: The change in optimal level of covenant strength with information asymmetry type 1 - behaviour of the project, when volatility increases from 5% to 15%

Therefore, the model implication regarding the optimal covenant strictness with information asymmetry is ambiguous. The conclusion depends on a state of the world which depends on the cost function magnitude. In other words, if the macroeconomic situation is favorable, renegotiation costs are not too high then, consequently, it is optimal to loosen covenant strictness when the volatility increases. On the other hand, when there is a bad state of the world, reflected in high renegotiation costs and extremely strict loan covenants in general, it is optimal for the bank to increase covenant strictness even more when the volatility increases.

A lot of models, both theoretical and empirical, use the state of the world or various macroeconomic variables as a key parameters to obtain certain model result. For example, Gârleanu and Zwiebel (2009) in their 2-period model account for the state of the world parameter when further investment decision has to be made. This parameter is unknown for both parties at time 0, but become known by the firm and the bank at time 1, when the decision has to be made on whether to make further investment in the project.

Although, according to Roberts (2015), the initial renegotiation doesn't depend on the macro conditions but rather on the firm leverage, the subsequent renegotiations depend on multiple factors. Hence, if we are considering the initial renegotiation, changes in macroeconomic situation should not influence the expected losses for a bank, according to Roberts (2015). However the renegotiation costs may differ, that in turn may influence the optimal covenant for a loan.

As we discussed in Section 2.2, there are different (and often opposing) implications of theoretical and empirical papers regarding covenant strictness in the information asymmetry setting. For example, Gertler (1992) in his theoretical paper argues that in asymmetric information setting covenant strength increases in bad states of the world. Asymmetric information in his paper implies that output is not publicly observable, the bank and third parties may only observe investment and capacity utilization. The empirical paper of Roberts (2015) states that when information asymmetry increases, renegotiations occur more frequently. The author explores the regression model of factors influencing renegotiations on covenant modifications. The renegotiation costs are not included in the model. The fact of importance of asymmetric information is based on the observations that the covenants are modified frequently and mostly as a response to changing borrowers' demands given that "the joint distribution of loan term modifications is fairly stable across renegotiation rounds". Dessein (2005) argues that although formal control increases when information asymmetry increases, real control decreases. We observe more sophisticated result in theoretical 2-period model with costly information acquisition by Gârleanu and Zwiebel (2009). Unlike other models based on outcomes of investment, this model considers transfers. The results of the paper are the following. With fixed probability p of good state of the world, more control goes to the lender. When the probability of good state of the world p goes down (meaning that the situation becomes worse), more control goes to the entrepreneur. When the probability p goes up, more control goes to the entrepreneur when information asymmetry increases and more control goes to the lender when costs increase.

Our model is novel in the sense that it is not straightforward equilibrium model with the unique optimal solution and obvious interpretation. Our model takes into account renegotiation costs and macroeconomic conditions influencing them. The conclusions of our model are more explanatory and sound and they are basically capable to explain differences in both theoretical and empirical findings about covenants. For example, the finding by Roberts (2015) that tight covenants need not lead to more ex post renegotiations can be explained by loans different macroeconomic conditions under investigation that lead to different conclusions.

2.4.3 Information Asymmetry - Behaviour of the Manager

When we consider the second case of information asymmetry we have two different sigmas in the model: one is project-specific σ_f and another one σ_b represents how the bank assess the manager behaviour when deciding whether to sell the covenant.

The comparison of the two cases of asymmetric information can be found on the Figure 2.7. In general, introducing two types of volatility σ leads to greater expected losses for the bank and, consequently, to less strict optimal covenant for the contract. For the Figure 2.7 we used $\sigma_f = 0.15$ and $\sigma_b = 0.25$.

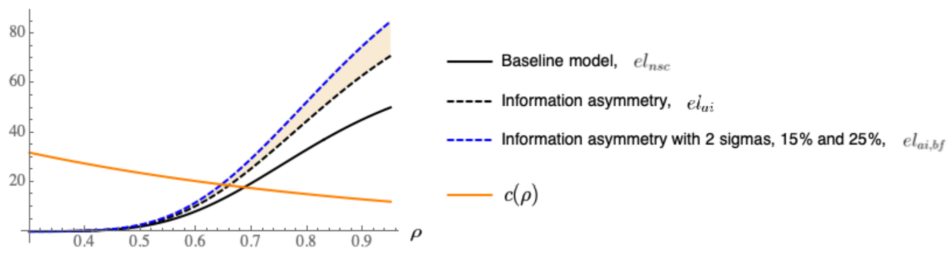

Figure 2.7: The comparison of the 2 types of information asymmetry under bigger volatility changes

Let's compare two figures: Figure 2.7 and Figure 2.5. Although they both indicate that the optimal covenant threshold should decrease, the magnitude is much more pronounced in case of increase in volatility in the first type of information asymmetry. This in turn shows that the risk of the project is more important than the risk of the manager behaviour.

However, the previous conclusion is not that straightforward. Let's consider increase in small values of volatility in the 2nd type of information asymmetry.

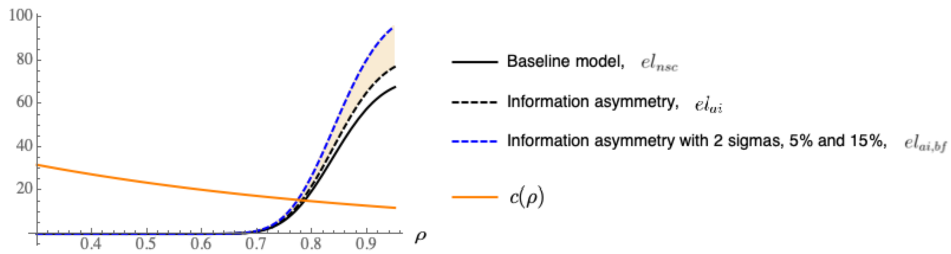

Figure 2.8: The comparison of the 2 cases of information asymmetry under smaller volatility changes

Figure 2.8 shows that the optimal covenant should decrease when we consider two small values of volatility: $\sigma_f = 5\%$ for the firm and $\sigma_b = 15\%$ for the bank. Let's compare Figure 2.8 and Figure 2.6. In the second type of information asymmetry, when we consider asymmetry based on the behaviour of the manager, we observe different magnitudes in changes of optimal covenant, but the main conclusion remains the same: the optimal covenant should decrease. It contrasts with the first type of the information asymmetry where there is ambiguity in the conclusion on the optimal covenant. When we consider information asymmetry based on the project behaviour, the optimal covenant strength can decrease in most cases, but can also increase if the project is in bad state (economic crisis for example) when renegotiation costs turned out to be extremely high and covenant strength is also high. Hence, in case of economic crisis it becomes optimal for a bank to increase covenant strength in order to obtain a minimum value of the loan.

Summarizing, we've investigated two source of risk resulting in information asymmetry in financial contracting: the risk of the project and the risk of the manager. In all states of the world the first one is shown to be the most important and the second one is only additional based on the magnitude of impact they cause. Consequently, the above conclusion supports the results provided in the empirical literature that the firm-specific uncertainty plays important role in allocating control rights in models with asymmetric information. As Roberts (2015) demonstrates, "it is interesting to note that firm-specific, as opposed to macroeconomic, uncertainty is what matters for the allocation of control rights between borrower and lender".

The evidence in the literature regarding the information asymmetry and control are controversial. Our model can be an explanation of these differences and not give the exact answer, but rather demonstrate that both conclusions can take place depending on the firm- and macro-factors (worth look at those differences in the papers, what period they take and what financial products consider).

2.5 Conclusion

In this chapter we consider a fundamental extension of our baseline model - a model of optimal covenant in debt contracts with asymmetric information. We compare it with our baseline model with symmetric information and investigate model behaviour and the main inferences with respect to dynamics in key model parameters. We show that when information asymmetry increase, the optimal covenant strictness should decrease. However, when we consider a bad state of the world with increased renegotiation costs, it becomes optimal for a bank to increase covenant strictness.

Hence, our result is state dependant. Moreover, this twofold result of our model may serve as a possible explanation of different implications of theoretical and empirical papers regarding covenant strictness in the information asymmetry setting discussed in the Section 2.2 and unite all the results if they were performed in consolidated settings.

Chapter 3

Simulations

In this section we introduce a methodology of assessing the consequences of covenant violation for a bank in Monte Carlo simulation setting that supports bank's decision-making process. First of all, we introduce a framework for measuring various risk-parameters of the project (average number of covenant violations per contract, frequency of covenant violation, frequency of loan repayment). Secondly, using these risk-characteristics we implement different decision rules for a bank and based on the risk-parameters we assess the effect of those decisions, that extends the traditional risk-analysis of a project. Moreover, we implement a recursive technique for determining the level of covenant strength that allows the bank to maintain the performance of a specific risk-parameter.

3.1 Introduction

There is always a trade-off between realism and simplicity of a dynamic model (Strebulaev and Whited, 2012a). On the one hand, simpler and easier solvable models may not reflect to the full the reality of things. Strebulaev and Whited (2012a) provides an example of using quadratic form of investment adjustment costs in investment Euler equations. However, despite the simplicity, it leads to smoothing the investment over time. On the other hand, one can make the model more realistic by using different methods of simulation.

Although the models in the first two chapters allow the bank to make a decision on the optimal covenant strength, the simulation model of this chapter gives the bank the possibility to assess the risks of the project once the covenant strength is chosen.

The model with Monte Carlo simulations allows us to assess empirically different characteristics of the project, that in turn reflects the potential risks and profits for the bank, such as the probability of covenant violation and the probability of repayment of the loan.

Moreover, the developed simulation model supports bank's decision making process. By assessing the risk-parameters of the project delivered as the results of our simulation, the bank improves the effectiveness of its decisions regarding the covenant strength, or regarding whether the waive the covenant violation at each rollover date, or whether to stop the contract in case of covenant violation. We describe a recursive technique of determining the optimal covenant strength that maintains certain levels of risk-parameters important for the bank, such as the probability of repayment of the loan to the bank.

Summarizing, the Monte Carlo simulation model developed in this chapter allows the bank to improve the methodology of risk-assessment and, consequently, the decision-making process.

3.2 Literature Review

In order to assess the uncertainty associated with the project, dynamic modelling, including various simulation techniques, are commonly used. Dynamic modelling in finance was studied in detail in the survey by Strebulaev and Whited (2012a). In their paper they present the main strands in the literature on dynamic corporate finance and try to expound simplified ideas underlying various types of models. Moreover, they describe main methods of estimation of dynamic models. Simulated method of moments and maximum simulated likelihood are among them. According to Borgonovo and Gatti (2013), the term "risk analysis" originated in the 60s and firstly Monte Carlo simulations were used to obtain the distribution of NPV or IRR.

Now there are various types of simulations in the literature: parametric simulations (Demerjian and Owens, 2014; Schroth et al., 2014; Arnold, 2014; Demerjian and Owens, 2016), path simulations (Strebulaev, 2007; Borgonovo and Gatti, 2013; Gündüz and Uhrig-Homburg, 2013; Liang et al., 2014; Jessen and Lando, 2015; Chang and Lee, 2013), computer simulation of the interactions in game theory models (Vanderschraaf, 2007).

Monte Carlo simulations are also used in the models with different financial products: along with private debt, we consider papers on public debt (Chang and Lee, 2013), asset-backed commercial papers or ABCP (Schroth et al., 2014), credit default swap (CDS) (Gündüz and Uhrig-Homburg, 2013).

3.2.1 Parametric Simulations

Demerjian and Owens (2014) build a comprehensive measure of covenant strictness that covers not only covenant slack, but also the dynamics of underlying parameters. In each period of time the authors simulate not only the dynamics of a certain financial ratio underlying the covenant, but all means, variances and covariances of all underlying parameters by using historical data.

Demerjian and Owens (2016) in their paper measure the probability that a borrower will violate financial covenants. They use real financial data in order to compute measures (means, variances, covariances) parameters that covenants are based on. In the next step the authors use those measures in order to simulate more data and calculate covenant probability on it.

Schroth et al. (2014) study debt runs in the market of asset-backed commercial papers or ABCP. The model is developed in order to explain the relation of increased yield spreads to future runs and dilution risk. The authors extract the determinants of runs among model parameters. Some parameters are used from the data available to the authors and some are simulated by using simulated method of moments (SMM). The simulated moments are generated in a way that minimizes the distance between the simulated parameters and actual ones. Thus, the larger simulated database is obtained. This simulated database allowed Schroth et al. (2014) to study sensitivities of bunk runs with respect to different parameters by measuring simulated run probabilities and comparing them to the actual data.

As it was underlined by Arnold (2014), since firms in real economy are different in their firm characteristics, it's important to simulate sample of firms with similar characteristics in order to compare parameters of simulated sample with real ones. Thus, they take as a basis S&P 500 firms and then replicate firm dynamics by varying such parameters as asset volatility, investment opportunities etc. They thus construct a simulated data set, which is used to test their hypotheses.

3.2.2 Path Simulations

Strebulaev (2007) in his paper develops the dynamic capital structure model. The author generates simulated paths of the firm's capital structure for 3000 firms in order to obtain a so-called simulated economy. Then, Strebulaev (2007) performs cross-sectional tests on this simulated data set.

Borgonovo and Gatti (2013) study in their paper the impact of covenant breaks on the risk analysis of investment projects. The authors use Monte Carlo simulation in order to obtain the net present value (NPV) distribution. Simulation is applied to a real project, for which authors have access to the financial model used for project evaluation. Namely, the authors simulate the distribution of the project valuation function Z, that take into account all project aspects, both financial and non-financial. Borgonovo and Gatti (2013) describe the simulation procedure step by step.

Borgonovo and Gatti (2013) distinguish material and technical covenant break in their model. Along with comparison of risk profiles with and without the covenant break, the authors estimate the probability of covenant violation using multiple scenarios.

In a comparative study Gündüz and Uhrig-Homburg (2013) explore the quality of prediction of the default time of structural and reduced form models. The implementation of this comparison is performed using simulation of credit default swap (CDS) pricing. Namely, the paths for the short rate and leverage are generated.

Inclusion of jumps in the contract modeling process from the one hand make models more realistic, but on the other hand requires the use of simplifying implementation techniques such as simulations due to the model complexity (Zhou, 1997; Jessen and Lando, 2015; Chang and Lee, 2013).

Chang and Lee (2013) develop two structural models of corporate risky debt with safety covenants under the double exponential jump-diffusion process. In their models the authors combine several enhancements from other papers. They introduce jumps in their structural models by modelling them according to double exponential jump-diffusion process instead of standard log-normal distribution. Moreover, the authors introduce two models with different assumptions regarding covenant boundaries. The first one incorporate the notion of so-called "caution time" when the firm can violate the covenant threshold for a certain period of time. The second model has two boundaries: one that triggers the default and the other one that triggers liquidation. Since valuation turns out to be extremely difficult, Chang and Lee (2013) use Monte Carlo simulation for corporate bond valuation. They use the modified Monte Carlo simulation in order to speed up the simulation of jump-diffusion process. Namely, the authors generate the jumps and use standard Monte Carlo simulation in between jumps.

Zhou (1997) build a jump-diffusion model for the firm value in modelling credit risk. In order to calculate bond price value, the authors use Monte Carlo simulation approach

The aim of the paper of Jessen and Lando (2015) is checking the robustness of distance-to-default — the measure of firm's credit risk that is widely used in the literature. They show that although this measure is quite successful in prediction of firm's default, it losses its robustness in presence of jumps and volatility shocks. The authors use simulations to make the sample for testing. They use asset value Monte Carlo simulations as well as iterative techniques in order to estimate parameters for distance-to-default calculation.

3.2.3 Computer Simulations in Game Theory

Vanderschraaf (2007) applies game theory approach in order to study an interplay between covenant violation and reputation. He performs computer simulation in order to determine conditions under which reputational issues may prevent covenant violation. Here computer simulation of a covenant game is developed in order to model a society and observe behaviour of two types of players: Humeans or Fooles. The author describes in detail the society settings as well as behaviour of those types of players. Vanderschraaf (2007) allows for a common knowledge in a society in a form of innocence or guilt markers for players. He concludes that in a setting with a common knowledge in a society reputation plays an important role in honoring covenants.

3.3 Methodology and Results

3.3.1 Model Setting Description

In this chapter we implement the model developed in previous chapters in a simulation framework. The value of the firm's assets (or investment project) X in this model follows a stochastic process (the same as in the baseline model from the Chapter 1)

$$dX_t = \mu X_t dt + \sigma X_t dW_t \tag{3.1}$$

with drift μ and volatility σ, where W_t is a standard Brownian motion that represents random shocks on the firm's assets value.

Assume that there are $n-1$ rollover dates, $t_1, \ldots, t_{n-1}$, at which the bank checks whether the covenant is violated ($t_n = T$ – maturity). For simplicity, we operate with equidistant intervals, i.e. $\Delta t = T/n$. For example, Liang et al. (2014) assumes the rollover frequency equal to 3 months. In the current study we assume that the rollover decisions have to be made monthly, i.e. $\Delta t = 1$ month.

For each of these dates t_i, let's denote each variable of the model with the corresponding subscript i. For example, the value of the debt at each rollover date is $S_{i+1} = e^{rs\Delta t} S_i = e^{rs(i+1)\Delta t} S_0$. The covenant threshold is denoted in the following way:

$$\alpha_i = \alpha(t_i) = e^{-r(T-t_i)} S_0 e^{rsT} \rho. \tag{3.2}$$

As it was discussed in the literature review in the Section 3.2, the Monte Carlo approach for simulations is common in the literature. Moreover, it is also the common practice in the literature to take an example of one project and apply Monte Carlo simulation to it (Zhou, 1997; Borgonovo and Gatti, 2013; Chang and Lee, 2013). In addition, as it was underlined by Arnold (2014), it's important to make separate simulations for firms with different characteristics in order to adequately compare simulated parameter values with real ones. Therefore, we use the simulation of one firm, estimate the probability of covenant violation and estimate the optimal covenant strength in this case.

3.3.2 Parameters Estimation

We employ a dynamic approach in the spirit of Borgonovo and Gatti (2013); Liang et al. (2014); Chang and Lee (2013) by simulating project value paths over time. We outline how the theoretical model from Chapter 1 is turned into a simulation procedure. The Monte Carlo approach to estimate the probability of covenant violation and the average number of covenant violations per contract is described below:

- Step 1. By assuming there are $n-1$ rollover dates, $t_1, \ldots, t_{n-1}$, at which the bank checks whether the covenant is violated ($t_n = T$ – maturity), we divide time interval $[0, T]$ onto n subperiods, i.e. $\Delta t = T/n$ (for simplicity, we operate with equidistant intervals).

- Step 2. Define the covenant threshold for each of these dates t_i in the following way

$$\alpha_i = \alpha(t_i) = e^{-r(T-t_i)} S_0 e^{r_S T} \rho. \tag{3.3}$$

- Step 3. Do Monte Carlo simulation of the project process with the given values of parameters of drift μ, volatility σ and initial project value X_0 for N ($j = 1, \ldots, N$) times.

- Step 4. On each step (or at each rollover date) check whether the covenant is violated. If the covenant is violated, set the value of the corresponding element in the matrix of indicators as 1, otherwise as 0.

- Step 5. Estimate covenant violation frequency f_{CV}.

- Step 6. Estimate the average number of covenant violations per contract ν_{CV}.

We denote the frequency of covenant violation (the estimator of the probability of covenant violation) as f_{CV} and calculate it as the number of contracts with at least one covenant violation n_{CV} to the total number of iterations in the simulation N, i.e.

$$f_{CV} = \frac{n_{CV}}{N}. \tag{3.4}$$

The average number of covenant violations per contract ν_{CV} is calculated as mean of all covenant violations in the contract through all the iterations in the simulation.

We use the same parameter values as in previous chapters: the drift $\mu = 1.56\%$ and the annual volatility $\sigma = 11.25\%$ for the value of the firm's assets (or investment project) X with the asset value at time t $X_t = 726.0$. For the risk-free rate, $r = 1.56\%$ will be used. The amount of debt $S_0 = 755.7$ is provided for the firm until the time $T = 3$ years (maturity) under the promised (continuously compound) rate of return $r_S = 5.72\%$.

In order to calculate the covenant threshold we need to set a covenant strength index. For our baseline simulation model we use the optimal covenant strength index from the results of Chapter 1, namely $\rho = 0.718379$. Thus, covenant threshold varies from $\alpha_0 = 615.84$ to $\alpha_T = 644.51$. The rollover decisions are assumed to be made monthly. Hence, the number of subperiods we use is $n = 36$.

The simulation algorithms were coded in Mathematica 10 and run on Macbook Pro (Processor 2.3 GHz Intel Core i5).

We first run the simulation for $N = 1\ 000$. The average timing needed to run an algorithm with this number of operations is less than a second (around 0.97 second). The average number of covenant violations per contract varies in average around 4.4. According to the recent empirical study by Roberts (2015), the average bank loan is renegotiated five times. Therefore, we obtain the lower frequency result than stated in the literature. The frequency of covenant violation is in turn estimated to be around 0.39.

If we run $N = 10\,000$ iterations, the evaluation time increases up to around 10 seconds. Whereas the timing increases the estimators for frequency of covenant violations and average number of violations per contract remain the same. When we run $N = 100\,000$ iterations, the calculation time equals to almost 6 minutes. The results are summarized in the Table 3.1

Parameter	Baseline Simulation
The average number of covenant violations per contract ν_{CV}	4.4
Frequency of covenant violation f_{CV}	0.39
Number of contracts with at least one covenant violation $n_{CV}(N = 100\,000)$	38 642

Table 3.1: The Results of Baseline Simulation Model

In contrast to the previous approach we can calculate the number of covenant violations in a different way. As opposed to calculating each covenant violation in each rollover date (every month or each day), we can count only each successive violations (an interval of violations) as 1 violation of the contract that requires renegotiation. For example, if covenant is violated 3 rollover dates in a row, we calculate it as 1 covenant violation instead of three using this approach. The results are summarized in the Table 3.2.

When implementing the described approach in Mathematica 10 software, we obtain the following results of simulations. The frequency of covenant violation is obviously remains the same (around 0.39) when using this approach. However, the average number of covenant violations per contract drops significantly to values around 0.7. Roberts

Parameter	Baseline Simulation	Interval Simulation
The average number of covenant violations per contract ν_{CV}	4.4	0.7
Frequency of covenant violation f_{CV}	0.39	0.39
Number of contracts with at least one covenant violation n_{CV}	38 616	38 616

Table 3.2: The Results of Baseline Simulation Model and Interval Simulation Model

(2015) reports the following renegotiation frequencies in his sample: whereas the average number of renegotiations per contract in the whole sample is 3.54, the same estimator for the subsample of the contracts with at least one renegotiation is 4.87. Therefore, we can conclude that in practice renegotiations indeed occur frequently and if not every violation is renegotiated, but close to it. On the other hand, we used the optimal covenant strength in our calculations, demonstrating less frequent covenant violations.

Another important indicator that can be calculated using Monte Carlo simulation method developed in this chapter is the estimator of the probability that the bank receives the loan back (or the frequency of loan repayment) f_{LR}. This estimator can be calculated simply by assessing on each round of simulation whether the value of the project at maturity is enough to pay the loan back. In other words, if the covenant is violated at maturity, the firm has not enough money to pay back the loan to the bank, otherwise the loan is repaid as planned.

Based on the data we use and after running our simulation $N = 10\ 000$ times, we receive the estimation of the above mentioned probability equal to 83.19%. The calculation time for this estimator is about 10 seconds. When we run the simulation $N = 100\ 000$, we obtain almost the same estimate of 83.06% probability of repayment. However, the

Parameter	Baseline Simulation	Interval Simulation
The average number of covenant violations per contract ν_{CV}	4.4	0.7
Frequency of covenant violation f_{CV}	0.39	0.39
Number of contracts with at least one covenant violation n_{CV}	38 616	38 616
Frequency of loan repayment f_{LR}	0.8319	0.8306

Table 3.3: The Results of Baseline Simulation Model and Interval Simulation Model

calculation time increases to 3.75 minutes. It's worth mentioning that due to the calculation algorithm, the values of the frequency of loan repayment are equal both for baseline simulation and interval simulation.

For assessing the effectiveness of Monte Carlo simulation run-length control is performed. In order to statistically compare the estimates, the standard errors are calculated. However, there is a possibility to calculate the standard errors only for the frequency of covenant violation (the estimator of the probability of covenant violation). The results are provided in Table 3.4.

Parameter	N = 1 000	N = 10 000	N = 100 000
Frequency of Covenant Violation f_{CV}	0.35	0.38	0.39
Relative Error	16.67%	9.52%	7.14%

Table 3.4: The Results of Run-Length Control for the Frequency of Covenant Violation

According to the analytical model from Chapter 1 the probability of default calculated at the origination of the contract is around 0.42. According to the simulations results for the frequency of covenant violation, the estimate is equal to 0.39. Hence, when performing run-length control, the relative error is 7.14% and it is decreasing when the number of rounds increases. Consequently, the accuracy of the estimates increases with the number of rounds in the simulation procedure.

3.3.3 Sensitivity Analysis

In this section we will explore how the results change with changes in the key parameters of our simulation model: the drift, the volatility and the parameter characterizing the rollover frequency (or in other words, how often the covenant maintenance is checked by the bank during the lifespan of the loan).

The dynamics with respect to the drift.

The decrease in the value of the project drift μ can result in financial distress of the firm, which in turn causes the difficulties with covenant maintenance and with paying off the financial obligations of the firm in general. Consequently, when the drift μ increases, all the measured of violation probability decrease. The results of the sensitivity analysis with respect to the parameter μ are shown in the Table 3.5.

According to the results from the Table 3.5, when μ decreases from the 1.56% to 0.56%, the average number of covenant violations per contract ν_{CV} increases from 4.4 to 7.9. Moreover, the number of contracts with at least one violation n_{CV} increase from 38 616 to 56 684 and the relative frequency of covenant violation f_{CV} increases from 0.39 to 0.57.

The dynamics with respect to the volatility.

The changes in the volatility σ can result in the difficulties with the covenant maintenance for the firm. As an indicator of the vulnerability of the project value, it may result

Parameter	$\mu = 0.0056$	$\mu = 0.0106$	$\mu = 0.0156$	$\mu = 0.0206$
Baseline Simulation				
ν_{CV}	7.9	5.9	4.4	3.1
f_{CV}	0.57	0.48	0.39	0.30
n_{CV}	56 684	47 506	38 616	30 366
f_{LR}	0.66	0.75	0.83	0.89
Interval Simulation				
ν_{CV}^{int}	1.0	0.9	0.7	0.6
f_{CV}^{int}	0.57	0.48	0.39	0.30
n_{CV}^{int}	56 684	47 506	38 616	30 366
f_{LR}^{int}	0.66	0.75	0.83	0.89

Table 3.5: The Dynamics with Respect to the Drift

in the excessive earnings as well as more risk for covenant violations and difficulties with loan repayment. The results of the sensitivity analysis with respect to the parameter σ are presented in the Table 3.6.

In general, the model is more responsive to the changes in the volatility, meaning that even small changes in the parameter σ result in the significant changes in the model estimators. For example, when σ increases from 11.25% to 15.25%, the average number of covenant violations per contract ν_{CV} increases from 4.4 to 8.2. Moreover, with increase in σ, the estimators of probability of covenant violation are also increase: the number of contracts with at least one violation n_{CV} increase from 38 616 to 59 145, the relative frequency of covenant violation f_{CV} increases from 0.39 to 0.59 compared to the baseline simulation model. This result is supported by the finding of (Liang et al., 2014), who

Parameter	$\sigma = 0.0525$	$\sigma = 0.0825$	$\sigma = 0.1125$	$\sigma = 0.1525$
Baseline Simulation				
ν_{CV}	0.10	1.6	4.4	8.3
f_{CV}	0.02	0.18	0.39	0.59
n_{CV}	1 708	17 925	38 616	59 145
f_{LR}	0.99	0.94	0.83	0.69
Interval Simulation				
ν_{CV}^{int}	1.0	0.9	0.7	0.6
f_{CV}^{int}	0.02	0.18	0.39	0.59
n_{CV}^{int}	1 708	17 925	38 616	59 145
f_{LR}^{int}	0.99	0.94	0.83	0.69

Table 3.6: The Dynamics with Respect to the Volatility

show that the total default risk increases in volatility.

The dynamics with respect to the rollover frequency.

Here we investigate how the risk of the project and the estimators of probability of covenant violation change when the rollover frequency changes. We show that when the bank checks the covenant violation more frequently, the frequency of covenant violations also increases. The results of the sensitivity analysis with respect to the parameter n are presented in the Table 3.7.

For example, when the bank starts to check the covenant violation twice a month ($n = 24$) compared to monthly checkouts ($n = 12$), the average number of covenant violations per contract ν_{CV} increase from 4.4 to 10.0. In addition, the number of contracts with at least one violation n_{CV} increase from 38 616 to 47 436, the relative frequency of

Parameter	$n = 4$	$n = 12$	$n = 24$
Baseline Simulation			
ν_{CV}	0.7	4.4	10.0
f_{CV}	0.19	0.39	0.47
n_{CV}	19 452	38 616	47 436
f_{LR}	0.89	0.83	0.84
Interval Simulation			
ν_{CV}^{int}	0.2	0.7	1.0
f_{CV}^{int}	0.19	0.39	0.47
n_{CV}^{int}	19 452	38 616	47 436
f_{LR}^{int}	0.89	0.83	0.84

Table 3.7: The Dynamics with Respect to the Rollover Frequency

covenant violation f_{CV} increases from 0.39 to 0.47. At the same time, the frequency of loan repayment f_{LR} stays almost the same and changes just slightly from 83% to 84%.

The increased frequency of covenant violations with the increased rollover frequency also supports the finding by Liang et al. (2014), who show that both probability of default due to illiquidity and total default probability increase with the increase in number of rollover dates. The authors explain it by more possibilities to run of financial institution that short-term creditors may have.

3.3.4 Decision of the Bank

As stated above, at each date the bank checks whether the covenant is violated. Suppose the covenant is violated at date t_k. In this case, the bank decides whether to waive the covenant violation or not. In case the bank decides not to waive the covenant violation, it sells the collateral (since the whole investment project value acts as a collateral). Provided the bank decides to waive the covenant violation, it may renegotiate the contract terms, including the covenant strictness. In this section we discuss different rules the bank can implement and the corresponding values of the estimations we consider with those rules in force.

Recursive Technique

As we mentioned before, the Monte Carlo simulation model developed in this chapter allows the bank to improve the methodology of risk-assessment and, consequently, the decision-making process. In order to determine the covenant strength that ensures maintaining specific risk-parameters on a certain level, we develop a recursive technique. Let's consider the probability of repayment of the loan to the bank as a risk-parameter of a particular interest for the bank (the algorithm of calculation of this particular parameter was described earlier in this chapter).

We further describe the recursive technique step by step:

- **Step 1.** Determine all necessary variables with their initial values (the initial value of the covenant index is set to 1).
- **Step 2.** Run the simulation of the process dynamics as Brownian motion for N times.
- **Step 3.** Determine the covenant threshold based on the given covenant index. Determine on which rollover dates the covenant threshold is maintained (not violated). Calculate the relative number of non violations at maturity and check whether this

ratio is greater or equal to a certain pre-specified level. If yes — stop the cycle, if not — reduce the covenant index by 0.05 and repeat Step 3 until the relative number of covenant non violations at maturity meets the requirement.

- **Step 4.** Return the resulting covenant index, the obtained relative number of covenant violations at maturity and the timing of the whole procedure.

We run the simulation for recursively obtaining covenant strength for $N = 10\ 000$ times. Assume the bank wants to maintain the probability of repayment of 85% level. As a result, we obtain the covenant strength index equal to 0.65. The calculation time of this algorithm in Mathematica 10 is around 15 seconds.

Caution Time

In practice, the bank can decide during renegotiations whether to close the contract or to waive the covenant violation. In order to investigate how the estimations of probability of covenant violations as well as the average number of violations per contract change let's implement different rules of bank behaviour.

By defining the first rule we would like to introduce the notion of a caution time (Chang and Lee, 2013). In their model for jump-diffusion process, Chang and Lee (2013) define the concept of "caution time" w in order to determine the default. In other words, the default occurs only if the firm stays below the threshold only beyond the agreed caution time period. Consequently, in our first rule we assume the default happens when the covenant is violated only after certain amount of violations in a raw.

Using this terminology, the caution time of our baseline simulation model is assumed to be $w = 0$. Now we change the caution time from 0 to 2/12. In means that with monthly rollover dates the bank waives the single month violations and counts violations occurring only two and more consecutive rollover dates. Using this rule for the bank to calculate the average number of violations per contract, the estimate drops even more, from 0.7 to 0.47. At the same time the calculation time increases for $N = 10\,000$ from about 10 seconds to 15 seconds.

Renegotiation of the Covenant Strength

It was reported in the literature that often renegotiations result in loosening the covenants (Roberts, 2015). In this subsection we'll implement the model in which the bank reduces the covenant strength each time the covenant threshold is violated. In order to investigate how the decision of the bank evolves and the estimator values change, we implement a model that allows covenant reduction after each violation. If, say, the covenant is violated at a certain rollover date t_k, the covenant index (and consequently, the covenant threshold itself) is reduced by 5% from the date t_{k+1} up to maturity T. Figure 3.1 demonstrates how the covenant threshold changes over the lifespan of the loan.

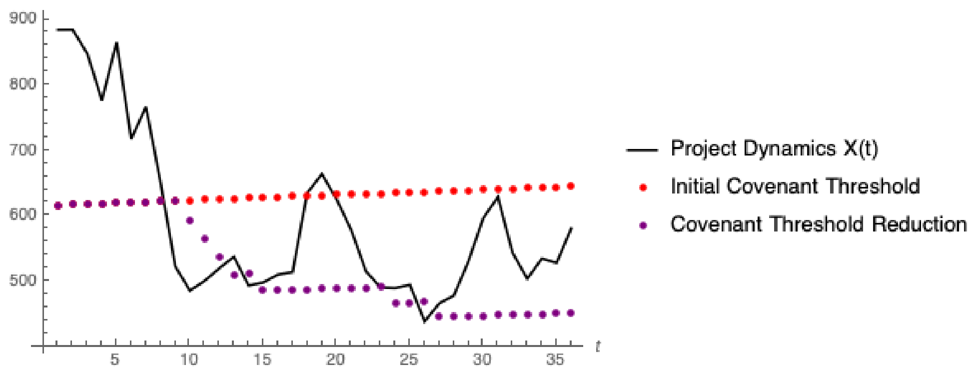

Figure 3.1: The simulation model with covenant threshold reduction in case of covenant violation

From the example on Figure 3.1 we can see how the project value evolves and how the covenant threshold changes during time. Due to the reduction in covenant strength, we observe many potential violations that the firm manages to avoid. Moreover, whereas the firm clearly defaults at maturity and violates initially set covenant, with covenant reduction the firm is able to meets the covenant at the end of the term of the loan agreement. Compared to the average number of violations per contract in a baseline simulation setting (4.4), with possibility of covenant reduction this estimation drops down to 2.2. The average probability of violation remains on the level of 0.39, as in the baseline simulation model. The calculation time increases from 10 to 12 seconds for $N = 10\ 000$ iterations.

3.4 Conclusion

In this chapter the simulation framework was build that allows us not only to assess different risk-parameters of the project, but also to implement various decision rules for a bank based on those parameters. After fixing the covenant strength index, our simulation model gives the bank the possibility to assess the risks of the project. The risks for the bank can be associated either with the covenant violation followed by costly renegotiation or with the probability of loan repayment in general. The parameters such as frequency of covenant violation or frequency of loan repayment can be used by the bank to assess those risks. If the obtained risk-profile of the project is acceptable for the bank, the chosen covenant strength may be included in the contract. Otherwise, the bank by altering the covenant strength may change the desired risk profile of the project. The sensitivity analysis is also performed in order to assess the dynamics of the simulation model with respect to key parameters.

Moreover, in this framework we implement a recursive technique for determining the covenant strength that ensures maintaining certain risk-parameter on a pre-specified level. The bank can use this technique either when signing the contract or during renegotiation process. The developed simulation framework can also allow the bank to implement different policies regarding the covenant renegotiation and assess the effectiveness of those policies. An example of reduction in covenant strength during renegotiation process is considered.

Thus, by using the simulation model the bank can improve the effectiveness of its decision making process regarding the covenant strength.

Appendices

Appendix B

Mathematica 10 Code for Simulations

Variable values definition:

```
NR = 100 000;
r = 0.0156;
T = 3;
n = 12;
m = 3;
dt = N[T/(n*m)];
sigma = 0.1125;
mu = 0.0156;
V0 = 875.8;
S0 = 755.7;
rS = 0.0572;
rho = 0.718379;
```

Covenant determination:

```
Cov = Table[NaN, {i, 1, n*m}];
Do[Cov[[i]] = E^(-r*(T - i*dt))*(S0 E^(rS T))*rho, {i, 1, n*m}];
```

The following algorithm calculates the estimates of the number of violations per contract and the average probability of violation for three cases: in the first one we calculate each violation at each rollover date, in the second case we calculate each intersection of the covenant threshold as violation, in the third case we calculate the estimates including caution time of the length 2/12 (including the calculation of the default at maturity).

Output: average number of violations per contract: every rollover date is counted; average probability of violation (relative number of contracts with violations to all contracts); number of contracts with at least one violated rollover date; average number of violations per contract: each intersection of the covenant threshold is counted; average probability of violation (relative number of contracts with violations to all contracts); number of contracts with at least one violation-intersection of covenant threshold; average number of violations per contract: each intersection of the covenant threshold is counted + caution time 2/12; average probability of violation (relative number of contracts with violations to all contracts); number of contracts with at least one violation-intersection of covenant threshold + caution time 2/12.

```
Timing[Do[
  Indsim8 = Table[NaN, {i, 1, NR}, {j, 1, n*m}];
  Indsim82 = Table[NaN, {i, 1, NR}, {j, 1, n*m - 1}];
  Indsimv8 = Table[NaN, {i, 1, NR}];
  Indsimv82 = Table[NaN, {i, 1, NR}];
  Indsimv83 = Table[NaN, {i, 1, NR}];
  V8 = Table[NaN, {i, 1, NR}, {j, 1, n*m}];
  NV = 0;
  NV2 = 0;
  NV3 = 0;
```

```
  Do[V8[[i]] =
    RandomFunction[
      GeometricBrownianMotionProcess[mu, sigma, V0], {1, n*m, 1}][[2,
     1, 1]], {i, 1, NR}];
  Do[If[V8[[i, j]] < Cov[[j]], Indsim8[[i, j]] = 1,
    Indsim8[[i, j]] = 0], {i, 1, NR}, {j, 1, n*m}];
  Do[Indsimv8[[i]] = Total[Indsim8[[i]]], {i, 1, NR}];
  Do[If[Indsimv8[[i]] > 0, NV++], {i, 1, NR}];
  Do[Indsim82[[i]] = Differences[Indsim8[[i]]], {i, 1, NR}];
  Do[Indsimv82[[i]] = Count[Indsim82[[i]], 1], {i, 1, NR}];
  Do[If[Indsimv82[[i]] > 0, NV2++], {i, 1, NR}];
  Do[Indsimv83[[i]] =
    Length[DeleteCases[Length /@
         Select[Split[Indsim8[[i]], First[#] == 1 &]),
      1]], {i, 1, NR}];
  Do[If[Indsimv83[[i]] > 0, NV3++], {i, 1, NR}];
  Do[If[Indsimv8[[i]] == 1 &&
        Last[Indsim8[[i]] == 1), NV3++], {i, 1, NR}];
  Print[N[Mean[Indsimv8], 2]];
  Print[N[NV/NR, 2]];
  Print[NV];
  Print[N[Mean[Indsimv82], 1]];
  Print[N[NV2/NR, 2]];
  Print[NV2];
  Print[N[Mean[Indsimv83], 1]];
  Print[N[NV3/NR, 2]];
  Print[NV3], {1}]]
```

The probability that the bank receives the loan back is calculated by the following algorithm:

```
Timing[Do[
  Indsim9 = Table[NaN, {i, 1, NR}, {j, 1, n*m}];
  V9 = Table[NaN, {i, 1, NR}, {j, 1, n*m}];
  NV4 = 0;
  Do[V9[[i]] =
    RandomFunction[
```

```
      GeometricBrownianMotionProcess[mu, sigma, V0], {1, n*m, 1}][[2,
     1, 1]], {i, 1, NR}];
  Do[If[V9[[i, j]] < Cov[[j]], Indsim9[[i, j]] = 1,
    Indsim9[[i, j]] = 0], {i, 1, NR}, {j, 1, n*m}];
  Do[If[Indsim9[[i, n*m]] > 0, NV4++], {i, 1, NR}];
  Print[N[1 - NV4/NR, 4]], {1}]]
```

Recursive technique of determining the optimal covenant strength that maintains certain levels of risk-parameters — probability of repayment

```
Timing[Do[
  Indsim10 = Table[NaN, {i, 1, NR}, {j, 1, n*m}];
  V10 = Table[NaN, {i, 1, NR}, {j, 1, n*m}];
  Cov10 = Table[NaN, {i, 1, n*m}];
  rho10 = 1;
  Do[V10[[i]] =
    RandomFunction[
      GeometricBrownianMotionProcess[mu, sigma, V0], {1, n*m, 1}][[2,
     1, 1]], {i, 1, NR}];
  While[True,
   NV5 = 0;
   Do[Cov10[[i]] = E^(-r*(T - i*dt))*(S0 E^(rS T))*rho10, {i, 1, n*m}];
   Do[If[V10[[i, j]] < Cov10[[j]], Indsim10[[i, j]] = 1,
     Indsim10[[i, j]] = 0], {i, 1, NR}, {j, 1, n*m}];
   Do[If[Indsim10[[i, n*m]] > 0, NV5++], {i, 1, NR}];
    If[NV5/NR <= 0.15, Break[], rho10 = rho10 - 0.05]];
  Print[rho10];
  Print[N[1 - NV5/NR, 4]], {1}]]
```

The algorithm in which the bank reduces the covenant strength each time the covenant threshold is violated

```
Timing[Do[
  Indsim11 = Table[NaN, {i, 1, NR}, {j, 1, n*m}];
  Indsim112 = Table[NaN, {i, 1, NR}, {j, 1, n*m - 1}];
```

```
Indsimv11 = Table[NaN, {i, 1, NR}];
Indsimv112 = Table[NaN, {i, 1, NR}];
Indsimv113 = Table[NaN, {i, 1, NR}];
V11 = Table[NaN, {i, 1, NR}, {j, 1, n*m}];
Cov11 = Table[NaN, {i, 1, n*m}];
Do[Cov11[[i]] = E^(-r*(T - i*dt))*(S0 E^(rS T))*rho, {i, 1, n*m}];
Cov112 = Table[NaN, {i, 1, NR}, {j, 1, n*m}];
Do[Cov112[[i, j]] = E^(-r*(T - j*dt))*(S0 E^(rS T))*rho, {i, 1,
  NR}, {j, 1, n*m}];
NV = 0;
NV2 = 0;
NV3 = 0;
Do[V11[[i]] =
  RandomFunction[
    GeometricBrownianMotionProcess[mu, sigma, V0], {1, n*m, 1}][[2,
   1, 1]], {i, 1, NR}];
Do[If[V11[[i, j]] < Cov112[[i, j]], Indsim11[[i, j]] = 1;
  Do[Cov112[[i, k]] = Cov112[[i, k]]*0.95, {k, j + 1, n*m}],
  Indsim11[[i, j]] = 0], {i, 1, NR}, {j, 1, n*m}];
Do[Indsimv11[[i]] = Total[Indsim11[[i]]], {i, 1, NR}];
Do[If[Indsimv11[[i]] > 0, NV++], {i, 1, NR}];
Do[Indsim112[[i]] = Differences[Indsim11[[i]]], {i, 1, NR}];
Do[Indsimv112[[i]] = Count[Indsim112[[i]], 1], {i, 1, NR}];
Do[If[Indsimv112[[i]] > 0, NV2++], {i, 1, NR}];
Do[Indsimv113[[i]] =
  Length[DeleteCases[Length /@
       Select[Split[Indsim11[[i]], First[#] == 1 &]),
    1]], {i, 1, NR}];
Do[If[Indsimv113[[i]] > 0, NV3++], {i, 1, NR}];
Do[If[Indsimv11[[i]] == 1 &&
      Last[Indsim11[[i]] == 1), NV3++], {i, 1, NR}];
Print[N[Mean[Indsimv11]]];
Print[N[NV/NR, 2]];
Print[N[Mean[Indsimv112]]];
Print[N[NV2/NR, 2]];
Print[N[Mean[Indsimv113]]];
Print[N[NV3/NR, 2]];
Print[NV];
Print[NV2];
```

```
Print[NV3], {1}]]
```

Chapter 4

Conclusion

In this study a theoretical framework is developed in order to determine an optimal level of covenant strength in loan contracts. The approach that utilizes dynamic contingent claim models is applied to this problem and is based on techniques developed in the derivatives pricing literature. The optimal covenant strength index is chosen as a tradeoff between the expected losses and renegotiation costs.

In order to explore our dynamic model in more realistic and accurate way, different modifications to the initial model are considered. We investigate the introduction of deadweight costs of distress or firesale price in the model as well as information asymmetry. The sensitivity analysis is performed in order to investigate the model dynamics and the behaviour of the optimal covenant strength index in financial distress. All the functions in the model are analyzed in detail with respect to key model parameters, including the drift and the volatility values, the credit amount, the market and the credit interest rates, as well as the parameters of the renegotiation costs function. In order to explore the model, numerical analysis is performed.

Moreover, we outlined how our dynamic model can be turned into a simulation procedure. We developed a Monte Carlo simulation framework that allows not only to access various risk-parameters of a project, but also to implement different decision rules for a bank. The model can support the development of more effective internal risk-management procedures for banks to assess their expected losses in order to protect themselves against incurring significant losses in cases of possible insolvencies of firms.

To summarize, the model can be used not only for explaining the differences described in the empirical studies, but also for computing the optimal covenant strength in order to minimize banks' risks and facilitate more effective monitoring.

Nevertheless, there are interesting avenues for further research. First of all, it's an important to empirically investigate and verify the proposed theoretical model for optimal covenants (both in baseline setting and under asymmetric information). One possible way of testing the model was proposed in Appendix A in the form of the regression model. Given the ambiguity of empirical findings on bank debt renegotiations and covenant amendments, testing our model empirically becomes even more important on a comprehensive data set including not only debt and firm characteristics together with renegotiations history, but also renegotiation costs and macroeconomic parameters.

Moreover, when having such a rich sample, it's also possible to apply neural network and machine learning techniques that are becoming more and more popular in finance.

There are still some possible improvements of our theoretical model, that can be implemented in order to enhance the realism of our model. First of all, it's possible to add cash amount M that the firm can also hold on the asset side and can invest at the risk-free rate r. Secondly, it may be interesting to explore our model with more complex debt structure by differentiating short-term and long-term debt. Finally, it is interesting to investigate the model behaviour under heterogeneity of borrowers. For example, we may consider high-leverage and medium leverage firms ($S0/X = 0.75$ and 0.5 respectively).

In addition, together with the general sources of information asymmetry embedded into the theoretical and empirical models of financial contracts, there is a particular type of the informational imperfection, namely on infrequent but sufficient losses caused by jumps in the value functions. The latter can be modeled by the Poisson process (Poisson random variable).

When developing our theoretical model, the assumption of rational behaviour of all agents was used. Consequently, we considered the positive values of bank expected losses, meaning that non selling the collateral is profitable for a bank. Therefore, the development of a model relaxing this assumption would be a logical extension of our model. In addition, it is interesting to apply the model to contracts in bond market.

Finally, investigation of the effect of credit constraints on real economy is an interesting avenue for further research. For example, the recent paper by Ersahin and Irani (2018) proves the increase in employment expenditures when the value of firms' collateral increases.

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