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Doctoral Thesis

Essays on Boundedly Rational Decision-making: Theory, Applications, and Experiments

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Contents

Outline of the Thesis

Standard microeconomic theory assumes that the the decision-maker (henceforth DM) is fully rational. This implies that she perceives all the given alternatives before deciding, has infinite computational capabilities, and always chooses the best alternative (alternatives) from any choice set according to an invariant preference relation. This assumption has being criticized as unrealistic and many studies have been carried out both at a theoretical and empirical level in order to provide alternative solutions (Conslik, 1996). One of the most important criticism is directed by Herbert Simon (1955; 1956), who proposes a list of simplifications in order to make the standard model more realistic. For instance, he argues that in the real world individuals do not perceive all the given alternatives before making the decision, but typically discover and analyze alternatives sequentially. Moreover, a remarkable amount of experimental evidence suggests that often subjects' behavior is not consistent with the full rationality hypothesis. Rather, experiments confirm that subjects exhibit attitudes that are closer to what Simon calls *bounded rationality* (Payne, Bettman and Johnson, 1993; Gigerenzer, Todd and the ABC Research Group, 1999; Caplin, Dean and Martin, 2009). In this thesis we analyze the impact of bounded rationality on various fields of microeconomic theory by defining boundedly rational agents as individuals who follow the so-called *satisficing heuristic* proposed by Herbert Simon. That is, individuals discover and analyze alternatives sequentially and stop searching as soon as they identify the first alternative that they judge to be satisfactory.

We first propose a theory of choice within the revealed preference approach in which the DM behaves consistently with the satisficing heuristic. We investigate the conditions under which we can infer from choices whether an alternative is considered to be satisfactory (revealed satisfaction), is revealed to attract attention (revealed attention), and is revealed to be preferred to some other alternative (revealed preference). We examine these identification issues under three different domains. Within each domain we make assumptions about what what variables are observable and what are not. For instance, if we assume that for each choice problem we can observe both choice set, choice made by the DM, and search order, then we are able to provide behavioral definitions of both satisfaction, attention, and preference. On the contrary, assuming that we can partially observe search order, then we are still able to infer satisfaction and preference, but attention only partially. We also provide an axiomatic characterization of our procedure under each domain. Finally, we analyze the relationships between our theory and a related model (Rubinstein and Salant, 2006).

We also examine what are the effects of bounded rationality on industrial organization. A notable amount of literature has been developed in this field suggesting that boundedly rational individuals are typically subject to exploitation (Spiegler, 2011). We investigate whether the fact that there is uncertainty about consumers' rationality increases their welfare. We analyze three market models: quality competition, signalling, and monopolistic screening and show that it is not obvious *a priori* whether uncertainty helps boundedly rationality. In particular, it depends on what market model we assume. For instance, uncertainty increases boundedly rational consumers' welfare in the quality competition model. On the contrary, uncertainty does not help bounded rationality under the monopolistic screening model. Moreover, the analysis of our results allows us to derive some suggestions for policy-makers.

Finally, we propose an experiment aimed at testing whether subjects' behavior is consistent with the satisficing heuristic. We ask subjects to solve a sequence of choice problems under time pressure, where each problem is represented by an incomplete algebraic sum. That is, at each problem only the result and the operators are visible to subjects that are financially incentivized to get as close as possible to the reported result by inserting numbers from a given set into the empty spaces. We record not only final choices, but also intermediate ones and decision time. We analyze the extent to which subjects commit mistakes (that is, fail to insert the combination of numbers that maximizes their material payoff) and choose sub-optimal alternatives and how allocate decision time and make intermediate choices. We show that on average subjects behavior is consistent with the satisficing heuristic. We also find that complexity has a stronger impact on subjects' performance than the variable *familiarity of the environment*. In addition, we derive some useful insights for modeling satisficing behavior. For instance, we find that the threshold that defines an alternatives to be satisfactory is not fixed, but vary across choice problems. Finally, we analyze the relationships between our experiment and a related study (Caplin, Dean and Martin, 2009).

This thesis is organized as follows. Chapter 1 introduces the topics investigated in this thesis and provides a brief review of the related literature; Chapter 2 proposes a theory of boundedly rational choice within the revealed preference approach; Chapter 3 investigates the effects of bounded rationality on industrial organization focusing on the effects of uncertainty; Chapter 4 proposes an experiment aimed at testing whether subjects' behavior is consistent with the Simon's idea of bounded rationality; Chapter 5 concludes. Proofs of proposition and theorems are given in the appendix.

Chapter 1

Bounded Rationality in Economics: Theory, Applications, and Experiments

1.1 Introduction

Standard microeconomic theory assumes that the DM is fully rational, knows perfectly the choice set before making the decision, and has infinite computational capability. This implies that she always identifies and selects the best alternative according to her preference relation. Many studies have been carried out in order to highlight the limitations of standard theory (Conslik, 1996). They challenge the assumption that the *economic man* is a realistic representation of economic agents and propose alternative solutions.

In this thesis we adopt the idea of *satisficing* proposed by Herbert Simon (1955; 1956), according to which individuals have limited computational capabilities and typically do not search for the optimum, but look for satisfactory alternatives. The goal of this chapter is to discuss the most important studies in economics that assume DMs to be boundedly rational. In particular we focus on choice theory, industrial organization, and experimental economics, topics within which we build our contribution. Section 1.2 summarizes Simon's satisficing theory and provides a brief discussion of the studies in psychology and marketing science on bounded rationality; section 1.3 proposes a review of the literature on choice theory about bounded rationality; section 1.4 discusses the most important studies in industrial organization that assume consumers to be boundedly rational; section 1.5 proposes a review of the experimental literature on boundedly rational individual decision-making; section 1.6 concludes.

1.2 Bounded vs Full Rationality

The most systematic and consistent attack to the fully rationality assumption is directed by Herbert Simon. He criticizes many aspects of standard theory and points out that in reality individuals do not have computational capabilities and access to information that the paradigm of rationality assumes. He proposes a list of possible simplifications in order to make the model more realistic. For instance, he suggests a simplified payoff function that has to be discrete rather than continuous. He illustrates a case in which the simplified function is equal either to 0 or to 1, which means that each outcome can be either 'satisfactory' (when the function is equal to 1) or 'unsatisfactory' (otherwise). Then, Simon argues that clear comparisons between alternatives are not always possible, namely payoffs are partially ordered. As an example, he reports that in some cases it is not possible to state which is the best option between two alternatives, because they may not be comparable. A further issue he addressed is that solutions to the consumer problem may not be unique or even exist. In particular, Simon argues that the assumption about the choice set is unrealistic and that individuals normally construct their choice set by discovering and exploring alternatives sequentially. Searching through the choice set, they select the first 'satisfactory' alternative, by which Simon means that people do not maximize a utility function, but look for sufficiently good solutions, that are not necessarily optimal (Simon, 1955: 104).[1] He also suggests that the aspiration level that defines an alternative to be satisfactory can be updated during the decision-making, depending on whether the searching process leads to new satisfactory alternatives or unsatisfactory ones (Simon, 1955: 104-112). In addition, he argues that the structure of the environment plays a central role in permitting further simplifications of the standard model. For instance, if the environment is relatively simple and familiar to the DM, then it is likely that she will end up choosing an alternative relatively closer to the optimum. On the contrary, if the environment is relatively complex and unfamiliar, then most probably the DM will be content with an alternative relatively far from the optimum (Simon, 1956).

The pioneer work by Simon gave rise to the concept of *bounded rationality* and opened a new research field. A huge number of studies have been carried out in both marketing, psychology, and economics.

If, on the one hand, the bounded rationality approach offers a list of advantages including a more realistic and adequate representation of economic agents, then, on the other hand, the neo-classical framework has served as useful tool for understanding and interpreting various relevant phenomena that occur in the real world. Before moving to the next section we provide a couple of examples in which standard theory does a good job in explaining observed behaviors.

Plott and Uhl (1981) simulate a market in the laboratory. Sellers and buyers were put into two separated rooms, could not communicate, and were given marginal cost function and reservation price, respectively. At every period four subjects, called *middlemen* (speculators), had first to enter sellers' room and buy inventories. After this first market, called A, was closed, middlemen had to move to the buyers' room and sell the products they previously bought from sellers. This market is called B. Both markets A and B were organized as an oral double auction. After market B was closed, mid-

[1] If DMs are particularly lucky, then the satisfactory solution is also optimal. This procedure has been called *satisficing heuristic*.

dleman returned to market A and a new period started. Products could not be carried forward to subsequent periods. If sellers and buyers were *maximizing* individuals and correctly estimated the probability of sales in market B, then prices in the two markets should have been the same and attained the level at which demand and supply intersect. Moreover, middlemen profits should have been zero. Plott and Uhl (1981) show that what happened in these markets is very well predicted by standard theory, including equilibrium price and the number of objects traded. It was as if middlemen did not exist and sellers and buyers were in the same market.

There is extensive evidence that suggests that subjects are altruistic. Consider the so-called *dictator game* according to which an individual, called dictator, receives an amount x of money. She has to decide how much of this amount to give to another player called *receiver*. Let g be this amount, with $g \in [0, x]$. Standard theory predicts that $g = 0$, because it is not rational to donate any amount of money. A notable amount of evidence has shown that typically a substantive fraction of dictators (around 60%) donate the 20% of amount x. This finding is clearly in contrast with standard theory. Cherry, Frykblom and Shogren (2002) design a dictator game that involves two stages. In the first stage dictators had to answer to some questions taken from a graduate admission test. After the test they were given an amount x of money that was linked with the performance in the test. That is, the greater the number of correct answers, the greater amount x. Cherry, Frykblom and Shogren (2002) show that the fact that dictators have to earn money makes them very selfish. In addition, if the experiment is made fully anonymous then altruistic behavior almost disappears.

1.2.1 Marketing Science

A consumer in a supermarket is typically confronted with hundreds of products (Schwartz, 2005). If she was fully rational, then she would consider all of them and pick the best product according the her preferences.

A well-established result in the marketing science literature is that consumer's behavior is not consistent with the full rationality assumption. Rather, the consumer typically follows a two-step procedure, in which in the first phase she restricts her attention on a subset of the available products, which is called *consideration set*. Then, she picks the best product from the consideration set according to her preferences. The way in which the consideration set is constructed does not necessarily depend on consumer's preferences.

There is strong evidence of this two-stage procedure (Wright and Barbour, 1977; Alba, Hutchinson and Lynch, 1991; Roberts and Lattin, 1997; Shum, 2004). Lussier and Olshavsky (1979), for instance, investigate how task complexity affects brand choice strategy. They find that at more complex problems subjects typically first eliminate alternatives that are judged to be unacceptable by means of a noncompensatory decision strategy. A strategy is noncompensatory whenever the DM does not make trade-offs between attributes. Then, among the remaining ones she chooses according to a compensatory decision strategy, according to which compensation between attributes takes place.

Hoyer (1984) studies laundry detergent purchases and finds that once that consumers identify the relevant shelf in the supermarket, the median number of products that they examine is one. In addition, unless promoted, consumers typically do not consider superior brands displayed on the same shelf, even though the products are new.

Finally, it is worth mentioning van Nierop et al. (2010), who propose a probabilistic model aimed at inferring the consideration set from household panel data.

1.2.2 Psychology

In psychology there are two main research programs aimed at studying individual decision-making: the *heuristics-and-biases* program initiated by Daniel Kahneman and Amos Tversky and the *fast-and-frugal-heuristic* program proposed by Gerd Gigerenzer, Peter Todd, and the ABC Research Group. These programs follow two distinct approaches.

The goal of the heuristics-and-biases program heuristics is to provide evidence that supports the hypothesis that individuals' lack of rationality leads them to behave in way that is not consistent with the laws of logic and probability. In other words, the use of heuristics allows individuals to save cognitive effort, but, on the other hand, implies that they do not behave according to the normative theory. As an example consider the well-known Monty Hall game (Chugh and Bazerman, 2007: 10-11).[2] Monty asked a participant to choose one of three doors. Behind one of these, there was the grand prize and behind the other two there were small prizes. In the 'Monty always opens' condition, Monty always opened an unchosen door and offered to the participant the possibility to change her previous decision. In this case the winning strategy, which is computed by using the Bayes' rule, is always to switch. In the 'Mean Monty' condition, Monty could decide either to end the game, or to open an unchosen door and offer the possibility to switch and his goal was to minimize the probability of winning. In this second case, the optimal strategy is not to switch. Clearly, the condition under which this game is played is crucial to calculate the optimal strategy. Nevertheless, there is extensive evidence suggesting that in the large majority of the cases participants decided not to switch.

On the other hand, the fast-and-frugal-heuristic program defines an heuristic to be accurate depending on how well subjects' behavior is predicted by the heuristic. Therefore, according to this approach the extent to which choices made according to an heuristic deviate from what the normative theory prescribes is not a matter of interest. As an example, consider Rieskamp and Hoffrage (1999), who examine experimentally adaptivity in individual decision-making in multi-attribute framework by analyzing both process and outcome data. They find that subjects are adaptive and willing to save cognitive effort and that, under time pressure, fast and frugal heuristics, such as

[2] The game is based on the American television game called 'Let us make a deal', whose first host was Monty Hall (from 1963 to 1977, and subsequently from 1980 to 1981 and from 1984 to 1986).

the lexicographic one, are much more accurate then the normative theory.

Analyzing the debate between these two schools is beyond the scope of this thesis. As to what concerns us, it is worth underlying that both programs agree on the assumption that DMs use simple heuristic to make decisions and a huge literature has been developed on boundedly rational individual decision-making (Kahneman and Tversky, 1974; 1979; Payne, Bettman and Johnson, 1993; Gigerenzer, Todd and the ABC Research Group, 1999). In addition, both approaches propose theories of choice in which they concept of consideration set is employed.

Tversky (1972), one of the founders of the heuristic-and-biases program, proposes a theory of choice called *elimination by aspects*. The idea is that a good may or may not include an "aspect".[3] At each stage of the choice process an aspect is selected with a certain probability, depending positively upon its weight. Given a certain choice of an "aspect", all alternatives that do not have the selected aspect are eliminated. This procedure goes on until one element remains (Tversky, 1972: 284-289). An interpretation of this process is that a good is defined as a bundle of characteristics (or attributes), a cutoff level is set for each characteristic, and attributes are ranked from the most to the least important.[4] Then, all elements that do not satisfy the cutoff level of the most important attribute are eliminated. This procedure goes on with the second most important attribute, with the third, and so on, until one alternative remains. The idea is that, if a good does not satisfy the cutoff

[3]Tversky (1972: 285) provides the following example: assume that a decision-maker wants to purchase a car. If the price of the car the decision-maker is considering is greater or equal than \$3000, then that car does not possess the aspect "it costs less than \$3000".

[4]This decision strategy is called *characteristic-based search* (CBS) because decision-makers compare attributes *across* alternatives, such as cost, weight, color, and speed. The idea is that decision-makers analyze the value of a single attribute of various alternatives before examining the next attribute. The *alternative-based search* decision strategy (ABS), instead, refers to a model in which decision-makers compare attributes *within* alternatives, that is they analyze multiple attributes of a given alternative before examining the next alternative (Payne, Bettman and Johnson, 1993: 29 and 31). The Elimination By Aspects procedure is a CBS noncompensatory decision strategy.

level of an attribute, then it does not posses an “aspect” (Payne, Bettman and Johnson, 1993: 27).

On the other hand, Gigerenzer and Goldstein (1996a), introduce the so-called *take-the-best heuristic*, which is a generalization of the well-known *lexicographic* heuristic. Attributes are ranked according to their validity, that is, according to how often each attribute has indicated that an option is correct or not. The DM compares alternatives according to the most valid attribute and eliminates all alternatives that are dominated on the selected attribute. Then, she examines the second most valid attribute and again eliminates all alternatives that are dominated on the selected attribute. This procedure goes on until one alternative is left.

1.2.3 Economics

There is a huge literature in economics about bounded rationality. Studies have been carried out in various fields, such as decision-making under uncertainty (Starmer, 2000), intertemporal choice (Frederick, Loewenstein and O'Donoghue, 2002), finance (Shleifer, 2000), other-regarding preferences (Rabin, 1993).[5]

The next three sections provide a review of the literature about bounded rationality in economics, focusing on choice theory, industrial organization, and experimental economics, respectively. The remaining part of this section proposes a brief history of the revealed preference theory, whose approach is employed in the second chapter.

Brief History of Revealed Preference Theory

Economists have developed two distinct approaches for investigating individual decision-making: the preference-based and the choice-based (revealed-preference). The former assumes that consumer preferences are primitive and imposes requirements of rationality on them. Then, given consumer

[5]See also Rubinstein (1998).

preferences, the consequences for choices are analyzed. In the choice-based approach, choices are treated as primitive and assumptions about consumer behavior are made.

There are several factors that make the second approach more appealing. First, the choice-based approach can incorporate a more general kind of choice behavior. Second, unlike the preference-based approach, the revealed preference one makes assumptions on observable objects (choices). Therefore, a theory of individual decision-making based on the the choice-based approach can have a behavioral foundation (Mas-Colell, Whinston and Green, 1995: 5).

The concept of *revealed preference* appears for the first time in Samuelson (1938). Paul Samuelson (1938) was critical about the preference-based approach. He thought that making certain assumptions about consumer behavior, such as increasing rate of marginal substitution, is an *ad hoc* strategy for explaining the kind of demand that is observed in markets. He was also concerned about the concept of utility that, in his view, cannot be defined as an entity independent of psychological elements. For this reason, he proposed an alternative approach in which the primitive elements of the theory are observable and independent of any introspective factor. This novel approach was not in contrast with the existing one. Rather Samuelson initiated a research field aimed at providing a behavioral foundation of the theory of consumer's behavior.

The first definition of *revealed preference* is contained in Samuelson (1938) and reads as follows (Varian, 2006: 2).[6]

Definition 1 (Revealed Preference) *Given some vectors of prices and chosen bundles* (p^t, x^t) *for* $t = 1, \ldots, T$, *we say that* x^t *is directly revealed preferred to a bundle* x $(x^t R_D x)$ *if* $p^t x^t \geq p^t x$. *We say that* x^t *is revealed preferred to* x $(x^t R x)$ *if there is some sequence* $r, s, t, \ldots, u, v$ *such*

[6] The initial terminology was *selected over*. Richter (1966) pointed out that *selected over* would have been better than *revealed preference*, because the former has the advantage that it avoids a circular definition of preference. See also Varian (2006).

that $p^r x^r \geq p^s x^s$, $p^s x^s \geq p^s x^t$, $\ldots$, $p^u x^u \geq p^u x$. *In this case,* R *is the transitive closure of* R_D.

The intuition behind this definition is the following. Consider two vector of prices and chosen bundles (p^t, x^t) and (p, x). The expenditures for (p^t, x^t) and (p, x) are $p^t x^t$ and px, respectively. Now, consider the vector (p^t, x), whose expenditure is $p^t x$. If $p^t x \leq p^t x^t$, then this implies that bundle x was available when x^t was chosen. However, the consumer did not choose to do so. Therefore, one can conclude that x^t was *selected over* x. That is, x^t was *revealed to be preferred to* x (Samuelson, 1938: 64-65).

Samuelson then introduced a consistency property on individual behavior that later became the well-known *Weak Axiom of Revealed Preference* (WARP).

Definition 2 (Weak Axiom of Revealed Preference) *If* $x^t R_D x^s$, *then it is not the case that* $x^s R_D x^t$

The axiom says that if some bundle x^t is chosen when another bundle x^s is available, then it cannot happen that x^s is chosen over x^t.

Ten years later Samuelson, motivated by the work of Little (1949), proposed a method for reconstructing indifference curves from the revealed preference relation. However, the proof was mainly graphical and for only two goods. Houthakker (1950), then, realized that in order to generalize Samuelson's result to n goods an extension of the concept of revealed preference from *direct* to *indirect* was necessary. He proposed the following condition.

Definition 3 (Strong Axiom of Revealed Preference) *If* $x^t R x^s$, *then it is not the case that* $x^s R x^t$.

Subsequently, Rose (1958) proved rigorously that Strong Axiom and Weak Axiom are equivalent in the two-commodity case.

The theory of consumer behavior based on the revealed preference approach was almost brought to completion with Samuelson (1953), even though not completely rigorous. Later studies, such as Newman (1960) and Uzawa (1960), added more formalism to the existing analysis. Finally, Marcel Richter (1966) provided pure set-theoretic foundations to the problems of integrability (recovering a utility function, given a demand function) and representability (recovering the preference relation that induces a given demand function). Formally, a budged space is a pair $\langle X, \mathcal{P}(X)\rangle$, where X is the grand set of alternatives (or set of bundles) and $\mathcal{P}(X)$ is a family of non-empty subsets (or budgets) of X. A DM (or a consumer) is a function that assigns to each $A \in \mathcal{P}(X)$ a non-empty subset $c(A) \subseteq A$. The interpretation is that $c(A)$ is the set alternatives chosen by the DM subject to the budget A (Richter, 1966: 635).[7]

Besides Richter (1966) many other studies have been carried out both at a theoretical and empirical level (Koo, 1963; Afriat, 1965; Sondermann, 1982; Varian, 1982; Andreoni and Miller, 2002). As to what concerns us in this study, it is worth mentioning the contributions by Kenneth Arrow (1959) and Amartya Sen (1971). The former proved that WARP is a necessary and sufficient property for the observed choice behavior to be rationalized by a rational preference relation. The latter showed that WARP is logically equivalent to two basic properties, called property α and β.[8]

1.3 Revealed Preference and Bounded Rationality

The key-concept in choice theory is the one of *rationalizability.* A choice function is rationalizable whenever there exists a preference relation such that the chosen alternatives from any given choice set are the best alterna-

[7]It is possible to impose $c()$ to be a singleton, in which case indifference is ruled out.

[8]Property α: given an alternative $x \in A \subseteq B$, if x is chosen from the big set B, then x has also to be chosen from the small set A. Property β: given two alternatives $x, y \in A \subseteq B$, if x is chosen from the small set A and y is chosen from the big set B, then x has to be chosen from the big set B (Sen, 1971: 313).

tives in that set according to that relation. The assumption at the basis of microeconomic theory is that individual preferences are rational. A preference relation is rational whenever it is complete and transitive. Completeness implies that, given two alternatives, the DM is always able to state whether she prefers the first to the second one, the second to the first one, or whether she is indifferent between the two. Transitivity, on the other hand, rules out cyclical preferences. For instance, if Beatles are judged to be at least as good as Rolling Stones and Rolling Stones at least as good as Doors, then, by transitivity, it cannot be that Doors are strictly preferred to Beatles. As we know from the previous section, Kenneth Arrow (1959) proved that WARP is a necessary and sufficient condition for a choice function to be rationalized by a rational preference relation, provided that the budget set contains all subsets of grand set up to three elements. Arrow's result is very important because it implies that any choice-theoretic model that departures from WARP does not assume full rationality.

In what follows we discuss the most relevant boundedly rational models in choice theory according to the kind of non-standard rationality that they assume. Kalai, Spiegler and Rubinstein (2002) observe that often in real-life situations WARP is violated. They argue that one possible explanation is that the DM does not use a single preference relation, but several, each applied to a subset of the grand set. Their interpretation is that each choice set encompasses information about its elements and by considering this information, the DM chooses what she judges to be the best alternative according to the appropriate rationale. Kalai, Spiegler and Rubinstein (2002) formalize this idea and focus on choice functions that employ the minimal number of rationales. Manzini and Mariotti (2007) propose a model in which the DM still uses multiple rationales, but, unlike Kalai, Spiegler and Rubinstein (2002), they assume that the DM applies them sequentially according to a given order. The so-called *Rational Shortlist Method* is a choice function sequentially rationalized by two rationales. The idea is that the DM follows a two-step procedure in which in the first phase she eliminates all alternatives that are dominated according to the first rationale. Then, among any remaining ones, she selects the best alternative according to the second rationale. Notice that the set of alternatives that survives the first stage can be interpreted as a consideration set, because it is from that restricted set that the DM makes her final decision. One of the major strengths of the rational shortlist method is that it can explain irrational choice patterns, such as cycles. Another study that employs the idea of sequential rationalizability is Apesteguia and Ballester (2009*a*), who provide a characterization of the class of models in which the DM sequentially applies a finite list of rationales. In addition Apesteguia and Ballester relate their model to other well-known boundedly rational procedures.

A notable amount of evidence suggests that subjects exhibit the so-called *status-quo bias* (Samuelson and Zeckhauser, 1988). That is, subjects tend to pay particular attention to a default option and evaluate it positively relative to other alternatives. Masatlioglu and Ok (2005) axiomatically characterize a model that incorporates standard choice theory as a special case in which the DM is affected by the status-quo bias.[9] According to this model the DM evaluates alternatives by means of various criteria. If she faces a choice problem without status-quo, then she selects the best alternatives according to an aggregator of these criteria. On the contrary, if she confronts a choice problem with status-quo, then she sticks with it, unless there are some alternatives that dominate the status-quo in all dimensions, in which case she chooses according to the aggregator of the above criteria. In the same spirit Apesteguia and Ballester (2009*b*) characterize a class of models in which the DM exhibits reference-dependent behavior.

Framing effects occur whenever the way in which the same problem is presented to the DM affects choices (Tversky and Kahneman, 1981; Kahneman and Tversky, 1984). Salant and Rubinstein (2008) propose a framework in which the choice function depends not only on the choice set, but also on some observable information, called *frame*, which is irrelevant from a stan-

[9]See also Masatlioglu and Ok (2009) for an extension of Masatlioglu and Ok (2005).

dard viewpoint. They investigate the conditions under which their model is equivalent to standard maximization and discuss the limitations of the model of choice correspondence when the DM is affected by framing effects. Bernheim and Rangel (2007) independently develop a similar model called *choice with ancillary conditions* aimed at analyzing welfare implications of bounded rationality.

Mandler, Manzini and Mariotti (2010), inspired by Tversky (1972) and others, propose a choice procedure according to which the DM makes a decision by using a checklist. That is, she has in mind a list of ordered properties and first eliminates all alternatives that do no possess the first property. Then, among the remaining ones, she eliminates all alternatives that do not possess the second property and so on until the survivor set does not shrinks anymore. Mandler, Manzini and Mariotti interestingly show that this procedure is equivalent to standard maximization. Manzini and Mariotti (2010*b*) extend checklists by assuming that the order with which the DM goes through properties can vary depending on the DM's mood.[10]

Simon (1956) argues that the structure of the environment plays a central role in determining the extent to which the standard framework can be simplified. In particular, if the environment is particularly complex, then it is likely that the DM will encounter relatively more difficulties in searching for a best alternative. Tyson (2008) addresses this issue and proposes a model in which the DM does not fully perceive her preferences by assuming that this bias increases in the complexity of the choice problem. He shows that, when complexity it is aligned with set inclusion (nestedness of preferences), then his choice procedure is equivalent to Sen (1971)'s β property.[11]

The concept of consideration set has attracted a lot of attention among choice theorists. Besides Manzini and Mariotti (2007), many other studies

[10]Another extension of Mandler, Manzini and Mariotti (2010) is provided by Manzini and Mariotti (2010*a*), who develop a theory of choice called *choice by lexicographic semiorders*.

[11]Property β: given two alternatives $x, y \in A \subseteq B$, if $x \in C(A)$ and $y \in C(B)$, then $x \in C(B)$. See section 1.3.

have been carried out in this field. Masatlioglu and Nakajima (2009) propose a model called *choice by iterative search* in which the DM starts by analyzing an exogenously given contemplation point, considers only those alternatives that are similar to it and, by means of an iterative search process, stops searching as soon as the contemplation point is the best available alternative in the consideration set. Masatlioglu, Nakajima and Ozbay (2009) propose a model in which the DM still picks the best alternative among those considered. However, unlike Masatlioglu and Nakajima (2009), they assume that the consideration set has the property that the removal of an alternative that the DM does not consider does not change the consideration set. Eliaz, Richter and Rubinstein (2011) axiomatically characterize three different procedures by which a consideration set can be constructed.

Simon (1955) criticizes the assumption that the DM perceives all alternatives before deciding. He argues that it would be more realistic to assume that the DM discovers and analyzes alternatives sequentially. In the second chapter of this thesis we address this issue and propose a model in which the DM behaves according to the satisficing heuristic. The closest studies to our model are Rubinstein and Salant (2006) and Caplin and Dean (2011).

Rubinstein and Salant (2006) propose a model in which the DM faces lists of alternatives and behaves according to the procedure $\langle R, \delta \rangle$, where R is a rational preference relation and δ is a priority indicator. Specifically, the DM selects either the first or the last maximal element from any list according to the preference relation R, depending on whether the priority indicator is equal to 1 or 2, respectively.

On the other hand, Caplin and Dean (2011) model a reservation-based search decision strategy by formalizing the concept of *choice process data*. That is, the DM picks the best alternative among the ones she has already explored at any given point in time and stops searching as soon as she identifies the first alternative that yields at least the reservation utility, given that searching is costly.

In the second chapter we discuss in detail the relationships between our model and Rubinstein and Salant (2006) and Caplin and Dean (2011).

1.4 Industrial Organization and Bounded Rationality

Plenty of studies have been carried out in order to model boundedly rational DMs.[12] More recently, a remarkable amount of literature has developed on applications of theories of boundedly rational choice to various economic fields. For instance, the analysis of welfare under the assumption that the DM is boundedly rational has attracted a lot of attention (Bernheim and Rangel, 2007; Apesteguia and Ballester, 2010; Rubinstein and Salant, 2010). In the third chapter we apply bounded rationality to another important area of economics: industrial organization (IO).

This research field is relatively new, as one of the pioneer studies was conducted by Glenn Ellison and Drew Fundenberg in 1993. Ellison and Fundenberg (1993) investigate the extent to which spread and dispersed information about product quality is aggregated at the population level and affects firms' decision about technology by assuming that boundedly rational players have limited capacity of perceiving what technology yields the highest payoff.[13] Despite its relatively young age, this research field has attracted a lot of attention and many studies have been carried out. Ellison (2006) proposes a nice survey in which he identifies three distinct traditions aimed at analyzing the relationships between bounded rationality and IO. The first one is called *rule-of-thumb* approach. This tradition, rather than characterizing equilibrium behavior, assumes that economic agents behave in some simple way. The second one is called *explicit bounded rationality* approach, assumes that cognition is costly, and derives second-best behaviors, given the costs. The third one models economic agents as individuals subject to biases

[12] See section 1.3.

[13] Ellison and Fundenberg (1995) propose a closely related model aimed at investigating how the structure of information flows affects the learning process.

typically detected in experimental economics and experimental psychology. The goal of this section is to discuss the most important studies in this field related to our work.

Standard contract theory typically assumes that individuals differ in preferences or cost parameters. Eliaz and Spiegler (2006), instead, assume that individuals have dynamically inconsistent preferences and are heterogeneous in the sense that they have different abilities in forecasting potential changes in their future tastes. For instance, more sophisticated types are more capable of perceiving changes in their preferences. Eliaz and Spiegler (2006) find that more naive individuals are typically subject to an higher exploitation and are associated with higher profits for the principal. Moreover, they use their results to interpret real-life contractual agreements in a several industries.[14] Non-standard preferences are also assumed in Spiegler (2010) that investigates optimal pricing in a monopolistic setting, where individuals are loss-averse according to a reformulation of the model by Koszegi and Rabin (2006).

There are other studies in the IO literature that support the hypothesis that boundedly rational consumers are subject to exploitation. Spiegler (2006*b*), for instance, defines boundedly rational consumers as individuals that follow an 'anedoctal' kind of reasoning. He shows that even market interventions aimed at pushing out low-quality firms do not improve welfare. Spiegler (2006*a*), instead, proposes a market model in which profit-maximizing firms compete in a multidimensional pricing framework by defining boundedly rational consumers as individuals that have limited capacity of understanding complex objects. In that model each firm to an increase in competition responds by putting in practice a confounding strategy rather than a strategy of more competitive prices. Finally, Rubinstein and Spiegler (2008) investigate vulnerability to exploitation of boundedly rational invididuals that have to decide whether or not to buy a lottery.

The idea of consideration set has attracted a lot of attention also in the

[14]See also Eliaz and Spiegler (2008).

field of bounded rationality and IO. Eliaz and Spiegler (2009), for instance, assumes that two firms compete in order to maximize market shares in a context in which consumers suffer from the effects of marketing. In particular, consumers apply their well-defined preferences only to the alternatives that belong to the consideration set, which, in turn, can be manipulated by firms by means of marketing strategies.[15] Another study that employs the idea of consideration set is Piccione and Spiegler (2010) that extend Bertrand competition by modeling a two-firm market in which the consumer is affected by framing effects. Firms have to decide not only the price of its product, but also a pricing structure, which is called *format*. The consumer is assigned randomly to one firm and the probability that she examines the other firm's product depends on the formats.

Finally, it is worth pointing out that Ran Spiegler is writing a book titled *Bounded Rationality and Industrial Organization* (Spiegler, 2011) in which he summarizes his main results in this field.[16]

1.5 Experiments on Individual Decision-making and Bounded Rationality

There is a huge experimental literature about boundedly rational individual decision-making both in economics and psychology. The large majority of these studies have in common a methodological feature: most of them make use of data enrichment techniques. That is, not only final choices are recorded, but also other variables of interests, such as decision time and intermediate choices, are taken into account. The idea is that those additional data can shed light on the process that leads the DM to make her decision.

Payne, Bettman and Johnson (1993) use verbal protocols and mouseLab to test adaptivity in individual decision-making. Verbal protocols require subjects to write down what they are thinking during the decision process.

[15]See also Eliaz and Spiegler (2010) for an extension of Eliaz and Spiegler (2009).

[16]This book should be available starting from January 2011.

MouseLab, instead, implies that subjects are presented on their screen with black boxes, behind which information is hidden. To access, subjects have to click on the box that they want to open. MouseLab allows experimenters to know how many times and how long subjects have looked at each piece of information. A similar analysis is performed by Rieskamp and Hoffrage (1999; 2008), who use mouseLab to test adaptivity under low and high time pressure. Those studies find that subjects change decision strategies as task complexity increases; that is, decision-makers are adaptive and willing to conserve cognitive effort.

MouseLab is also employed by Gabaix et al. (2006), who analyze aggregate information acquisition patterns in multi-attribute and multi-alternative choice problems (maximize an algebraic sum). What they find is that the directed cognition model (Gabaix and Laibson, 2005) seems to fit the experimental data. In the directed-cognition model DMs do not explore all possible alternatives, but only those that have the highest ratio of benefit to cost. Furthermore, when their model and the fully rational one differ, the directed-cognition model better predicts subjects' information acquisition patterns.

Reutskaja et al. (2010), instead, use eye-tracking to investigate consumer search dynamics in a context characterized by time pressure. Eye-tracking records eye movement images and pupil dilatation by means of a camera placed on the computer screen. Eye movement images allow experimenter to infer subject information acquisition patterns, whereas pupil dilatation yields information about arousal, pain, and cognitive difficulty. In that experiment subjects have to choose among snack food items. Reutskaja et al. (2010) show that first subjects tend to choose the optimal alternative among the discovered ones. Second, search behavior is compatible with an hybrid of the optimal search and the satisficing model. Third, subjects seem to search more often in certain areas of the monitor.

Another study that uses eye-tracking is Arieli, Ben-Ami and Rubinstein (2010) that tests whether subjects use an ABS or CBS decision strategy while choosing binary lotteries.[17] Arieli, Ben-Ami and Rubinstein (2010) find that whenever the computation of the expected value is difficult, subjects tend to use a CBS decisions strategy. Differently, when computations are easier, subjects behavior is consistent with an hybrid of a CBS and ABS decision strategy.

In the fourth chapter we propose an experiment aimed at testing whether subjects behavior is consistent with the satisficing heuristic. The closest study to our experiment is Caplin, Dean and Martin (2009).

Caplin, Dean and Martin (2009) propose an experiment in which they analyze the source of choice errors by using a data enrichment technique called *choice process data*. Subjects have to choose the highest sum of money expressed in terms of a sum of natural numbers. According to choice process data, not only final choices but also intermediate ones that change with the contemplation time are recorded. Caplin, Dean and Martin (2009) show that subjects behavior is consistent with the satisficing procedure proposed by Herbert Simon (1955).

The relationships between our experiment and Caplin, Dean and Martin (2009) are discussed in detail in chapter 4.

1.6 Concluding Remarks

This chapter provides a review of the literature in economics about bounded rationality focusing on choice theory, industrial organization, and experimental economics.

Equipped with the tools provided in this chapter, one should be able to better understand not only the content of the coming chapters, but also what is our contribution relative to the existing literature.

[17] ABS and CBS stand for alternative-based and characteristic-based, respectively. See section 1.2.2 for definitions.

Chapter 2

Satisficing Choice Procedures

2.1 Introduction

The problem of choice is the basis of microeconomic theory. Standard models of decision theory assume that a choice problem is composed of two elements: a family $\mathcal{P}(X)$ of nonempty subsets of the grand set of alternatives X and a choice function $c : \mathcal{P}(X) \to X$. The problem of choice entails picking one element from a choice set $A \in \mathcal{P}(X)$ that is known in advance (Richter, 1966; Mas-Colell, Whinston and Green, 1995).

This framework has been criticized as unrealistic. According to Herbert Simon,

> in most global models of rational choice, all alternatives are evaluated before a choice is made. In actual human decision-making, alternatives are often examined sequentially. We may, or may not, know the mechanism that determines the order of procedure. When alternatives are examined sequentially, we may regard the first satisfactory alternative that is evaluated as such as the one actually selected. (Simon, 1955: 110)

Plausibly, in the real world the DM does not evaluate all alternatives before making the decision. Rather, she discovers and explores new alternatives while the choice-process is taking place. We propose a model within the revealed preference approach in which we assume that the DM discovers and analyzes alternatives sequentially. There are various circumstances in the real world in which alternatives are presented to the DM in form of a sequence. For instance, alternatives may be disposed horizontally on some shelf and the DM examines them from left to right. In this case there is a spatial constraint that prevents the DM from perceiving all alternatives before deciding. Alternatively, the DM may receive job offers sequentially in time. There are also circumstances in which constraints are neither spatial nor temporal, but the DM examines alternatives sequentially, because alternatives come to the DM's mind according to some ordering. As an example think of a person who has to decide where to spend her summer vacation. Typically, an individual does not have in mind a complete set of potential destinations. Rather, she discovers and analyzes new options by searching on the web and referring to travel agencies while the decision process is taking place.[1]

We assume that the DM behaves according to the well-known *satisficing heuristic* (Simon, 1955). That is, she explores alternatives sequentially, stops searching as soon as she identifies the first satisfactory alternative, and selects the best alternative among those discovered. If there is no satisfactory alternative in the choice set, then the DM reconsiders all alternatives and picks the best available one. Of course there are other ways in which one could model satisficing behavior. For instance, suppose that the DM is particularly *lucky* and identifies the first satisfactory alternatives in relatively little time. Then one could assume that since she has exerted little cognitive effort to identify the first acceptable alternative, then she can manage to keep searching in order to find better alternatives. This would mean to design a model in which the DM does not necessarily stop as soon as the first acceptable alternative is discovered, but the extent to which she keeps searching depends on the length of the search history.

[1]See Rubinstein and Salant (2006) for further discussion and examples.

We investigate the conditions under which we can infer from choices whether or not an alternative is revealed to be satisfactory (revealed satisfaction), to attract attention (revealed attention), and to be preferred to some other alternative (revealed preference). We explore these issues under three different domains. We first assume that, for each choice problem, we can observe not only the choice set and the choice made by the DM, but also the entire menu sequence according to which she examines alternatives. In this case we are able to provide behavioral definitions of both attention, satisfaction, and preference. In addition we show that our procedure is equivalent to an axiom called *Full Attention WARP*, which is a weakening of standard WARP. In particular Full Attention WARP requires the DM to choose consistently with standard WARP restricted to those alternatives to which she pays attention. Our model includes the standard maxization procedure as a special case, because if the DM pays attention to all alternatives at all choice problems, then Full Attention WARP reduces to WARP.

Under the second domain, we assume that we cannot observe the entire menu sequence, but only its first stage. In this case we provide behavioral definitions of satisfaction and preference and we show that attention can be inferred only partially. We also axiomatically characterize our procedure by using a slightly modified version of Full Attention WARP.

Finally, we assume that the order according to which the DM examines alternatives is unobservable. Under this domain we show that we can infer satisfaction and also preference, but only over unsatisfactory alternatives. In addition we demonstrate that our model is a special case of the standard maximization procedure.

The model is related to the literature about attention and consideration set (Manzini and Mariotti, 2007; Eliaz and Spiegler, 2009; Masatlioglu, Nakajima and Ozbay, 2009).[2] The closest studies to our work are Rubinstein and Salant (2006) and Caplin and Dean (2011). Rubinstein and Salant (2006) develop a model called *choice function from lists* in which they assume that the

[2]See sections 1.3 and 1.4 for more details.

DM chooses from sequences of alternatives. Unlike the proposed model, they assume that the DM necessarily examines alternatives one by one. The relationships between our model and Rubinstein and Salant (2006) are analyzed in detail in a section below.

Caplin and Dean (2011) propose a reservation-based search decision strategy by formalizing the concept of *choice process data*. That is, the DM picks the best alternative among the ones she has already explored at any given point in time and stops searching as soon as she identifies the first alternative that yields at least the reservation utility, given that searching is costly. Unlike their model, we do not make use of choice process data, but consider only final choices and assume that search order is observable or partially observable. We also investigate the case in which search order is unobservable as Caplin and Dean (2011) do, but the two models still differ because unlike them we assume that in this case the choice correspondence records the choices made by the DM under multiple menu sequences.

The chapter is organized as follows: Section 2.2 develops the formal model; Section 2.3 provides an axiomatic characterization of the model and behavioral definitions of revealed satisfaction, attention, and preference under three different domains; Section 2.4 investigates the relationship between the our model and and Rubinstein and Salant (2006). Section 2.5 concludes.

2.2 The Model

Let X be a finite grand set of alternatives, where $\mathcal{P}(X)$ represents the set of all non-empty subsets of X. As we know from chapter 1, in standard choice theory a choice problem is simply a choice set $A \in \mathcal{P}(X)$. In this framework we define an extended choice problem as a pair $(A, \{A_j\})$, where $A \in \mathcal{P}(X)$ is a choice set and $\{A_j\}$ represents a sequence with which the DM examines the alternatives in A. We call $\{A_j\}$ *menu sequence* and define it as follows.

Definition 4 *A* menu sequence *of the set $A \in \mathcal{P}(X)$ is a sequence $\{A_j\}_1^N$ such that $A_j \subseteq A_k \subseteq A$, for all $k > j$ and $A_N = A$.*

We call each element A_j of the menu sequence $\{A_j\}$ *stage* of the menu sequence $\{A_j\}$.

Let an extended choice function be a choice function defined on the domain $\mathcal{D}_1 = \{(A, \{A_j\}) | A \in \mathcal{P}(X)$ and $\{A_j\}$ is a menu sequence of $A\}$. We assume that $c(A, \{A_j\})$ is non-empty and picks one alternative from A.

Let $\succ$ be a strict linear order on the set X representing DM's preferences.[3] Let $x^s \in X$ be a distinguished alternative (or an aspiration level) such that all alternatives to which x^s is $\succ$-preferred are considered to be unsatisfactory. Let $UC_{\succ}(A; x^s) = \{x \in A | \neg(x^s \succ x)\}$ be the upper-contour set of x^s according to $\succ$, representing the set of satisfactory alternatives available in the set A.[4] Let $\max(A; \succ) = \{x \in A | \nexists y \in A \text{ s.t. } y \succ x\}$ be the set of maximal alternatives according to the relation $\succ$ in the set A. Since $\succ$ is assumed to be a linear order, then the set $\max(A; \succ)$ is always a singleton.

Definition 5 *c is a* Satisficing Choice Function *(SCF) if and only if there exist a strict linear order $\succ$ on X, an alternative $x^s \in X$, and a consideration set mapping $\Gamma_{(A,\{A_j\})} \subseteq A$ such that*

$$\{c(A, \{A_j\})\} = \max(\Gamma_{(A,\{A_j\})}; \succ)$$

where

$$\Gamma_{(A,\{A_j\})} = \begin{cases} A_{\overline{j}} & \textit{if } UC_{\succ}(A; x^s) \neq \emptyset \\ A & \textit{otherwise} \end{cases}$$

and $\overline{j} = \min\{j | A_j \cap UC_{\succ}(A; x^s) \neq \emptyset\}$.

[3] A binary relation $\succ$ is irreflexive whenever $(x, x) \notin \succ$. Given $x, y, z \in X$, a binary relation $\succ$ is transitive, whenever $(x, y) \in \succ$ and $(y, z) \in \succ$ imply that $(x, z) \in \succ$. Given $x, y \in X$ such that $x \neq y$, a binary relation $\succ$ is complete whenever either $x \succ y$ or $y \succ x$. Given $x, y \in X$, a binary relation $\succ$ is asymmetric whenever $(x, y) \in \succ$ implies that $(y, x) \notin \succ$. A strict linear order is a transitive, asymmetric, irreflexive, and complete binary relation.

[4] Since $x^s \in X$, then we assume that $x \in UC_{\succ}(A; x^s)$ whenever $x \in A$ and $\neg(x^s \succ x)$ because we want also the aspiration level x^s to be part of the upper-contour set at all $A \in \mathcal{P}(X)$ such that $x^s \in A$.

Our interpretation is that the DM has in mind a preference relation $\succ$ and a distinguished alternative x^s. She judges satisfactory those alternatives that are at least as good as x^s and unsatisfactory those that are worse than x^s. The DM searches through the choice set A according the menu sequence $\{A_j\}$ and if she identifies at least one satisfactory alternative in the first stage, then she stops searching at A_1, which becomes the consideration set, and selects the $\succ$-maximal alternative available in A_1. Otherwise, she explores the second stage. Then, again if she identifies at least one satisfactory alternative in the second stage, then she stops searching at A_2, which becomes the consideration set, and selects the $\succ$-maximal alternative available in A_2. Otherwise, she keeps searching and the procedure is the same as before. If there is no satisfactory alternative in the choice set A (i.e., $UC_{\succ}(A; x^s) = \emptyset$), then she selects the $\succ$-maximal unsatisfactory element from A. In this case the consideration set coincides with the choice set A.

2.2.1 Explained Behaviors

The proposed procedure can be defined also in terms of choices with frames (Salant and Rubinstein, 2008).[5] Let a frame be weak order O on X, representing an ordering with which the DM examines alternatives. We identify the choice function with frames $c_O(A)$ with the extended choice function $c(O|A)$, where $O|A$ is the menu sequence induced by restricting the order O to the choice set $A \in \mathcal{P}(X)$. An SCF, which we denote by $c_{SCF}(O|A)$, can then be defined as:

$$c_{SCF}(O|A) = \max(A; \succ_O)$$

where $\succ_O \equiv (\succ \setminus \{(x,y) \in \succ \,|x,y \in UC_{\succ}(X;x^s)\}) \cup \{(x,y) \in \succ \,|x,y \in UC_{\succ}(X;x^s) \text{ s.t. } xOy \text{ and } (\neg(yOx) \text{ or } x \succ y)\}$

Clearly, any $c_{SCF}(O|A)$ satisfies WARP. Therefore, violations of WARP can take place only when the DM mixes two or more frames. Since the

[5] See section 1.3.

proposed model does not place any restriction on menu sequences, then this implies that an SCF can explain a variety of irrational choice patterns. The next examples illustrate that the proposed procedure can explain cyclical choice patterns and menu effects.

Example 1 (Cycles) *Let $x, y, x \in UC_{\succ}(X; x^s)$ and assume $z \succ x \succ y$. Let $(A, \{A_j\})$, $(B, \{B_j\})$, and $(C, \{C_j\})$ such that $A_1 = \{x\}$, $y \in A$, $B_1 = \{y\}$, $z \in B$, $C_1 = \{z\}$, and $x \in C$. Since the DM stops searching as soon as she identifies the first satisfactory alternative, then $x = c(A, \{A_j\})$, $y = c(B, \{B_j\})$, and $z = c(C, \{C_j\})$. This choice pattern clearly exhibits a cycle.*

Example 2 (Menu Effects) *Assume that $x, y \in UC_{\succ}(X; x^s)$ and $x \succ y$. Let $(A, \{A_j\})$, $(B, \{B_j\})$, and $(C, \{C_j\})$ such that $A_1 = A = \{x, y\}$, $B_1 = B = \{x, z\}$, $C_1 = \{y, z\}$, and $C = \{x, y, z\}$. If c is an SCF, then $x = c(A, \{A_j\})$, $x = c(B, \{B_j\})$, and $y = c(C, \{C_j\})$. This choice pattern exhibits menu effects, because x is chosen in binary comparison over y and over z. However, when the choice set encompasses both x, y, and z, the DM chooses y.*

Despite this feature, the satisficing procedure cannot explain any choice pattern, as the next example illustrates.

Example 3 (Violations of Satisficing) *Assume that $x \in UC_{\succ}(X; x^s)$. Let $(A, \{A_j\}) \in \mathcal{D}_1$ be such that $A_1 = \{x\}$. Since x is satisfactory, then the DM must stop searching at A_1 and chooses x. Therefore, any c that does not select x from $(A, \{A_j\})$ is not an SCF.*

2.3 Axiomatic Characterization

Suppose that we observe the DM making choices. Our concern is to find behavioral definitions of satisfaction (x^s), attention ($\Gamma_{(A,\{A_j\})}$), and preference ($\succ$). Moreover, we are interested in identifying the conditions under which her behavior is consistent with the satisficing procedure. We explore these identification issues under three different domains.

2.3.1 Full Domain

Under the full domain $\mathcal{D}_1$, we assume that, for every choice problem, we can observe not only the choice set and the choice made by the DM, but also the entire menu sequence.

Assume that c is an SCF. How can we tell whether an alternative is satisfactory of not? A satisficing DM always stops searching as soon as she identifies a satisfactory alternative. Therefore, it must be that if an alternative x is satisfactory, then the DM never discovers x and then explores further the menu sequence. In other words, if x is acceptable, then it cannot happen that the DM discovers and chooses some alternative y at some stage of the menu sequence, given that at an earlier stage she explored x.

Formally, define $A_m^{c(A,\{A_j\})}$ as the first stage of the menu sequence $\{A_j\}$ to which the chosen alternative $c(A,\{A_j\})$ belongs, where $m = \min\{j : c(A,\{A_j\}) \in A_j\}$. If x is acceptable, then it cannot happen that $x \in A_{m-1}^{c(A,\{A_j\})}$. The next proposition states that this condition is not only necessary, but also sufficient for x to be satisfactory.

Proposition 1 (Revealed Satisfaction) *Suppose that c is an SCF. Then, $x \in UC_{\succ}(X; x^s)$ if and only if there exists no $(B, \{B_j\}) \in \mathcal{D}_1$ such that $x \in B_{m-1}^{c(B,\{B_j\})}$. Whenever this occurs we say that x **is revealed to be satisfactory**.*

Next, we analyze the conditions under which we can unambiguously state what alternatives are considered by the DM. Assume the DM chooses some alternative y from $(A, \{A_j\})$ and we are instersted in verifying whether she considers x. If $x = y$, then obviously she considers x. Next, assume that $x \neq y$ and that $x \in A_m^{c(A,\{A_j\})}$. Then, once again we can conclude that the DM considers x. The reason is that since y is chosen, then it must be that all alternatives that precede y in the sequence (or are discovered at the same time as y) are considered. Finally, assume that $x \notin A_m^{c(A,\{A_j\})}$. This implies that the chosen alternative y precedes x in the menu sequence. The only way in which the DM can explore x is that the chosen alternative y is not a satisfactory alternative. We already know, by proposition 1, that this occurs whenever there exists a choice problem $(B, \{B_j\})$ such that $y \in B_{m-1}^{c(B,\{B_j\})}$. Whenever one of these three cases occur we say that x *attracts attention at* $(A, \{A_j\})$. The next proposition states this concept formally and provides a behavioral definition of revealed attention.

Proposition 2 (Revealed Attention) *Suppose that c is an SCF. Then, $x \in \Gamma_{(A,\{A_j\})}$ if and only if either*

1. $y = c(A, \{A_j\})$ *and* $x \in A_m^{c(A,\{A_j\})}$ *or*

2. $y = c(A, \{A_j\})$ *and there exists a* $(B, \{B_j\}) \in \mathcal{D}_1$ *such that* $y \in B_{m-1}^{c(B,\{B_j\})}$.

Whenever this occurs we say that x ***attracts attention at*** $(A, \{A_j\})$.

We are also interested in investigating whether, given two alternatives $x, y \in X$, x is revealed to be preferred to y or vice versa. Assume that $x = c(A, \{A_j\})$ and that $y \in A$. This information is not enough for ensuring that $x \succ y$. To see why, assume that y is preferred to x, $x = c(A, \{A_j\})$, and $y \in A$. This does not lead to a contradiction, because it might have happened that the DM stopped searching before exploring y and chose x. Therefore, in order to conclude that $x \succ y$ we must also require that y attracts attention at $(A, \{A_j\})$. The next proposition states this result formally.

Proposition 3 (Revealed Preference) *Suppose that c is an SCF. Then, $x \succ y$ if and only if there is some $(A, \{A_j\}) \in \mathcal{D}_1$ such that $x = c(A, \{A_j\})$ and y attracts attention at $(A, \{A_j\})$. Whenever this occurs we say that x* ***is revealed to be preferred to*** y.

Now we move to the axiomatic characterization.

Standard WARP requires that given two alternatives $x, y \in A \cap B$, if $x = c(A)$, then $y \neq c(B)$ without distinguishing between alternatives that the DM considers and alternatives that she does not. We introduce a weaker version of this property, which we call Full Attention WARP. This property requires the extended choice function to satisfy WARP, provided that the DM pays attention to x and y at both choice problems according to the definition provided in proposition 2.[6]

Full Attention WARP (FAWARP). Assume that x and y attract attention at $(A, \{A_j\})$ and $(B, \{B_j\})$. Then, if $x = c(A, \{A_j\})$, then $y \neq c(B, \{B_j\})$.

The main result of this section is stated in the following theorem.

Theorem 1 *An extended choice function c is an SCF if and only if c satisfies Full Attention WARP.*

Equipped with the results of this subsection, we are able to infer non-parametrically whether DM's behavior is consistent with the satisficing procedure simply by testing the axiom that characterizes the SCF. In addition, by using propositions 1, 2 , and 3 we can infer both revealed satisfaction, attention, and preference.

Finally, notice that if x attracts attention at $(A, \{A_j\})$ for any $x \in A$ for all $(A, \{A_j\}) \in \mathcal{D}_1$, then FAWARP reduces to WARP and c is rationalizable by a strict linear order. It is easy to see that this happens if and only if $\{x^s\} = \max(X; \succ)$. That is, a necessary and sufficient condition for the extended

[6]Masatlioglu, Nakajima and Ozbay (2009) introduce an axiom called 'WARP with Limited Attention'. The difference between FAWARP and WARP(LA) is in the definition of 'paying attention to'. Masatlioglu, Nakajima and Ozbay (2009) define a set to be a consideration set whenever the removal of one alternative that the DM does not consider does not change the consideration set. On the contrary, in this paper the consideration set is defined according to the definition of proposition 2.

choice function to be equivalent to standard maximization is that the minimal satisfactory alternative is maximal in the grand set. This observation is intuitive: given that the DM is satisfied only when she chooses a satisfactory alternative, if there is only one satisfactory alternative in the grand set, then it is as if the DM got satisfied only at the optimum.

2.3.2 Partially Observable Menu Sequences

There are circumstances in which assuming that we can always observe the entire menu sequence for any choice problem is too demanding. For instance, consider a consumer who analyzes the items allocated on a certain shelf. It is hard to assume that the order with which she examine alternatives is exogenously given. She may explore items from left to right or form to top to the bottom. She may even mix multiple search heuristics, in which case the above assumption would hardly hold. On the contrary, it seems more realistic to assume that we can identify the set of items at which she starts her search process. In the above example the observable starting point could be the set of items placed at the head of the shelf that the consumer necessarily considers.

In order to capture this kind of situations we introduce the domain $\mathcal{D}_2 = \{(A, A_1)|A_1 \subset A \in \mathcal{P}(X)\}$. We interpret (A, A_1) as a choice problem, where A is the choice set and A_1 is the first (observable) stage of the menu sequence. The remaining stages (if any) are assumed to be unobservable. The next definition formalizes the satisficing procedure on the domain $\mathcal{D}_2$.

Definition 6 *c is a* Satisficing Choice Function under $\mathcal{D}_2$ *(SCF$_2$) if and only if there exist a strict linear order $\succ$, an alternative x^s, and a consideration set mapping $\Gamma_{(A,A_1)} \subseteq A$ such that*

$$\{c(A, A_1)\} = \max(\Gamma_{(A,A_1)}; \succ)$$

where

$$\Gamma_{(A,A_1)} = \begin{cases} A_1 & \textit{if}\, UC_{\succ}(A_1; x^s) \neq \emptyset \\ \bar{A} & \textit{if}\, UC_{\succ}(A \setminus A_1; x^s) \neq \emptyset = UC_{\succ}(A_1; x^s) \\ A & \textit{otherwise} \end{cases}$$

and $A_1 \subset \bar{A} \subseteq A$.

Our interpretation of the SCF_2 is analogous to the SCF's. The only difference is that if the DM does not find any satisfactory alternative in the first stage of the menu sequence, then she keeps exploring the next stages until she identifies a satisfactory alternative, in which case she stops searching and selects the $\succ$-maximal alternative. Since all stages of the menu sequence apart from the first one are unobservable, then in this case the consideration set is given by $\bar{A}$, where $\bar{A}$ is some subset of A such that $A_1 \subset \bar{A} \subseteq A$.

Now we move to the behavioral definitions of satisfaction, attention, and preference. The definitions of revealed satisfaction and preference are analogous to the ones of previous section.

Proposition 4 (Revealed Satisfaction) *Suppose that* c *is an* SCF_2*. Then,* $x \in UC_{\succ}(X; x^s)$ *if and only if there exists no* $(A, A_1) \in \mathcal{D}_2$ *such that* $x \in A_1$ *and* $c(A, A_1) \notin A_1$*. Whenever this occurs we say that* x ***is revealed to be satisfactory****.*

Proposition 5 (Revealed Preference) *Suppose that* c *is an* SCF_2*. Then,* $x \succ y$ *if and only if there is some* $(A, A_1) \in \mathcal{D}_2$ *such that* $x = c(A, A_1)$ *and* $y \in A_1$*. Whenever this occurs we say that* x ***is revealed to be preferred to*** y*.*

It is worth observing that in order to infer $\succ$, we have to required $|A_1| > 1$ for some suitable $(A, A_1) \in \mathcal{D}_2$. To see why, suppose not and assume that all alternatives in the grand set are satisfactory. In this case the DM always stops searching at the first stage and selects its alternative. This implies that it is impossible to infer whether $x \succ y$ for all $x, y \in X$. On the contrary, revealed satisfaction can be inferred even without this assumption.

Providing a behavioral definition of revealed attention is more complicated, because some stages of the menu sequence are not observable. In particular we can infer that x is part of the DM's consideration set only if two cases occur. First, either x belongs to the first stage of the menu sequence or x is chosen. Second, the chosen alternative is revealed to be unsatisfactory and therefore the DM considers all alternatives in the choice set, including x. These conditions are not necessary for x to be part of the DM's consideration set, because it can happen that y is satisfactory and $x \notin A_1$, but $x \in \bar{A}$. In this case x is part of the consideration set. However, since $\bar{A}$ is not observable, it is impossible to infer from choices whether this occurs or not. The next proposition summarizes this observation.

Proposition 6 (Revealed Attention) *Suppose that c is an SCF$_2$. Then, $x \in \Gamma_{(A,A_1)}$, only if either*

1. $x \in A_1 \vee x = c(A, A_1)$ *or*
2. $y = c(A, A_1)$ *and there exists a* $(B, B_1) \in \mathcal{D}_2$ *such that* $y \in B_1$ *and* $c(B, B_1) \notin B_1$.

However, in order to provide an axiomatic characterization of the SCF$_2$ we need a definition of attention. The only case in which we have problems in inferring attention is when the chosen alternative is satisfactory and $x \notin A_1$. In this case x is part of the DM's consideration set if and only if $x \in \bar{A}$, where $\bar{A}$ is unobservable. To solve this problem we use the following trick: assume that the chosen alternative is revealed to be satisfactory and $x \notin A_1$. Then, we assume that x attracts attention at (A, A_1) whenever x is never revealed to be preferred to the chosen alternative. In this way, even though x was not part of the DM's consideration set, assuming it to be considered does not affect neither revealed preference nor revealed satisfaction, because we require that x is always revealed to be inferior to y. The next definition formalizes this idea.

Definition 7 *We say that x **attracts attention** at (A, A_1) whenever*

1. $x \in A_1 \vee x = c(A, A_1)$ *or*

2. $x \notin A_1$, $y = c(A, A_1) \in A_1$, *and there is some* $(B, B_1) \in \mathcal{D}_2$ *such that* $y \in B_1$ *and* $c(B, B_1) \notin B_1$.

3. $x \notin A_1$, $y = c(A, A_1) \notin A_1$, *and there is no* $(C, C_1) \in \mathcal{D}_2$ *such that* $y \in C_1$ *and* $x = c(C, C_1)$.

The axiom that characterizes the SCF_2 is very similar to FAWARP. The difference is that this version of FAWARP makes use of the concept of attention provided in definition 7.

Full Attention WARP under $\mathcal{D}_2$ (FAWARP_2). Assume that x and y attract attention at (A, A_1) and (B, B_1). Then, if $x = c(A, A_1)$, then $y \neq c(B, B_1)$.

Theorem 2 *An extended choice function c is an SCF_2 if and only if c satisfies $FAWARP_2$.*

We believe that this is an interesting result, because requiring to observe much less data than under $\mathcal{D}_1$, we are still able to characterize the proposed procedure and provide behavioral definitions of satisfaction and preference. The only sacrifice, as proposition 6 suggests, is that under the domain $\mathcal{D}_2$ we are able to infer attention only partially.

2.3.3 Unobservable Menu Sequences

There are also circumstances in which it is also hard to assume that the first stage of the menu sequence is observable. Consider a consumer who searches for a new t-shirt in a market place. Assuming that in this case there is a set of items that she necessarily considers appears to be unrealistic. Since there are many entries to the market place, there are multiple ways in which she can starts analyzing the products. In order to capture this kind of situations we introduce another domain, $\mathcal{D}_3$, in which we assume that for each choice problem we do not observe menu sequences, but only the choice set and the choice made by the DM, as in the standard model.

Let $\mathcal{D}_3 = \mathcal{P}(X)$. Let $C(A) \subseteq A$ be a choice correspondence defined on $\mathcal{D}_3$. For each $A \in \mathcal{P}(X)$, let $\mathcal{A}$ be the set of menu sequences of the choice set A. We define C to be a *Satisficing Choice Correspondence* whenever C records the choices made by the DM who follows the proposed procedure under multiple menu sequences. The next definition expresses this idea formally.

Definition 8 *C is a* Satisficing Choice Correspondence *(SCC) if and only if*

$$C(A) = \bigcup_{\{A_j\} \in \mathcal{A}} c(A, \{A_j\})$$

and $c(A, \{A_j\})$ is an SCF.

We first investigate the conditions under which we can unambiguously state that an alternative is satisfactory. If an alternative x is acceptable, then there is always a menu sequence at which x is chosen. This implies that x is chosen for all $A \in \mathcal{D}_3$ such that $x \in A$, or, equivalently, $x \in C(X)$. The next proposition states that this condition is not only necessary, but also sufficient for x to be a satisfactory alternative.

Proposition 7 (Revealed Satisfaction) *Suppose that C is an SCC. Then, $x \in UC_{\succ}(X; x^s)$ if and only if $x \in C(X)$. Whenever this occurs we say that x* ***is revealed to be satisfactory****.*

Proposition 7 suggests that an alternative is satisfactory whenever it is chosen from all choice sets to which it belongs. Suppose that a choice set contains at least two satisfactory alternatives. By proposition 7, all satisfactory alternatives have to be chosen. This implies that under this domain we cannot identify what is the ranking between two satisfactory alternatives. On the contrary, we can infer preferences over unsatisfactory alternatives. Assume that $A = \{x, y\}$ and assume that x and y are revealed to be unsatisfactory. In this case, independently of the menu sequence, the DM always pays attention to both alternatives. Therefore, if she prefers x to y, then $\{x\} = C(A)$. Conversely, if she prefers y to x, then $\{y\} = C(A)$. Proposition 8 generalizes this idea and provides a behavioral definition of preference over unsatisfactory alternatives. Let $\succ_{\neg S}$ be the restriction of $\succ$ to unsatisfactory alternatives.

Proposition 8 (Revealed Preference) *Suppose that c is an SCC. Then, $x \succ_{\neg S} y$ if and only if there is some $A \in \mathcal{P}(X)$ such that $\{x\} = C(A)$ and $y \in A$ and some $B \in \mathcal{P}(X)$ such that $x \notin C(B)$.*

Now we move to the axiomatic characterization.

We propose two properties. The first one is the well-known Weak Axiom of Revealed Preference. The second one is called *Maximal Indifference* and requires that if and alternative x is jointly chosen with another alternative y, then x has to be chosen from all choice sets to which x belongs. The idea behind this axiom is that y represents a kind of reference alternative, where what makes y special is the fact that it is chosen with another alternative. If x is chosen when also y is chosen, then this means that also x is a kind of superior alternative. Therefore, it has to be always chosen.

Weak Axiom of Revealed Preference (WARP). Given $x, y \in A \cap B$, if $x \in C(A)$ and $y \in C(B)$, then $x \in C(B)$.

Maximal Indifference (MI). If $x, y \in C(A)$, then $x \in C(B)$ for all $B \in \mathcal{P}(X)$ such that $x \in B$.

In appendix A we show that WARP and Maximal Indifference are independent. The next proposition establishes that these two properties are necessary and sufficient for C to be an SCC.

Proposition 9 *C is an SCC if and only if it satisfies WARP and Maximal Indifference.*

We draw two conclusions from this subsection. First, as proposition 9 suggests, the SCC model is a special case of standard maximization. In particular C is not rationalizable by any weak order, but only by the weak order that admits indifference only among maximal alternatives. Second, by assuming that for each choice problem we cannot observe the menu sequence, but only the choice set and the choice made by the DM, we are still able to infer revealed satisfaction and revealed preference below the threshold.

2.4 Relationships with Choice From Lists

Rubinstein and Salant (2006: 5) propose a model called *choice from lists* in which the DM does not perceive all alternatives in the choice set before deciding, but examines alternatives one by one. Formally, given any two stages A_j, A_{j+1} of a menu sequence $\{A_j\}$, $\{A_j\}$ is defined to be *linear* whenever $|A_j| = |A_{j+1}| - 1$ for all $j = 1, \ldots, N-1$. Let $\mathcal{D}_L \subset \mathcal{D}_1$ be the domain $\mathcal{D}_1$, where all menu sequences in $\mathcal{D}_1$ are linear. A Rubinstein and Salant's list is a linear menu sequence.

Rubinstein and Salant (2006: 6-10) characterize the set of choice functions that maximize some weak preference relation, where indifference is resolved according the position that alternatives take up in the lists: either the first or the last maximal alternative is selected. Formally, let R be a preference relation on X and let $\delta : X \to \{1, 2\}$ be a priority indicator, where $\delta(x) = \delta(y)$ whenever xIy. Given any list $(A, \{A_j\}) \in \mathcal{D}_L$, let $D_{R,\delta}$ be a choice function that chooses from A either the first or the last R-maximal element of the list $\{A_j\}$ depending on whether the δ-value of the R-maximal set is 1 or 2, respectively. Rubinstein and Salant (2006) show that an axiom called *List Independence of Irrelevant Alternatives* is necessary and sufficient for the choice function from lists to be a $D_{R,\delta}$.[7]

The next theorem establishes the relationship between the choice function from lists $D_{R,\delta}$ and the satisficing procedure proposed in this chapter.

Let $\mathcal{I}_{(R;\setminus \max)}$ be the set of all indifference classes in the grand set X according to the relation R apart from the class $\max(X; R)$.

Theorem 3 *If the extended choice function c is an SCF, then the restriction of c to $\mathcal{D}_L$ is a $D_{R,\delta}$. Moreover, a $D_{R,\delta}$ can be extended to an SCF if and only if $\delta = 1$ and the sets in $\mathcal{I}_{(R;\setminus \max)}$ are singletons.*

This result provides new insights into the link between the satisficing principle and the choice function from lists $D_{R,\delta}$. Rubinstein and Salant (2006: 8) argue that $D_{R,\delta}$ is consistent with it when R induces two indifference sets of satisfactory and unsatisfactory alternatives being the δ-value equal to 1 and 2 for satisfactory and unsatisfactory alternatives, respectively. That is, the DM chooses the first satisfactory alternative, if any, from any list. Otherwise, she chooses the last alternative. An alternative interpretation that emerges from Theorem 2 is that there can be more than two indifference sets, provided that the non-maximal ones are singletons, all R-maximal alternatives in the grand set X are satisfactory and, since $\delta = 1$, the DM stops searching as soon as she identifies the first one that she encounters in the

[7] List Independence of Irrelevant Alternatives: assume that $x = c(A, \{A_j\})$. Then, removing any alternative $y \neq x$ from A does not change the choice, *ceteris paribus* (Rubinstein and Salant, 2006: 6-7).

list and chooses it. If there is no satisfactory alternative, then she selects the unique R-maximal one in the list.

The procedure proposed by Rubinstein and Salant (2006) encompasses a more general class of models than ours. We limit ourselves to model the satisficing heuristic. On the contrary, as explained in the previous paragraph, Rubinstein and Salant (2006) propose a procedure that includes the satisficing heuristic as a special case. Nevertheless, there are several aspects of our model that makes it complementary to theirs.

First, we generalize list to menu sequences. That is, we do not assume that the DM necessarily examines alternatives one by one. There are many real world examples that fit within our model, but do not within theirs. For instance, assume that a person receives job offers sequentially in time. It may happen that she receives two or more offers simultaneously. Alternatively, think of a driver that has to go past a sequence of junctions. At any junction the driver compares two alternatives simultaneously. In addition, there is extensive evidence from experimental psychology that the visual field may include more than one item. For instance, Pylyshyn and Storm (1988) provide evidence that suggests that subjects are able to track up to a subset of 5 items from a set of 10 identical randomly-moving objects in order to distinguish a change in a target from a change in a distractor.[8]

Second, unlike Rubinstein and Salant (2006), our model allows for inferring preferences above the threshold x^s, not only under $\mathcal{D}_1$, but also when the domain is $\mathcal{D}_2$ and only the first stage of the menu sequence is assumed to be observable.

In order to make the analysis of the relationships between our model and Rubinstein and Salant (2006) even richer further work could be done. On the one hand, it would be interesting to characterize the SCF by using List Independence of Irrelevant Alternatives. On the other hand, a generalization of their results to menu sequences would shed further light on what are the

[8]Further evidence is provided by Pylyshyn and Annan (2006) and Franconeri et al. (2008).

implications of assuming sequences rather than lists.

2.5 Concluding Remarks

This chapter proposes a theory of boundedly rational choice within the revealed preference approach in which the DM's behavior is consistent with the satisficing heuristic. We believe that this chapter has four main strengths.

First, we axiomatically characterize our model and investigate behavioral definitions of satisfaction, attention, and preference under three different domains. We think that this provides a rich and complete analysis, because it allows us to understand how much of DM's behavior we can infer depending on how many data we can observe. Our results are summarized in table 2.1.

	Domains		
	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$
Rev.Sat.	yes	yes	yes
Rev.Att.	yes	partially	no
Rev.Pref.	yes	yes	only below x^s
Axioms	FAWARP	FAWARP$_2$	WARP and MI

Figure 2.1: A Summary of the Results

Under $\mathcal{D}_1$ we assume that for each choice problem we can observe not only the choice set and the choice made by the DM, but also the entire menu sequence. Under this domain we are able to infer both satisfaction, attention, and preference and we show that Full Attention WARP is equivalent to the proposed procedure, which incorporates standard maximization as a special case. If we assume that we cannot observe the whole menu sequence, but only the first stage, then we are able to fully infer satisfaction and preference, but attention only partially. In this case FAWARP$_2$ is shown to be equivalent to our model. Finally, under $\mathcal{D}_3$ we assume that search order is unobservable. In this case we are able to infer satisfaction and preference, but only below the threshold x^s. We also show that under $\mathcal{D}_3$ our procedure is a special case of standard maximization. In fact it is equivalent to WARP and Maximal Indifference.

Second, we generalize the framework proposed by Rubinstein and Salant (2006). Moreover, we show that even though their procedure encompasses a more general class of models than ours there are some feature that make our model complementary to theirs. For instance, unlike Rubinstein and Salant (2006), our model allows us to infer preferences above the threshold not only under $\mathcal{D}_1$, but also under $\mathcal{D}_2$.

Third, the issue of inferring the consideration set has become increasingly important and studies have been carried out in various fields, such as marketing science and psychology.[9] For instance, Masatlioglu, Nakajima and Ozbay (2009) approach the problem from a theoretical perspective and van Nierop et al. (2010) propose a probabilistic model by using household panel data. We provide a method for inferring the consideration set when DM's behavior is consistent with the satisficing heuristic.

Fourth, there is extensive evidence suggesting that often subjects do not behave as if they were fully rational. Rather, as if they used simple heuristics to make decisions (Payne, Bettman and Johnson, 1993; Gabaix et al., 2006).[10] Moreover, several experimental studies support the hypothesis that subjects' behavior is consistent with the satisficing heuristic. For instance, Caplin, Dean and Martin (2009) propose an experiment in which they analyze the source of choice errors by using choice process data and find that that subjects behavior is consistent with a reservation-based model of sequential search. Reutskaja et al. (2010) use eye-tracking to investigate consumer search dynamics in a context characterized by time pressure. They show that subjects tend to choose the optimal alternative among the discovered ones and that search behavior is compatible with an hybrid of the optimal search and the satisficing model.

Our work could be further extended by assuming that the threshold is

[9]See sections 1.3, 1.4, and 1.5.

[10]See section 1.5.

not fixed, but can vary depending on the complexity of the choice problem. The more complex the problem, the more the threshold depreciates. On the contrary, the simpler the problem, the more the threshold appreciates. We think that this would make the model more realistic.

Chapter 3

Does Uncertainty Help Bounded Rationality? An IO study

3.1 Introduction

Standard decision theory assumes that the decision-maker is fully rational. That is, she knows all alternatives in the choice set before deciding and always picks the best alternative according to her preference relation. However, many experimental studies have shown that often decision-makers are not *maximizers*, but use simple heuristics to make decisions.[1] In response to this growing literature, theorists have proposed new models that assume decision-makers to be boundedly rational.[2] We believe that the natural following step is to apply the assumption of bounded rationality to more concrete economic problems.

We decided to analyze the impact of bounded rationality on industrial organization (IO) for two main reasons. First, as Spiegler (2011: 4) argues,

[1]See section 1.5.
[2]See section 1.3.

IO is a very important area of economics.[3] Second, in recent years an increasing number of studies have been carried out in this field (Eliaz and Spiegler, 2009; Piccione and Spiegler, 2010; Spiegler, 2011).

As Ran Spiegler (2011) points out,

> bounded rationality is another potential source of market friction. When some agents have limited understanding of their market environment (including their own behavior in certain circumstances), limited ability to process information, and preferences that are highly unstable, context-dependent and malleable, market outcomes may differ in interesting and economically significant ways from the rational-consumer benchmark. Moreover, introducing boundedly rational agents into our market models may challenge conventional wisdom regarding the welfare properties of market interactions (Spiegler, 2011: 2).

We focus on analyzing the effects of uncertainty on bounded rationality. We define a consumer to be boundedly rational whenever she behaves consistently with the 'satisficing' heuristic (Simon, 1955). That is, she discovers and analyzes alternatives sequentially and stops searching as soon as she identifies the first alternative that she judges to be acceptable. In contrast, a fully rational consumer knows all alternatives before deciding and always picks the best available good.

We expect that if firms know with certainty the consumer's type, then they will exploit this informative advantage to maximize profits by always supplying an optimal alternative to the fully rational consumer and the minimal acceptable alternative to the boundedly rational one. The hypothesis that boundedly rational consumers are subject to exploitation is well-documented in the literature (Spiegler, 2006*b*;*a*; Rubinstein and Spiegler, 2008).[4] We are interested in investigating whether the fact that firms are

[3] Spiegler (2011) has not been published yet, but the introduction is available on-line at **h**ttp://www.tau.ac.il/ rani/briocontents.pdf.

[4] See also section 1.4.

uncertain about the consumer's type increases consumers welfare. That is, whether uncertainty induces firms to supply the optimal alternative, given that there is a positive probability that consumers are boundedly rational. We believe that it is worth performing such an analysis because it might lead to interesting policy implications.

We first investigate a model of quality competition. A store sells $n \geq 2$ products and is restocked by n firms. Each firm produces one good and has to decide its quality. The higher the quality of a good, the higher its production cost. Whereas fully rational consumers know all alternatives that are sold in the store in advance and always pick the best alternative, boundedly rational ones examine the products sequentially and stop searching as soon as they identify the first satisfactory alternative. We show that if consumers are fully rational then in equilibrium firms supply the good with the highest quality. On the contrary, if consumers are boundedly rational, firms supply the minimal satisfactory alternative, which can be non-optimal. We find that under uncertainty firms supply the optimal product only if the probability that the consumer is fully rational is above some threshold. If the threshold is not met, firms supply some intermediate alternative between the optimal and the minimal satisfactory one. In this model uncertainty makes the boundedly rational consumer better off.

Secondly, we analyze a strategic interaction between a consumer and a firm that play a sequential game. The consumer has to decide first whether or not to enter the firm. Then, if the consumer decides to enter, the firm can choose to supply either an optimal or a satisfactory product, provided that the production cost of the former is greater than the latter's. We assume that the fully rational consumer buys only optimal products, whereas the boundedly rational one buys also satisfactory goods. The main result is that the fully rational consumer is better off when there is certainty about consumers' rationality, because in equilibrium she always gets the optimal good. On the contrary, when there is uncertainty, she gets either the optimal product or she does not buy anything. The boundedly rational consumer, instead, is better off under uncertainty because if the posterior probability that the consumer is fully rational is sufficiently high, then she gets the optimal product in equilibrium.

Thirdly, we investigate how bounded rationality affects the equilibrium outcome in a model of monopolistic screening. A monopolist supplies a two-attribute good. There are two types of consumers: the fully rational one follows a compensatory and the boundedly rational the satisficing decision strategy, which is noncompensatory. The former refers to those individuals that make tradeoffs between attributes and the latter refers to individuals that do not. The firm does not know with certainty whether consumers are of the first or of the second type. We show that whereas the boundedly rational consumer gets always the minimal satisfactory alternative, the fully rational one is better off under uncertainty because under certain conditions she gets more than her reservation utility. In particular this happens whenever the threshold of the boundedly rational consumer are particularly high. In this model uncertainty does not help bounded rationality.

We then conclude the chapter by providing some suggestions for policy-makers. For instance, in the monopolistic screening model we found that the fact that fully rational consumer's preferences are compensatory prevents the boundedly rational consumers from getting something more than the minimal satisfactory alternative. That is, compensatory preferences always allow firms to give the minimum to the boundedly rational consumer and, by moving along indifference curves, the reservation utility to the fully rational one. Given this result, we suggest to develop a policy aimed at reducing the probability that compensation between attributes takes place. As an example, policy-makers could incentivize advertisement strategies, in which advantages and disadvantages of similar products are clearly highlighted in order to minimize the probability that consumers perceive those as substitutes.

This study is closely related to the literature on bounded rationality and

IO.[5] To the best of our knowledge, there are no studies that investigate the effects of uncertainty on IO by defining boundedly rational consumers as individuals who behave according to the satisficing heuristic.

This chapter is organized as follows. Section 2.3 investigates a model of quality competition; Section 3.3 examines a signalling game; Section 4.4 analyzes a model of monopolistic screening; Section 5.5 concludes. Throughout the chapter the acronym 'FRC' stands for 'fully rational consumer' and the acronym 'BRC' for 'boundedly rational consumer'.

3.2 Quality Competition

Consider the following example. Suppose that a consumer wants to buy a new guitar. There is a big music store in the city, called 'Hendrix Music Store', that sells more than hundred different kinds of guitar. The set of all guitars in the store represents the choice set. If the consumer is an FRC, then she knows all products in the store before making the decision. On the other hand, if she is a BRC and follows the satisficing heuristic, she discovers and analyzes alternatives sequentially. Moreover, since she is not aware of how the choice set looks like *a priori*, she explores it and stops searching as soon as she identifies the first acceptable alternative. Assume, for instance, that she does not want to spend more than 2.500 Euros, but needs an high-quality musical instrument.[6] Let a guitar with these characteristics be called *satisfactory*. Whereas the FRC's choice is the best guitar of the store according to her preferences, which is, for instance, the *Gibson Les Paul Traditional Desert Burst* that costs 1.630 Euros, the BRC's depends on the extent to which she explores the choice set. For instance, assume that she examines the department of guitars from left to right and suppose that the

[5] See section 1.4 for a brief literature review.

[6] High-quality guitars are, for instance, 'Fender', 'Gibson', 'Ibanez', etc.

first satisfactory alternative she encounters is the *Fender Showmaster Elite Cherry Sunburst/Ebony* that costs 2.200 Euros. Since this product is judged to be satisfactory, then the BRC stops searching and buys it.

The above story highlights the fact that if a consumer follows the satisficing heuristic, then she discovers and analyzes alternatives sequentially and may end up with an inferior product. Indeed, the 'Gibson' is Pareto-superior to the 'Fender', because it is of the same quality but costs less. The goal of this section is to investigate the extent to which the fact that consumers can be either FRCs or BRCs affects an n-firm market that compete within a given store.

We first introduce some notation and definitions. A binary relation $\succeq$ on a set X is defined as a subset of the Cartesian product of X with itself. A binary relation $\succeq$ is complete whenever either $x \succeq y$ or $y \succeq x$ or both. Given $x, y, z \in X$, a binary relation $\succeq$ is transitive, when $x \succeq y$ and $y \succeq z$ imply that $x \succeq z$. A weak order is a transitive and complete binary relation. Let $\succ = \{(x, y) \in X \times X | (x \succeq y) \wedge \neg(y \succeq x)\}$ denote the asymmetric part of $\succeq$ and let $\sim = \{(x, y) \in X \times X | (x \succeq y) \wedge (y \succeq x)\}$ denote its symmetric part.

Let X be a finite set of alternatives. The market is composed of identical firms that have to decide simultaneously which product $x \in X$ to produce. Let $N = \{1, 2, \ldots, n\}$ be the set of players (firms), where $n = |N| \geq 2$ is the number of firms acting in the market. Let $S_i \subseteq X$ be firm i's set of strategies, with $i \in N$. A firm i's pure strategy is denoted by $x_i \in S_i$. A strategy profile is a vector $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbf{S} = \times_{i=1}^{n} S_i$. All products $x \in X$, if sold, yield the same level of revenue, which is normalized to 1. Let $c : X \to \Re_+$ be the cost function, where $c(x)$ is the cost of producing one alternative $x \in X$. The goal of firms is to maximize profit.

We assume that there is a unique consumer, that can be either an FRC or a BRC. Let $\succeq$ be a weak order on X that represents consumer's preferences. Let $x^{\max} \in \{x \in X | \nexists y \in X$ such that $y \succ x\}$ be a $\succeq$-maximal alternative in X and let $x^{\min} \in \{x \in X | \nexists y \in X$ such that $x \succ y\}$ be a $\succeq$-minimal one. Let $x^s \in X$ be a $\succeq$-minimal satisfactory alternative available in X. That

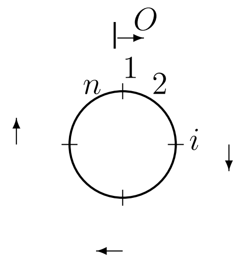

Figure 3.1: Alternatives are ordered according to the strict linear order O, where $1O2O\ldots On$.

is, x^s is a lowest-quality alternative that is judged to be satisfactory by the BRC and all alternatives that are at least as good as x^s are considered to be satisfactory as well. Obviously, the FRC does not care about x^s because she always searches for the optimum.

We also assume that the higher the quality of good x the higher its production cost, that is, $c(x) > c(y)$ if and only if $x \succ y$, provided that $c(x^{\max}) < \frac{1}{n}$.[7] Let $\pi_i(\mathbf{x})$ be firm i's profit (or payoff) when the played strategy profile is $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbf{S}$ with $i \in N$.

Finally, we assume that the products supplied by the n firms are ordered according to the strict linear order O.[8] In particular, given $i, j \in N$, if iOj, then the alternative supplied by firm i precedes j's. We can think of O as some spatial ordering according to which alternatives are arranged.[9] Figure 3.2 shows an example.

If the consumer is an FRC, then she knows the characteristics of all alternatives in advance and picks the maximal available alternative according to $\succeq$. If all firms produce a good of the same quality, the FRC chooses randomly. On the other hand, if the consumer is a BRC and follows the

[7] We assume that $c(x^{\max}) < \frac{1}{n}$ in order to make the strategy of supplying some alternative feasible.

[8] A strict linear order is a transitive, complete, and antisymmetric binary relation. A binary relation is antisymmetric whenever, given $x, y \in X$, $x \succeq y$ and $y \succeq x$ imply $x = y$.

[9] Notice that O is defined on N and not on X. Nevertheless, we interpret it as an ordering over supplied alternatives, because each firm i supplies exactly one good x_i.

satisficing heuristic, then she explores alternatives sequentially according to O. In particular, we assume that she starts with firm i's product with probability $\frac{1}{n}$. If it is satisfactory, then she stops searching and buys it. Otherwise, she examines firm j's product, provided that iOj and there is no $k \in N$ such that $iOkOj$. Again, if this product is satisfactory, she stops searching and buys it. Otherwise, she goes one and the procedure is the same as before. If she examines the product supplied by the O-minimal firm in N and finds it not to be satisfactory and there is some $j \in N$ such that jOi, then she examines the goods supplied by the firms in the set $\{i \in N | jOi\}$ still according to the relation O. If all alternatives are explored and are of the same quality, then the BRC chooses randomly.

Formally, let $LC(O;i) = \{j \in N | iOj\}$ and $UC(O;i) = \{j \in N | jOi\}$ be the lower and the upper contour-set of i according to the relation O, respectively. Let $O_{LC,i} = O \cap (LC(O;i) \times LC(O;i))$ and $O_{UC,i} = O \cap (UC(O;i) \times UC(O;i))$. Let $(O_i)_{i \in N}$ be a family of binary relations on N, where O_i satisfies:

- $\{i\} = \max(N; O_i)$,
- $O_i \cap O_{LC,i} = O_{LC,i}$,
- $O_i \cap O_{UC,i} = O_{UC,i}$, and
- given $k, j \in N$, kO_ij whenever $k \in LC(O;i)$ and $j \in UC(O;i)$, provided that $LC(O;i) \neq \emptyset \neq UC(O;i)$.

In short the BRC examines alternatives according to O_i with probability $\frac{1}{n}$ for all $i \in N$. See figure 3.2 for an example.

Clearly, there are many other ways in which the BRC could explore alternatives. In this study we focus on the above search procedure, but extensions of the model could encompass other procedures by which the consideration set is constructed. For instance, more advertised products may have an higher probability of being discovered first.

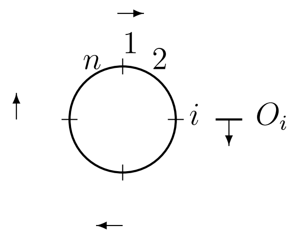

Figure 3.2: The BRC examines alternatives according to the strict linear order O_i with probability $\frac{1}{n}$, for all $i \in N$.

Let this game be called 'n-firm game'. The analysis of the equilibrium market is restricted to pure-strategy equilibria.

3.2.1 Certainty About the Consumer's Type

Assume first that firms know that the consumer is an FRC.

If the consumer is an FRC, then she knows the products that all firms produce in advance. She picks the best alternative according to her preference relation and if products are of the same quality then she chooses randomly.

Given a strategy profile $\mathbf{x}$, let $n(i, \mathbf{x}) = |\{j \in N | x_j \sim x_i\}|$ be the number of firms that supply an alternative x_j of the same quality as x_i. Formally, firm i's payoff function is defined as follows.

$$\pi_i(\mathbf{x}) = \begin{cases} \frac{1}{n(i,\mathbf{x})} - c(x_i) & \text{if } x_i \succeq x_j \text{ for all } j \in N \\ -c(x_i) & \text{otherwise} \end{cases}$$

Assume that firm i supplies an alternative $x_i \succeq x_j$ for all $j \in N$. The magnitude of profits depends on how many firms supply alternatives of exactly the same quality as x_i. For instance, if $x_i \succ x_j$ for all $j \in N$, then $n(i, \mathbf{x}) = 1$ and $\pi_i(\mathbf{x}) = 1 - c(x_i)$. On the other hand, if $x_i \sim x_j$ for all $j \in N$, then $n(i, \mathbf{x}) = n$ and $\pi_i(\mathbf{x}) = \frac{1}{n} - c(x_i)$. Whenever $1 < n(i, \mathbf{x}) < n$,

$\pi_i(\mathbf{x}) = \frac{1}{n(i,\mathbf{x})} - c(x_i)$. Hence, the greater $n(i, \mathbf{x})$, the lower profits.

Suppose then that $x_i \prec x_j$ for some $j \in N \setminus \{i\}$. Then, independently of how many firms supply an alternative superior to x_i firm i's revenues are zero, i.e., $\pi_i(\mathbf{x}) = -c(x_i)$, because the FRC never buys a dominated alternative.

The game of figure 3.3 represents the n-firm game in which the consumer is an FRC, there are two firms ($n = 2$), and $X = \{x^{\min}, x', x^{\max}\}$, where $x^{\min} \prec x' \prec x^{\max}$.

	$x^{\min}$	x'	$x^{\max}$
$x^{\min}$	$\frac{1}{2} - c(x^{\min})$, $\frac{1}{2} - c(x^{\min})$	$-c(x^{\min})$, $1 - c(x')$	$-c(x^{\min})$, $1 - c(x^{\max})$
x'	$1 - c(x')$, $-c(x^{\min})$	$\frac{1}{2} - c(x')$, $\frac{1}{2} - c(x')$	$-c(x')$, $1 - c(x^{\max})$
$x^{\max}$	$1 - c(x^{\max})$, $-c(x^{\min})$	$1 - c(x^{\max})$, $-c(x')$	$\frac{1}{2} - c(x^{\max})$, $\frac{1}{2} - c(x^{\max})$

Figure 3.3: The n-firm game, where the consumer is an FRC, $n = 2$, $X = \{x^{\min}, x', x^{\max}\}$, and $x^{\min} \prec x' \prec x^{\max}$

The row player is firm 1 and the column player is firm 2. For instance, suppose that firm 1 plays $x^{\max}$ and firm 2 $x^{\min}$. Since $x^{\max} \succ x^{\min}$, then the FRC buys $x^{\max}$. Therefore, firm 1's profits are $1 - c(x^{\min})$ and firm 2's are $-c(x^{\min})$.

The next proposition characterizes the equilibrium of the n-firm game, where the consumer is an FRC.

Proposition 10 *In the n-firm game in which the consumer is an FRC, X is finite, and $n \geq 2$, there exists a unique pure-strategy Nash equilibrium in which firms play $x^{\max}$ and earn profits equal to $\frac{1}{n} - c(x^{\max})$.*

If firms play $x^{\max}$, then they obtain the lowest level of profit and consumers get their most preferred object. Moreover, $\lim_{n \to +\infty} \frac{1}{n} - c(x^{\max}) = 0$, that is, if the number of firms in the market tends to infinity, then firms' profits tend to zero. For this reason the equilibrium characterized in Proposition 10 ($\mathbf{x}^{\max} = (x_i^{\max})_{i \in N}$) is called *competitive outcome*.

This result is not surprising. Since the FRC knows all alternatives before deciding, then she always picks the best available good. This implies that there cannot be equilibria in which firms supply a non-maximal alternative. The reason is that each firm has an incentive to raise a little bit the quality in order to attract the FRC. This mechanism pushes the quality upwards and leads firms to play $x^{\max}$, the product with the highest quality.

Moreover this result resembles Bertrand competition, that is, two firms are sufficient for the equilibrium market to be perfectly competitive.

Assume now that firms know that the consumer is a BRC.

A BRC follows the satisficing heuristic. That is, she examines alternatives sequentially and stops searching as soon as she identifies the first satisfactory alternative. Moreover, if all products are discovered and are of the same quality, then she chooses randomly.

Assume firm i's perspective. Since the BRC examines alternatives sequentially according to O_j with probability $\frac{1}{n}$ for all $j \in N$, then the probability that the BRC actually examines firm i's product crucially depends on what the firms that O_j-precede firm i do. For instance, assume that some firm that O_j-precedes i supplies a satisfactory alternative. In this case, the BRC does not even consider firm i's product at O_j, because she stops searching before encountering it. In the definition of the payoff function we have to consider this possibility.

Let $P_j(i) = \{k \in N | k O_j i\}$ for any $j \in N \backslash \{i\}$. This set is never empty and contains all firms whose products O_j-precede firm i's at O_j.[10] Next, given a strategy profile $\mathbf{x}$, we introduce an indicator function $I_j(i, \mathbf{x})$ that tells us whether or not firm i's product belongs to the BRC's consideration set at O_j. In particular, if $I_j(i, \mathbf{x}) = 1$, then all products supplied by the firms that O_j-precede firm i are unsatisfactory and the BRC considers x_i at O_j. On the

[10]We do not need to consider the case in which $i = j$, because the first alternatives that the BRC examines under O_i is precisely x_i.

contrary, if $I_j(i, \mathbf{x}) = 0$, then there is at least one firm that O_j-precede firm i that supplies a satisfactory product and, therefore, x_i is not part of the BRC's consideration set at O_j. Let $UC_{\succeq}(X; x^s) = \{x \in X | x \succeq x^s\}$ be the upper-contour set of x^s according to $\succeq$, representing the set of satisfactory alternatives available in the grand set X. Formally, $I_j(i, \mathbf{x})$ is defined as follows: for any $j \in N \setminus \{i\}$,

$$I_j(i, \mathbf{x}) = \begin{cases} 1 & \text{if } x_k \notin UC_{\succeq}(X; x^s) \text{ for all } k \in P_j(i) \\ 0 & \text{otherwise} \end{cases}$$

Since the BRC examines alternatives sequentially according to O_k with probability $\frac{1}{n}$ for all $k \in N$, then the probability that x_i is part of her consideration set is $\frac{\sum_{j \neq i} I_j(i,\mathbf{x})}{n-1} + \frac{1}{n}$. The second addendum captures the state of the world in which the BRC starts by exploring firm i's product (x_i is always part of BRC's consideration set at O_i). Let $q = \frac{\sum_{j \neq i} I_j(i,\mathbf{x})}{n-1} + \frac{1}{n}$. Notice that $q \in [\frac{1}{n}, 1]$, where $q = \frac{1}{n}$ when all firms supply a satisfactory product and $q = 1$ when only firm i supplies a satisfactory good.

Formally, firm i's payoff function is defined as follows.

$$\pi_i(\mathbf{x}) = \begin{cases} q\,(1 - c(x_i)) - (1 - q)\,(c(x_i)) & \text{if } x_i \succeq x^s \\ \frac{1}{n(i,\mathbf{x})} - c(x_i) & \text{if } (x_i, x_j \notin UC_{\succeq}(X; x^s)) \\ & \text{ and } (x_i \succeq x_j\ ) \forall j \in N \\ -c(x_i) & \text{otherwise} \end{cases}$$

Assume first that x_i is satisfactory. In this case, firm i's revenues are equal to 1, only if the BRC considers x_i. We know that this happens with probability q. Conversely, its revenues are equal to zero with probability $(1 - q)$, because if $(1 - q) > 0$, then this implies that at some O_j there is a firm, which precedes firm i, whose product is satisfactory. Hence, firm i's profits are $q\,(1 - c(x_i)) - (1 - q)\,(c(x_i))$.

Next, suppose that all firms supply an unsatisfactory product, but $x_i \succeq x_j\ \forall j \in N$. In this case, the BRC's consideration set includes all the supplied alternatives. That is, the BRC behaves as if she was an FRC. Since x_i is at least as good as all the supplied products, then firm i's profits are $\frac{1}{n(i,\mathbf{x})} - c(x_i)$.

Finally, notice that if either x_i is unsatisfactory and some other firm j supplies a satisfactory alternative (i.e., $x_i \notin UC_{\succeq}(X; x^s) \ni x_j$ for some j) or all the supplied alternatives are unsatisfactory and x_i is worse than some other alternative x_k (i.e., $(x_j \notin UC_{\succeq}(X; x^s) \forall j \in N)$ and $(x_k \succ x_i$ for some $k \neq i)$), then firm i's revenues are zero. Hence, its profits are equal to $-c(x_i)$.

The game of figure 3.4 represents the n-firm game in which the consumer is a BRC, there are two firms $(n = 2)$, $X = \{x^{\min}, x^s, x^{\max}\}$, $x^{\min} \prec x^s \prec x^{\max}$, and $UC_{\succeq}(X; x^s) = \{x^s, x^{\max}\}$.

	$x^{\min}$	x^s	$x^{\max}$
$x^{\min}$	$\frac{1}{2} - c(x^{\min})$, $\frac{1}{2} - c(x^{\min})$	$-c(x^{\min})$, $1 - c(x^s)$	$-c(x^{\min})$, $1 - c(x^{\max})$
x^s	$1 - c(x^s)$, $-c(x^{\min})$	$\frac{1}{2} - c(x^s)$, $\frac{1}{2} - c(x^s)$	$\frac{1}{2} - c(x^s)$, $\frac{1}{2} - c(x^{\max})$
$x^{\max}$	$1 - c(x^{\max})$, $-c(x^{\min})$	$\frac{1}{2} - c(x^{\max})$, $\frac{1}{2} - c(x^s)$	$\frac{1}{2} - c(x^{\max})$, $\frac{1}{2} - c(x^{\max})$

Figure 3.4: The n-firm game, where the consumer is a BRC, there are two firms $(n = 2)$, $X = \{x^{\min}, x^s, x^{\max}\}$, $x^{\min} \prec x^s \prec x^{\max}$, and $UC_{\succeq}(X; x^s) = \{x^s, x^{\max}\}$

Notice that the game of figure 3.4 (in which the consumer is a BRC) is different, *ceteris paribus*, from the game of figure 3.3 (in which the consumer is an FRC). For instance, assume that firms play $(x^s, x^{\max})$. In the case in which the consumer is an FRC firm 1's profits are $-c(x^s)$ and firm 2's are $1 - c(x^{\max})$. On the other hand, if the consumer is a BRC, then firm 1's profits are $\frac{1}{2} - c(x^s)$ and firm 2's are $\frac{1}{2} - c(x^{\max})$. The reason is that for the BRC all products that are at least as good as x^s are satisfactory. Obviously, if $x^{\max} \sim x^s$, then the BRC is actually an FRC and the two games coincide. Therefore, from now on we assume that $UC_{\succeq}(X; x^s)$ contains at least two indifference classes.

The next proposition characterizes the equilibrium of the n-firm game in which the consumer is a BRC.

Proposition 11 *In the n-firm game in which the consumer is a BRC, X is finite, and $n \geq 2$, there exists a unique pure-strategy Nash equilibrium in which firms play x^s and earn profits equal to $\frac{1}{n} - c(x^s)$.*

Proposition 11 suggests that bounded rationality on the demand side implies that firms supply precisely the minimal satisfactory alternative x^s, which can be non-optimal. The fact that the consumer follows the satisficing heuristic attenuates the mechanism of competition and induces firms not to supply the maximal good.

The intuition behind this result is that as long as firms supply a good worse than x^s, then the same mechanism of competition described above is at work. That is, each firm i, independently of its positioning according to O_i, has an incentive to deviate and raise the quality in order to capture the BRC. The reason is that if firm i produces a good inferior to x^s, then the BRC keeps searching looking for a better alternative and acts as if she was an FRC.

On the contrary, assume that firms supply some good y $\succ$-superior to x^s. In this case each firm i's profits are $1 - c(y)$ with probability $\frac{1}{n}$ and $-c(y)$ with probability $1 - \frac{1}{n}$. Notice that as long as $y \succeq x^s$ all products are satisfactory and, therefore, the BRC always stops searching at $\max(N, O_i)$ for all i. This implies that each firm i has an incentive to deviate to z, provided that $y \succ z \succeq x^s$, because its profits are equal to $1 - c(z)$ with probability $\frac{1}{n}$ and to $-c(z)$, otherwise, given that $c(y) > c(z)$. This argument and the one described in the previous paragraph imply that firms play x^s in equilibrium.

Let $\mathbf{x}^{\min} = (x_i^{\min})_{i \in N}$ be called *collusive outcome* because firms playing this strategy profile make the highest amount of profits and consumers get their least preferred alternative.

Notice that if $x^s \notin \min(X, \succ)$ and $x^s \notin \max(X, \succ)$, that is, if the minimal satisfactory alternative is neither minimal nor maximal in X, then the

Nash equilibrium of the n-firm game played by the BRC lies between the collusive and the competitive outcome: firms earn less than in the collusive outcome and more than in the competitive one. The BRC, instead, buys an intermediate good with respect to the relation $\succ$. This result resembles the Nash equilibrium of the Cournot model.

3.2.2 Uncertainty About the Consumer's Type

Suppose that firms do not know with certainty whether the consumer is an FRC or a BRC. In particular, let ρ be the probability that the consumer is an FRC. Let $\pi_i^u = \rho\pi_i^{FRC} + (1-\rho)\pi_i^{BRC}$ be firm i's profit function under uncertainty, where π_i^{FRC} and π_i^{BRC} are firm i's profit functions when the consumer is an FRC and a BRC, respectively.

The next proposition characterizes the equilibrium of the n-firm game in which there is uncertainty about whether the consumer is an FRC or a BRC.

Proposition 12 *In the n-firm game in which the consumer is an FRC with probability ρ and a BRC with probability $(1-\rho)$, X is finite, and $n \geq 2$, there exists a unique pure-strategy Nash equilibrium in which firms play $x^{\max}$ and earn profits equal to $\frac{1}{n} - c(x^{\max})$ only if $\rho \geq \bar{\rho} = n(c(x^{\max}) - c(x^s))$. Moreover, if $\rho < \bar{\rho}$ firms do not supply alternatives $\succeq$-worse than x^s in equilibrium.*

Proposition 12 suggests that the BRC is better off under uncertainty, because under certainty she gets a minimal satisfactory alternative x^s. On the contrary, under uncertainty she gets her most preferred good $x^{\max}$ if $\rho \geq \bar{\rho}$ and at least x^s, otherwise.

If the supplied alternatives are unsatisfactory, then the FRC and the BRC behave exactly in the same way in this model. They are distinguishable only when some satisfactory alternatives are supplied. In particular, unlike the FRC, the BRC stops searching as soon as she identifies the first satisfactory alternative, even though it is not optimal. On the contrary, the FRC is satisfied only when she chooses precisely the optimum. The intuition behind the result of proposition 12 is that for a sufficiently high ρ, the FRC's desire to get the optimum has an higher impact on the profit function π^u then BRC' satisfycing attitudes. Therefore, firms find more convenient to supply $x^{\max}$ rather than x^s.

The threshold $\bar{\rho} = n(c(x^{\max}) - c(x^s))$ increases in n and $c(x^{\max})$ and decreases in $c(x^s)$. On the one hand, it seems intuitive that the threshold decreases as the difference between $c(x^{\max})$ and $c(x^s)$ decreases. The reason is that if $c(x^{\max})$ and $c(x^s)$ get closer and closer to each other, then x^s gets closer and closer to $x^{\max}$ in terms of preferences $\succeq$. This makes the BRC progressively more and more similar to the FRC, because the set $UC(\succeq, x^s)$ shrinks and less and less alternatives are considered to be satisfactory. Hence, a lower $\bar{\rho}$ is needed to induce firms to supply $x^{\max}$.

On the other hand, the fact that the threshold increases in n seems counterintuitive, because typically one is led to think that consumer welfare increases as the market becomes more and more competitive. Our interpretation is that to an increase in the number of firms acting in the market corresponds an higher probability that the BRC stops searching before exploring the whole choice set. This increases the chances that the optimal alternative $x^{\max}$ is not part of the BRC's consideration set and, consequently, reduces firms' incentives to supply $x^{\max}$. This is why an higher ρ is required in order to induce firms to supply $x^{\max}$.

Finally, notice that the FRC is worse off under uncertainty. In fact, if ρ is below the threshold, then $\mathbf{x}^{\max}$ is not an equilibrium. The worst alternative that the FRC can get in equilibrium is x^s. Firms, instead, are clearly better off under certainty when the consumer is a BRC.

3.3 Signalling

Consider the following example. Assume that a consumer needs to buy a new camera. There is a beautiful store in the center of the city that sells either high-quality or intermediate-quality cameras. Suppose that she decides to visit it. The shop assistant knows that there are mainly two types of consumers. The first one buys only if she finds high-quality cameras. The second one buys even though she finds intermediate-quality ones. The shop assistant has to decide what kind of camera to show first. He is perfectly aware of the fact that the consumer's opinion about the store is strongly influenced by the first product he shows. For instance, if the first showed camera is a swindle, then the consumer is led to think that the store does not sell good products and that it is better to change place. The shop assistant is also aware of the fact that showing the intermediate-quality camera requires little time, because this kind of product is pretty simple to use. On the other hand, explaining how the high-quality camera works requires a lot of effort, more time, and concentration of the consumer. Moreover, the cost-opportunity of doing that is high, because it prevents him from carrying out other important tasks in the store. His problem is to choose what kind of cameras to show first, given that he does not know whether the consumer he is facing is of the first or of the second type.

The goal of this section is to analyze a situation analogous to the one described in the above example. That is, a firm has to decide whether to supply either an optimal or a satisfactory good. There are two types of consumers: FRCs and BRCs. FRCs are willing to buy products only if they are maximal according to their preference relation. On the other hand, BRCs buy not only maximal products, but also satisfactory ones. The firm's objective is to maximize profits, given that she does not know whether consumers are FRCs or BRCs and the production cost of the optimal good is greater than the satisfactory good's one.

Formally, let $N = \{1, 2\}$ be the set of players, where player 1 is the consumer and player 2 is the firm. The consumer has to decide first whether to enter or not the firm. The consumer's pure strategy space is $S_1 = \{E, NE\}$, where E stands for "enter" and NE "don't enter". Then, if she decides to enter, then the firm has to choose whether to supply either an optimal or a satisfactory product. The firm's pure strategy space is $S_2 = \{x^{\max}, x^s\}$, where the strategy $x^{\max}$ represents the choice of producing the optimal good and x^s represents the choice of supplying the satisfactory one. The cost $c(x^{\max})$ of producing $x^{\max}$ is assumed to be greater than the cost $c(x^s)$ of producing x^s, where $c(x^{\max}) < 1$. The consumer always prefers $x^{\max}$ to x^s. If the consumer chooses NE, then the consumer gets the reservation utility and the firm the reservation profit $\bar{\pi}$. A sold product yields a positive level of revenues, which is normalized to 1. The goal of the firm is to maximize profits.

Consider first the case in which the consumer is a FRC. If the FRC chooses E and the firm supplies $x^{\max}$, then the FRC buys $x^{\max}$, her utility is $u(x^{\max})$, and firm's profits are $1 - c(x^{\max})$. On the contrary, if she chooses E and the firm supplies x^s, then the FRC does not buy x^s, her utility is $u(x^s)$, and firm's profits are $-c(x^s)$. In this case the FRC does not buy anything because x^s is not optimal. Moreover, since she is wasting time in the store, then we assume that $u(x^{\max}) > \bar{u} > u(x^s)$. The firm, on the other hand, makes negative profits because x^s is not sold.

Next, consider the case in which the consumer is a BRC. If the firm supplies x^s, then the BRC buys x^s, her utility is $v(x^s)$, and firm's profits are $1 - c(x^s)$. On the other hand, if the firm supplies $x^{\max}$, then the BRC still buys the supplied product, her utility is $v(x^{\max})$, and firm's profits are $1 - c(x^{\max})$. Since the consumer prefers the optimal to the satisfactory good, then we assume that $v(x^{\max}) > v(x^s)$. However, since the consumer is a BRC and x^s is satisfactory, then $v(x^s) > \bar{v}$.

The analysis is restricted to pure-strategy equilibria.

3.3.1 Certainty About the Consumer's Type

Consider first the situation in which the consumer is a FRC. Figure 3.5 represents the game described above, where the consumer is a FRC.

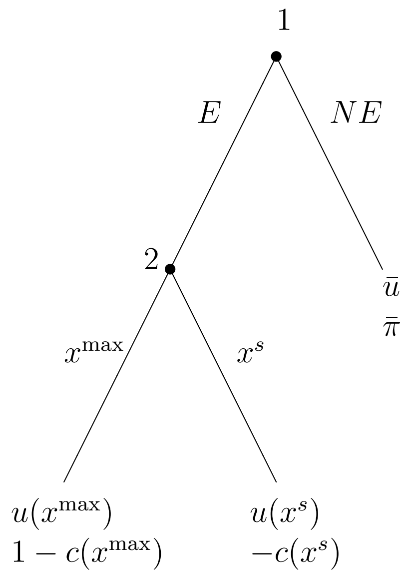

Figure 3.5: The game in which the consumer is a FRC

The firm supplies the maximal good $x^{\max}$ if and only if $1 - c(x^{\max}) > -c(x^s)$. Since this inequality is always true, then the firm's optimal choice is $x^{\max}$. Hence, the consumer's best response is to enter, because $u(x^{\max}) > 0$. The conclusion is that there is a subgame perfect Nash equilibrium in which the consumer buys and the firm supplies the maximal alternative.

Consider now figure 3.6 that represents the game described above, where the firm knows that the consumer is a BRC.

The firm supplies x^s if and only if $1-c(x^s) > 1-c(x^{\max})$, that is, $c(x^{\max}) > c(x^s)$. Since this inequality is always true, then the optimal strategy for the

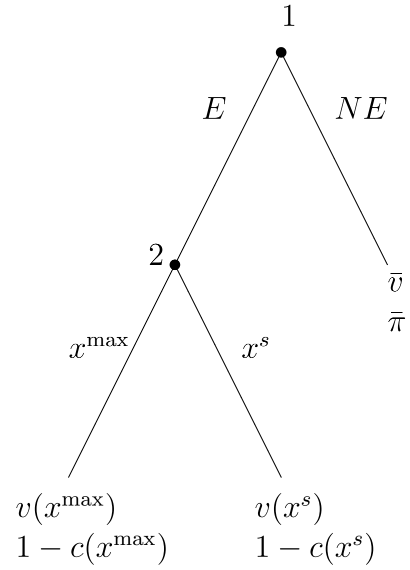

Figure 3.6: The game in which the consumer is a BRC

firm is to supply x^s. Hence, the consumer's best response is to enter because $v(x^s) > \bar{v}$. The conclusion is that the subgame perfect Nash equilibrium of the game of figure 3.6 is (E, x^s).

Notice that certainty about consumer's rationality induces the firm to put into practice a sort of *contingent strategy*: (i) supply the maximal product when facing a FRC and (ii) supply the satisfactory alternative when facing a BRC. This result is not surprising because, contingent on the specific situation, the firm simply maximizes its profits.

3.3.2 Uncertainty About the Consumer's Type

Suppose now that the firm does not know with certainty whether the consumer is a FRC or a BRC. In particular assume that the nature selects with a certain probability what is the type of consumer. If the state is FRC, then the game of figure 3.5 is played. On the contrary, if the state is BRC, then the game of figure 3.6 is played. The firm has a prior, denoted by $p(\cdot)$, about what is the state of nature, which is common knowledge between players.

This situation is depicted in figure 3.7.

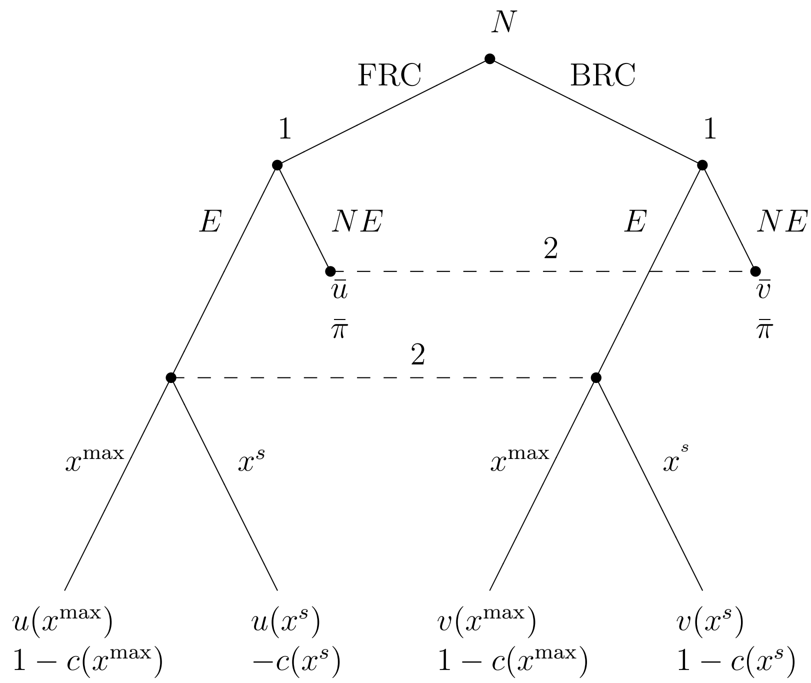

Figure 3.7: The game in which the firm does not know with certainty whether the consumer is a FRC or a BRC

Notice that the game of figure 3.7 is a signalling game.[11]

At this point one could wonder what kind of strategy the firm will follow.

[11] In a basic signalling game player 1 is the sender (of a signal), has private information about her type, and chooses a strategy contingent on her type. Player 2, whose type is common knowledge, observes the strategy played by player 1 and chooses a strategy. Let Θ be the set of player 1's types. It is assumed that player's 2 prior probability about player's 1 type is denoted by $p(\theta)$ and is common knowledge. Let $x_i \in S_i$ be a pure strategy, where S_i is player i's strategy space, with $i = 1, 2$. A player 1's mixed strategy $\sigma_1(\cdot|\theta)$ prescribes a probability distribution over the set S_1 for each type $\theta \in \Theta$. A player 2's mixed strategy $\sigma_2(\cdot|\sigma_1)$ prescribes a probability distribution over S_1 for each strategy σ_1 played by player 1. Player i's payoff is denoted by $u_i(\sigma_1, \sigma_2, \theta)$ (Fundenberg and Tirole, 1991*a*: 324-325).

In order to answer this question we compute the perfect Bayesian equilibria of the game of figure 3.7.[12] Remember that under certainty the firm behaves according to the contingent strategy, that is, playing $x^{\max}$ when the consumer is a FRC and x^s when she is a BRC.

The next proposition characterizes the equilibria of the game of figure 3.7.

Proposition 13 *In the game of figure 3.7, there are two sets of pure-strategy perfect Bayesian equilibria:*

(i) the FRC plays E, the BRC plays E, the firm supplies $x^{\max}$*, and the posterior probabilities are* $p(FRC|E) \in (c(x^{\max}) - c(x^s), 1]$ *and* $p(FRC|NE) \in [0, 1]$*;*

(ii) the FRC plays NE, the BRC plays E, the firm supplies x^s*, and the posterior probabilities are* $p(FRC|E) = 0$ *and* $p(FRC|NE) = 1$*.*

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[12] A perfect Bayesian equilibrium of a signalling game is a strategy profile σ^* and posterior beliefs $\mu(\cdot|x_1)$ such that

$$(i) \qquad \forall \theta \in \Theta,\ \sigma_1^* \in \arg\max_{\sigma_1} u_1(\sigma_1, \sigma_2^*, \theta)$$

$$(ii) \qquad \forall x_1, \sigma_2^*(\cdot|x_1) \in \arg\max_{\sigma_2} \Sigma_\theta \mu(\theta, x_1) u_2(x_1, \sigma_2, \theta)$$

and

$$\mu(\theta|x_1) = p(\theta)\sigma_1^*(x_1|\theta) / \Sigma_{\theta' \in \Theta} p(\theta')\sigma_1^*(x_1|\theta')$$

$$\text{if } \Sigma_{\theta' \in \Theta} p(\theta')\sigma_1^*(x_1|\theta') > 0$$

and $\mu(x_1|\theta)$ is any probability distribution over Θ if $\Sigma_{\theta' \in \Theta} p(\theta')\sigma_1^*(x_1|\theta') = 0$ (Fundenberg and Tirole, 1991*a*: 325-326). Since the game of figure 3.7 is a signalling game with two types, then a strategy profile is a perfect Bayesian equilibrium if and only if it is a sequential equilibrium (Fundenberg and Tirole, 1991*b*).

In the first set of perfect Bayesian equilibria of Proposition 13 both types of consumer enter and the firm supply the optimal good $x^{\max}$ and in the second the FRC stays out, the BRC enters, and the firm supplies the satisfactory good x^s. This result has different implications on FRC's and BRC's welfare. In equilibrium the FRC is worse off under uncertainty than under certainty, because under certainty the contingent strategy implies that she gets $u(x^{\max})$ for sure and under uncertainty she gets either $u(x^{\max})$ or $\bar{u}$, where $\bar{u} < u(x^{\max})$. On the contrary, in equilibrium the BRC is better off under uncertainty. The reason is that under certainty she gets $v(x^s)$ for sure and under uncertainty she gets either $v(x^s)$ or $v(x^{\max})$, where $v(x^s) < v(x^{\max})$. That is, in equilibrium the BRC' payoffs under uncertainty are Pareto-superior to the BRC' payoffs under certainty. The intuition behind this result is that under uncertainty as long as the posterior probability of the consumer being a FRC is sufficiently high ($p(FRC|E) > c(x^{\max}) - c(x^s)$), then it is too risky for the firm to play x^s: if the firm chose x^s, the FRC would punish the firm by not buying inducing a negative profit of $-c(x^s)$. In order to prevent this outcome, the firm chooses the optimal alternative $x^{\max}$ even though there is a positive probability that the consumer is a BRC.

The firm, instead, maximizes its profits when it faces a BRC with certainty, because in equilibrium it gets $1 - c(x^s)$. On the contrary, whenever if it deals with a FRC its profits are $1 - c(x^{\max})$, where $c(x^s) < c(x^{\max})$. Moreover, whenever there is uncertainty about whether the consumer is an FRC or a BRC, it gets in equilibrium either $1 - c(x^{\max})$ or $1 - c(x^s)$, which is worse than $1 - c(x^s)$.

3.4 Monopolistic Screening

A decision strategy is called *compensatory* whenever decision-makers make tradeoffs between attributes. On the other hand, a decision strategy is *non-*

compensatory whenever DMs do not (Payne, Bettman and Johnson, 1993: 29 and 31). A compensatory decision strategies is, for instance, standard maximization. On the contrary, the satisfycing heuristic is an example a noncompensatory strategy. In this section a model of monopolistic screening is illustrated in which two types of consumers act: the first one relies on a compensatory and the second one on a noncompensatory decision strategy.

Consider the following example. Assume a consumer is interested in buying a new car. Suppose that she considers only two characteristics, that is, price and level of emissions (expressed in Light Duty Vehicle European standards[13]). The first type of consumer, called FRC, prefers to spend as little as possible and to own a car that does not pollute that much. Her main feature is that she makes tradeoffs between attributes. On the other hand, the second type of consumer is called BRC. Her preferences over attributes are similar to FRC's, however, she judges an alternative to be satisfactory only if the levels of its attributes are at least as good as some threshold. Suppose that the BRC judges a car to be satisfactory only if it costs at most 15.000 Euros and its standard is at least Euro 3.

Notice that FRC's and BRC's preferences may diverge. For instance, consider car $y = (1.000, 1)$, where the first element represents the attribute 'price' and the second 'emissions' in terms of the standard 'Euro x' with $x \in \{1, 2, 3, 4, 5\}$.

According to BRC's preferences, y is not acceptable, because its standard is less than Euro 3. On the other hand, since FRC's preferences allow for compensation, then there exists a car z that is indifferent to y. Suppose that $z = (14.000, 5)$. Notice that whereas $z \sim_{FRC} y$, it turns out that $z \succ_{BRC} y$, because z is satisfactory.

The goal of this section is to investigate the extent to which the fact that there are FRCs and BRCs affects the equilibrium outcome of a market in

[13]In the European Union there are five standards: Euro 1, Euro 2, Euro 3, Euro 4, and Euro 5. The principle is that the less a car pollutes the highest 'Euro' it gets. The cars that conform to Euro 1 are the ones that pollute more. Currently, Euro 5 is the standard that certifies the highest level of environmental sustainability.

which a monopolist supplies a multi-attribute good.

Let $X = \Re^0_+ \times \Re_+$ be a subset of the two-dimensional Euclidean space. Each vector $x = (p_x, q_x) \in X$ represents a two-attribute alternative, where $p_x > 0$ represents the attribute 'price' and $q_x \geq 0$ represents a generic attribute different from price.

We assume that there is a monopolist that produces good x whose profit function is $\pi(x) = p_x - c(q_x)$, where $c(q_x)$ is the cost of producing good x with characteristic q_x. It is assumed that $c(q_x) = \alpha q_x$, where $\alpha \in (0, 1)$. The goal of the monopolist is to maximize profits.

On the demand side, there are two kinds of consumer: the FRC and the BRC. The FRC is an expected utility maximizer whose Bernoulli utility function is defined as $u(x) = u(g(q_x) - p_x)$, where the function g measures the utility of the characteristics q_x in monetary units. Let $g(q_x) = \ln(q_x + 1)$. This functional form implies that the FRC evaluates more and more characteristic q_x as it increases, but marginal utility is decreasing. Furthermore, the FRC is assumed to be risk averse and her reservation utility is $\bar{u}$, where $\bar{u} > u(0)$.

The BRC is not a maximizer, but follows the satisficing heuristic. Specifically, she judges a good $y \in X$ to be satisfactory only if $p_y \leq \bar{p}$ and $q_y \geq \bar{q}$, where $(\bar{p}, \bar{q}) \in X$ is a minimal satisfactory alternative.

The goal of the FRC is to maximize her utility and the goal of the BRC is to get at least a satisfactory product. Throughout this chapter we denote by $x = (p_x, q_x)$ the product that the monopolist supplies to the FRC and by $y = (p_y, q_y)$ the product that the monopolist supplies to the BRC.

3.4.1 Certainty About the Consumer's Type

We assume first that the firm knows that consumers are FRCs. This implies that the firm maximizes its profits subject to the so called 'reservation utility constraint'. That is, the utility of good x must be greater or equal than the reservation utility. Formally, $u(\ln(q_x + 1) - p_x) \geq \bar{u}$, or, equivalently, $\ln(q_x + 1) - p_x \geq u^{-1}(\bar{u})$. Thus, the firm has to solve the following problem.

PROBLEM 1

$\max_{p_x, q_x} p_x - \alpha q_x$

s.t. (i) $\ln(q_x + 1) - p_x \geq u^{-1}(\bar{u})$

The next proposition characterizes the solution of problem 1. Let the solution be denoted by $x^c = (p_x^c, q_x^c)$, where c stands for *certainty.*

Proposition 14 *Problem 1 has a unique solution* $x^c = (p_x^c, q_x^c) = (ln(\frac{1}{\alpha}) - u^{-1}(\bar{u}), \frac{1}{\alpha} - 1)$.

Proposition 14 suggests that the FRC gets precisely the reservation utility under certainty. In fact, plugging the optimal solution into constraint (i), we get $\ln(\frac{1}{\alpha} - 1 + 1) - ln(\frac{1}{\alpha}) + u^{-1}(\bar{u}) \geq u^{-1}(\bar{u})$, which implies that $\bar{u} = \bar{u}$.

Suppose now that the firm knows that consumers are BRCs. This implies that the firm maximizes its profits subject to the constraint that the supplied product y has to be satisfactory, that is, $p_y \leq \bar{p}$ and $q_y \geq \bar{q}$. Let these constraints be called 'satisficing constraints'. Thus, the firm has to solve the following problem.

PROBLEM 2

$\max_{p_y, q_y} p_y - \alpha q_y$

s.t. (i) $p_y \leq \bar{p}$

(ii) $q_y \geq \bar{q}$

The next proposition characterizes the solution of problem 2. Let the solution be denoted by $y^c = (p_y^c, q_y^c)$, where c stands for *certainty.*

Proposition 15 *Problem 2 has a unique solution* $y^c = (p_y^c, q_y^c) = (\bar{p}, \bar{q})$.

If the consumer is a BRC, then the firm supplies precisely the minimal satisfactory alternative under certainty. This result confirms what we found in the quality competition and signalling game models.

3.4.2 Uncertainty About the Consumer's Type

Assume now that the firm does not know with certainty whether consumers are FRCs or BRCs. Let $\rho \in (0,1)$ be the probability that consumers are FRCs. The firm has to find two goods $x = (p_x, q_x) \in X$ and $y = (p_y, q_y) \in X$ such that the expected profit $\rho\,(p_x - \alpha q_x) + (1-\rho)\,(p_y - \alpha q_y)$ is maximized, where x maximizes FRC's utility and y is at least satisfactory. However, the optimal solution must satisfy not only the reservation utility and the satisficing constraints, but also the so called 'incentive compatible' ones. These constraints require that, first, alternative x must yield at least the same level of utility of good y. Second, good x does not have to be more than satisfactory, that is, either $p_x \geq \bar{p}$ or $q_x \leq \bar{q}$ or both. In order to ensure the second incentive compatible constraint to hold we impose $p_x \geq \bar{p}$. Thus, the firm has to solve the following problem.

PROBLEM 3

$\max_{p_x,p_y,q_x,q_y} \rho\,(p_x - \alpha q_x) + (1-\rho)\,(p_y - \alpha q_y)$

s.t. (i) $\ln(q_x + 1) - p_x \geq u^{-1}(\bar{u})$

(ii) $p_y \leq \bar{p}$

(iii) $q_y \geq \bar{q}$

(iv) $\ln(q_x + 1) - p_x \geq \ln(q_y + 1) - p_y$

(v) $p_x \geq \bar{p}$

The next proposition characterizes the solution of problem 3. Let the solution be denoted by $x^u = (p_x^u, q_x^u)$ and $y^u = (p_y^u, q_y^u)$, where 'u' stands for *uncertainty*.

Proposition 16 *The solution of Problem 3 is characterized as follows.*

1. *Assume $\bar{u} \geq u(\bar{p}, \bar{q})$.*

   (a) *if $\bar{p} < p_x^c$, then $x^u = (\ln\left(\frac{1}{\alpha}\right) - u^{-1}(\bar{u}), \frac{1}{\alpha} - 1)$ and $y^u = (\bar{p}, \bar{q})$;*

(b) *if* $\bar{p} \geq p_x^c$, *then* $x^u = (\bar{p}, e^{\bar{p}+u^{-1}(\bar{u})} - 1)$ *and* $y^u = (\bar{p}, \bar{q})$.

2. *Assume* $\bar{u} < u(\bar{p}, \bar{q})$.

   (a) *if* $\bar{p} < p_x^c$, *then* $x^u = (\ln\left(\frac{1}{\alpha}\right) - \ln(\bar{q}+1) + \bar{p}, \frac{1}{\alpha} - 1)$ *and* $y^u = (\bar{p}, \bar{q})$.

   (b) *if* $\bar{p} \geq p_x^c$, *then* $x^u = (\bar{p}, \bar{q})$ *and* $y^u = (\bar{p}, \bar{q})$.

The solution of problem 3 suggests first that even though the firm is uncertain about the consumer's type, once again the BRC gets nothing more than the minimal satisfactory alternative $(\bar{p}, \bar{q})$.

Second, the characteristics of the FRC's good depend on whether the utility of minimal satisfactory alternative $(\bar{p}, \bar{q})$ is greater or equal or less than the reservation utility $\bar{u}$ and on whether the level of threshold $\bar{p}$ is greater or less than, or equal to p_x^c, the FRC's optimal price under certainty. If $\bar{u} \geq u(\bar{p}, \bar{q})$, then the FRC gets precisely the reservation utility. On the contrary, if $\bar{u} < u(\bar{p}, \bar{q})$, then the FRC gets more than $\bar{u}$.

Third, the characterization of the solution of problem 3 clearly highlights the compensatory feature of FRC' preferences. We know that under certainty the FRC gets $x^c = (\ln\left(\frac{1}{\alpha}\right) - u^{-1}(\bar{u}), \frac{1}{\alpha} - 1)$, which yields precisely the reservation utility $\bar{u}$. If the FRC follows a compensatory decision strategy, then it must be the case that if the monopolist supplies a good x whose price p_x is greater than p_x^c, then in order to leave the FRC indifferent between x and x^c it must set the quality q_x of good x above q_x^c. Consider now the case in which there is uncertainty and suppose that $\bar{u} > u(\bar{y})$ and $\bar{p} > p_x^c$ (case 1b of proposition 16). In this case, $x^u = (\bar{p}, e^{\bar{p}+u^{-1}(\bar{u})} - 1)$ and $y^u = (\bar{p}, \bar{q})$. Moreover, notice that the FRC gets precisely the reservation utility, because

$$
\begin{aligned}
\ln(q_x^u + 1) - p_x^u &= \ln(e^{\bar{p}+u^{-1}(\bar{u})} - 1 + 1) - \bar{p} \\
\ln(q_x^u + 1) - p_x^u &= u^{-1}(\bar{u}) \\
u(p_x^u, q_x^u) &= \bar{u}
\end{aligned}
$$

Since $p_x^c = \bar{p}$ and the FRC gets precisely her reservation utility, then the quality of good x must be greater than q_x^c, which is the optimal level of quality for the FRC under certainty. That is, she gets $q_x^u = e^{\bar{p}+u^{-1}(\bar{u})} - 1 > \frac{1}{\alpha} - 1 = q_x^c$ that compensates her for the higher level of price $p_x^u = \bar{p} > \ln(\frac{1}{\alpha}) - u^{-1}(\bar{u}) = p_x^c$. In other words, the FRC moves along the indifference curve that yields the reservation utility $\bar{u}$.

Finally, in terms of welfare the firm entirely bears the cost of informational asymmetries, which increases in the level of the thresholds $\bar{p}$ and $\bar{q}$. On the demand side, the FRC is better off under uncertainty, because if the thresholds of $(\bar{p}, \bar{q})$ are particularly high, then she gets more than the reservation utility $\bar{u}$. If not, she gets in any case $\bar{u}$. On the other hand, uncertainty does not increase BRC's welfare. The fact that the FRC makes trade-offs between attributes prevents the BRC from getting something more than the minimal satisfactory alternative. That is, the compensatory feature of FRC' preferences always allows the firm to give the minimum to the BRC and, combining properly the attributes, an unsatisfactory good that yields the reservation utility to the FRC. Figure 3.8 illustrates graphically this intuition.

Assume that $\bar{p} > p_x^u$ and that $\bar{q} > q_x^c$, so that both x^c and y^c are satisfactory and x^c is Pareto-superior to y^c. In this case, one expects that under uncertainty the monopolist makes the BRC better off by supplying to the FRC and to the BRC two goods x^u and y^u, respectively, such that $x^u = y^u = x^c$. Instead, the optimal solution is $x^u = (\bar{p}, e^{\bar{p}+u^{-1}(\bar{u})} - 1)$ and $y^u = (\bar{p}, \bar{q})$. That is, the monopolist *pushes* x^u towards north-east along the indifference curve that yields $\bar{u}$ until it reaches the border of the satisficing area, so that $x^u = (\bar{p}, e^{\bar{p}+u^{-1}(\bar{u})} - 1)$. In this way x^u is unsatisfactory and yields the reservation utility $\bar{u}$, $(\bar{p}, \bar{q})$ is not Pareto-dominated by x^u, and the monopolist is free to supply the minimal satisfactory alternative $(\bar{p}, \bar{q})$ to the BRC.[14] We believe that this is an interesting result, because it pro-

[14]Technically, $x^u = (\bar{p}, e^{\bar{p}+u^{-1}(\bar{u})} - 1)$ is satisfactory and x^u is still Pareto-superior to y^u, because $p_x^u = \bar{p}$. However, since we are on the continuum, we interpret $p_x^u = \bar{p} + \epsilon$,

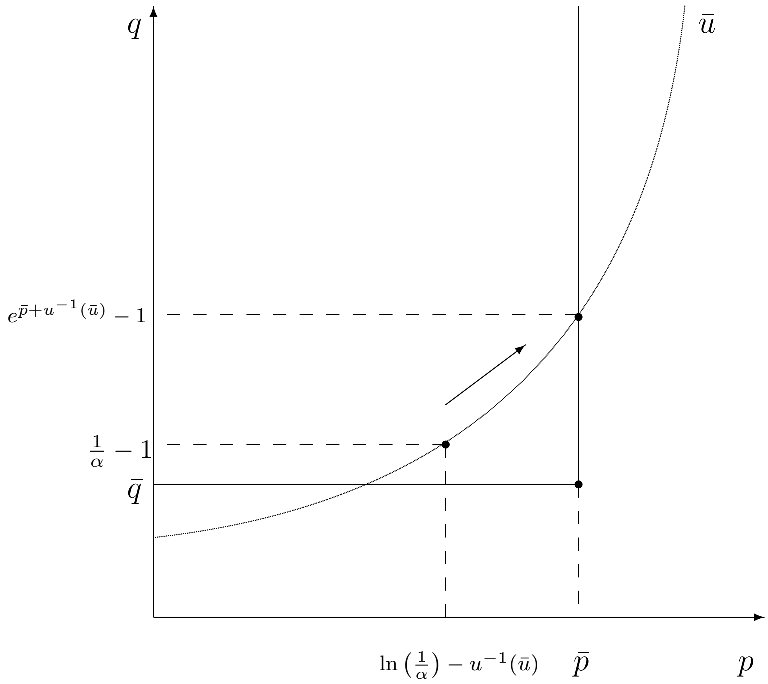

Figure 3.8: Compensatory preferences prevent the BRC from getting something more than $(\bar{p}, \bar{q})$

vides a clear intuition of why in this case uncertainty does not help bounded rationality.

3.5 Concluding Remarks

This chapter examines what are the effects of uncertainty on bounded rationality in a variety of IO models. Results are summarized in the parametric space of figure 3.9. The x-axis identifies the different models, the y-axis differentiates between certainty and uncertainty, and the z-axis measures potential extensions of the proposed models.

where $\epsilon > 0$ is arbitrarily small, so that x^u is unsatisfactory.

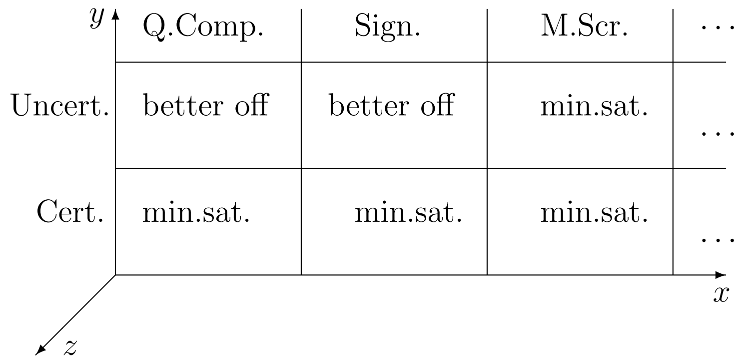

Figure 3.9: A parametric space representing BRC's welfare

Under certainty the BRC gets nothing more than the minimal satisfactory alternative (min.sat.). This implies that firms prefer to face BRCs rather than FRCs, because supplying to BRCs increases profits. However, whenever there is uncertainty about whether consumers are FRCs or BRCs, it is not obvious *a priori* whether or not the BRC derives benefits. For example, in the models of quality competition and signalling, the BRC is better off. In particular, if the probability that the consumer is an FRC is above some threshold, then in equilibrium firms supply the optimal product $x^{\max}$. Interestingly, we found that in the quality competition model the threshold increases in the number of firms acting in the market. Our interpretation is that the more competitive the market, the higher the probability that the optimal alternative is not part of the BRC's consideration set. This implies that firms are less and less incentivized to supply $x^{\max}$. Therefore, an higher threshold is required to induce them to supply an optimal alternative. On the other hand, in the signalling game the fact that there is a (even small) probability that the firm faces a FRC rather than a BRC prevents it from supplying a non-optimal good in order to avoid a punishment (and, therefore, a drastic reduction of its profits) by the FRC.

In the monopolistic screening model uncertainty does not help BRCs instead. In particular BRCs get the minimal satisfactory alternative even when there is uncertainty. The intuition behind this result is that in a multi-attribute framework the compensatory feature of FRC's preferences prevents the BRC from getting something more than the minimal satisfactory alternative. Firms give the minimum to the BRC and moving along indifference curves at least the reservation utility to the FRC.

In the light of these results, we provide some suggestions to policy-makers aimed at enhancing consumer welfare. First, firms should be induced to believe that consumers are mainly FRCs. In this way the probability that all consumers get the optimal alternative is increased. Second, in the long run FRC's preferences should be made noncompensatory in order to prevent firms from giving the minimum to BRCs by moving along FRCs indifference curves. As an example, advertisement strategies devoted to highlight advantages and disadvantages of products among which consumers are likely to compensate should be incentivized. In this way, it would be more probable that rather than perceiving those products as substitutes, consumers rank them according to a strict preference relation. Third, mechanisms aimed at facilitating the construction of the consideration set should be developed.

With reference to figure 3.9, further work could be done in various directions. For instance, concerning the x-dimension, the hypothesis that uncertainty help bounded rationality could be tested on other models, such as Cournot or von Stackelberg. Moreover, the proposed analysis could be further refined by adding more complexity/realism to the models (z-dimension). For instance, in the quality competition model alternative procedures to form the consideration set could be investigated.

Chapter 4

An Experimental Study on the Satisficing Heuristic

4.1 Introduction

The second chapter of the thesis proposes a theoretical model within the revealed-preference approach in which DMs behave consistently with the satisficing heuristic. The third chapter applies this model to more concrete economic situations, such as markets in which boundedly and fully rational consumers interact with firms. This chapter provides an empirical test aimed at verifying whether subjects behavior is consistent with the satisficing heuristic.

Standard theory assumes that individuals have infinite computational capabilities, know all alternatives before deciding, and always select the best alternative(s) from any choice set. In contrast, Simon (1955) argues that individuals are boundedly rational. This means that they have bounded computational capabilities, discover and analyze alternatives sequentially, and often, rather than choosing the optimal alternative, select a satisfactory one. In addition, he points out that these attitudes depend on the the environment in which individuals are asked to operate. If the environment is relatively complex and new to individuals, then it is likely that they will be content with alternatives relatively far from the optimum. On the contrary, when the environment is relatively simple and familiar to them, then presumably they will choose alternatives relatively closer to the optimum.

In order to test Simon (1955)'s model we propose an experiment in which subjects are asked to solve five problem sets under time pressure. Each problem set encompasses 3 choice problems of different complexity. Each choice problem is an algebraic sum where only the operators and the result are visible. The spaces between operators and before the 'equal sign' are empty. Subjects are financially incentivized to insert into each algebraic sum the combination of numbers (from a given set) such that the actual result of the algebraic sum is as close as possible to the reported result. They are allowed to change the inserted numbers as many times as they want before the time expires. Moreover, after an insertion is made, they are informed about what is the updated actual result of the algebraic sum. We record not only final choices, but also intermediate ones and decision time.

We investigate the extent to which subjects commit mistakes (i.e., fail to choose the combination of numbers that maximizes their material payoff) and choose sub-optimal alternatives and how allocate decision time and make intermediate choices. Moreover, we analyze how these variables behave relative to the environment in which subjects are asked to operate. In particular, how they are affected by the extent to which the environment is complex/simple and familiar/unfamiliar to subjects. We define the variable *complexity* as the length of an algebraic sum (the longer the sum, the more complex the problem) and the variable *familiarity of the environment* as the temporal order with which subjects solve problem sets (the first problem set that subjects face constitutes a less familiar environment with respect to the second and so on).

We found that in general subjects behavior is consistent with the satisficing heuristic. Data on errors and final choices confirm that subjects are sensitive to the environment in which they are asked to operate. However, complexity seems to have a stronger impact on performance then the variable *familiarity of the environment*. Decision time data reveal that an increase in the complexity of the choice problem does not necessarily cause an increase in the time spent in trying to solve it. Similarly, for the number of intermediate choices. Finally, results provide useful insights for modeling satisficing behavior. For instance, unlike what several existing models assume (Caplin and Dean, 2011), we found that the threshold that defines an alternative to be satisfactory is not fixed, but vary across choice problems.

There is a huge literature on individual decision-making in experimental economics (Payne, Bettman and Johnson, 1993; Gigerenzer, Todd and the ABC Research Group, 1999; Gabaix et al., 2006; Reutskaja et al., 2010).[1] However, the closest study to this work is Caplin, Dean and Martin (2009). As explained in the first chapter, Caplin, Dean and Martin design an experiment aimed at testing whether subjects behavior is consistent with an RBS model in which search order is not observable (Caplin and Dean, 2011). Each alternative is an algebraic sum and a choice set is a list of algebraic sums. Subjects are asked to solve them and choose the one that yields the highest payoff, given that there is a one-to-one correspondence between results and material payoff. Caplin, Dean and Martin (2009) employ the so-called *choice process data*, according to which subjects are allowed to change their choice as many times as they want before the time expires and the amount of time pressure is random. In addition, at any point in time subjects can see what alternative they have provisionally chosen. In contrast, we do not list alternatives and subjects do not see their provisional choice at any point in time. In this study alternatives are different objects: an alternative is a combination of numbers that plugged into an algebraic sum yields a certain result. Once that an inserted number is replaced, the result changes, and, therefore, the previous choice is not visible anymore. Finally, the amount of time pressure is not random, but fixed at 75 seconds. We believe that in the real-world there are both choice problems in which alternatives are listed and

[1] See also section 1.5.

choice problems in which alternatives are not. As an example of the former, think of a Google search. As an example of the latter, imagine a consumer who looks for a new t-shirt in a marketplace. T-shirts are not listed, but the consumer walks back and forth around the stalls in order to search for her most preferred alternative. In view of this difference, we think that these studies are complementary.

This chapter is organized as follows. Section 4.2 explains design and implementation; Section 4.3 provides a formalization of the kind of choice problem that subjects are required to solve in this experiment; Section 4.4 formulates hypothesis and shows and discusses the results; Section 4.5 and section 4.6 examine how performance is affected by the order with which subjects approach choice problems within a problems set and by individual features, respectively; Section 4.7 concludes.

4.2 Design and Implementation

Each subject had to solve five problem sets randomly drawn from a dataset of 15 problem sets. Each problem set was presented in one screen shot as figure 4.1 illustrates. On the left-hand part there was a list of six natural numbers (in the box *numeri che puoi utilizzare*) ordered in descending ordered. In the middle there was a list of three algebraic sums (in the box *formule da risolvere*), where only the results (called *Goals*, written in black) and the operators were visible. On the right-hand part, for each algebraic sum there was a number in red in the box *risultati parziali*, which we call *Intermediate Result*. At the bottom there was a box in which the amount of time left to solve the screen shot was reported both numerically (in seconds) and visually (the horizontal bar shrank).

Subjects had to insert the numbers available from the box *numeri che puoi utilizzare* in the empty spaces between operators of each algebraic sums in such a way that the actual result of the algebraic sum was as close as possible to the respective Goal. A number could not be repeated within each algebraic

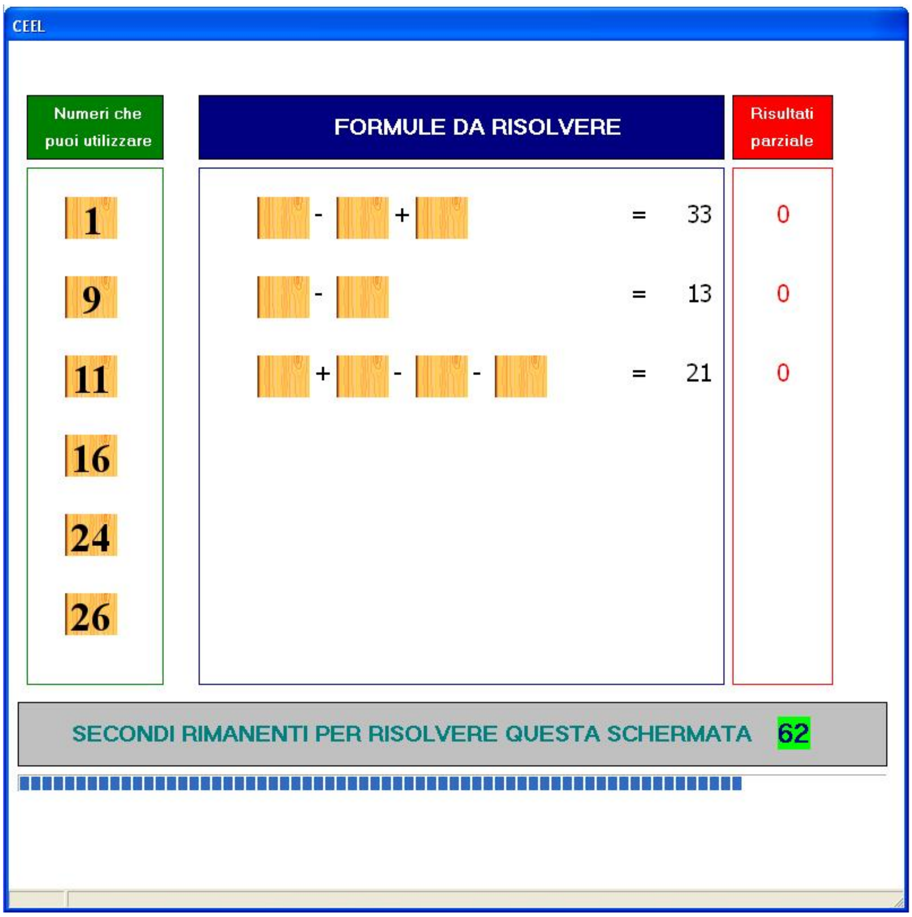

Figure 4.1: An Example of a Screen Shot

sum, but could be used more than once across algebraic sums. The three algebraic sums had different length: one had one operator (a minus), one had two (a plus and a minus), and one had three (a plus and two minuses). The order with which algebraic sums of different length appeared to the subject within each screen shot was random. After a number was inserted in an algebraic sum, the correspondent Intermediate Result was updated automatically and showed what was the result of the algebraic sum at that point in time.

In order to insert a number in a sum, subjects had to click on the number they wanted to select and an horizontal bar appeared just below the number to confirm its selection. Then, they had to click on the position in which they wanted to insert the number. There was the possibility to replace an inserted number simply by repeating this procedure. Subjects had 75 seconds for solving one screen shot.

The show-up fee was 2.00 euros. One round was selected at random for calculating the payoff performance. They payoff of the selected screen shot was calculated as follows.

$$\pi \equiv 2.00 + 4\sum_{i=1}^{3} \frac{d_i^{\max} - d_i^{act}}{d_i^{\max}}$$

where index i identifies the ith algebraic sum of the selected screen shot, d_i^{max} measures the absolute maximal distance between the Goal and the Intermediate Result of the ith algebraic sum, and d_i^{act} measures the absolute actual distance between the Goal and the Intermediate Result of the ith algebraic sum when the time expires.

If, when the time expires, the Intermediate result of the ith algebraic sum is equal to the Goal, then the respective d_i^{act} is zero. Therefore, $\frac{d_i^{\max} - d_i^{act}}{d_i^{\max}} = 1$. On the contrary, if, when the time expires, the distance between the Intermediate result and the Goal of the ith algebraic sum is maximal, then $d_i^{act} = d_i^{\max}$. Therefore, $\frac{d_i^{max} - d_i^{act}}{d_i^{\max}} = 0$. This implies that in general the ratio $\frac{d_i^{\max} - d_i^{act}}{d_i^{\max}} \in [0, 1]$ and the closer the Intermediate Result to the Goal, the closer the ratio to one, the higher the payoff. Conversely, the further the Intermediate Result to the Goal, the closer the ratio to zero, the lower the payoff. Algebraic sums were equally important for the calculation of the payoff. Since $\pi = 2.00 + 4\sum_{i=1}^{3} \frac{d_i^{\max} - d_i^{act}}{d_i^{\max}}$, subjects could earn at most 14 euros and at least 2 euros.

Subjetc were clearly told that only completed algebraic sums (sums without empty spaces between operators) were taken into account for the calculation of the payoff. In order to implement this rule, each d_i^{act} associated with an incomplete algebraic sum was automatically set equal to $d_i^{\max}$.

Before the experiment subjects were given five minutes to read the instructions by themselves. An experimenter then red them loudly. Subsequently, subjects were required to complete a questionnaire aimed at verifying whether or not they understood the instructions. Subjects were allowed to proceed only if they answered correctly to all questions. Then the experiment started and took on average 8 minutes. After the experiment, general questions were asked to participants, such as age, sex, etc. Besides final choices, collected data include intermediate choices, decision time, time taken to complete the questionnaire, and general information about subjects (sex, age, etc.).

A total of 60 subjects were recruited and the experiment was conducted at the Computable and Experimental Economics Laboratory (University of Trento). Three sessions of 20 subjects each were implemented on Wednesday, 06 October 2010.

4.3 Choice Problem and Preference

Each algebraic sum of a given screen shot can be viewed as a choice problem, where, in order to maximize material payoff, the right combination of numbers has to be inserted into each algebraic sum.

Consider, for instance, the problem of figure 4.1. The set of available numbers is $\{1, 9, 11, 16, 24, 26\}$ and the three Goals, as they appear on the screen shot, are $\{33, 13, 21\}$. Consider first the simplest algebraic sum, the one with 1 operator (the second of figure 4.1). In order to maximize material payoff one needs to plug into the algebraic sum the ordered pair of numbers $(a, b) \in \{(x, y)|x, y \in \{1, 9, 11, 16, 24, 26\}$ and $x \neq y\}$ such that $a-b = 13$. In principle there are $\frac{6!}{(6-2)!} = 30$ ways in which one can complete this sum. The choice problem under consideration is therefore composed of 30 alternatives. Since each ordered pair (a, b) is associated with a certain ratio $\frac{d_i^{\max}-d_i^{act}}{d_i^{\max}}$, then we can rank the alternatives according to how much each alternative yields in terms of payoff, by starting from the best alternative and getting down progressively to the worst one. In this example the pair $(24, 11)$ is the only alternative associated with a $d^{act} = 0$. Indeed, the Goal is equal to 13,

$24 - 11 = 13$, and, therefore, the ratio is equal to one. Notice that there is no pair for which $d^{act} = 1$. It is easy to see that there are three pairs for which $d^{act} = 2$, that is, $(16, 1), (24, 9)$, and $(26, 11)$. The associated ratio is $\frac{38-2}{38} = 0.95$, because $d^{\max} = 38$. Proceeding in this way, we can construct a preference relation $\succeq$ defined on $\{(x, y)|x, y \in \{1, 9, 11, 16, 24, 26\}$ and $x \neq y\}$ under the assumption that subjects prefer to earn as much as possible: $(24, 11) \succ (16, 1) \sim (24, 9) \sim (26, 11) \succ \ldots$.

Similarly, we can get the same kind of information from the other algebraic sums (with 2 and 3 operators) of figure 4.1 and from any other screen shot that has this structure.

The algebraic sums that subjects faced during this experiment are characterized according to table 4.2.

No of Operators	Cardinality	Prob of $\succeq -\max$
1	30	$\frac{1}{30}$
2	120	$\frac{1}{60}$
3	360	$\frac{1}{90}$

Figure 4.2: Main Characteristics of the Algebraic Sums

A choice problem associated with an algebraic sum with 1, 2, and 3 operators is composed of 30, $\frac{6!}{(6-3)!} = 120$, and $\frac{6!}{(6-4)!} = 360$ alternatives, respectively. Moreover, screen shots are designed in such a way that if a subject inserted numbers at random in the algebraic sums, then the probabilities that she chooses the $\succeq$-maximal alternative would be $\frac{1}{30}$, $\frac{1}{60}$, and $\frac{1}{90}$ for algebraic sums with 1, 2 and 3 operators, respectively.

4.4 Hypotheses and Results

4.4.1 Errors

Suppose that a fully rational individual was asked to participate to this experiment. Because of the full rationality assumption, she would always select the $\succeq$-maximal alternative from any choice problem. That is, she would plug into each algebraic sum the ordered sequence of numbers such that $d^{act} = 0$.

However, according to Simon (1955)'s bounded rationality, this should not be always the case. Subjects do not have infinite computational capabilities and are likely to commit mistakes. Moreover, they do not know all alternatives before deciding. Rather, they discover and analyze options sequentially. The first hypothesis assumes that subjects do not always choose the alternative that maximizes their material payoff.

Hypothesis 1 *Subjects do not choose the $\succeq$-maximal alternative in* 100% *of the cases.*

In order to test hypothesis 1, we computed the number of times each subject chose an alternative different from the $\succeq$-maximal one and then we averaged across subjects. Since each subject solved 15 choice problems (3 algebraic sums for each of the 5 screen shots), then averages are numbers between 0 and 15. It turns out that on average subjects failed to choose the optimum 10.7 times, which, in percentage, is the 71.33% of the cases. If one looks at this measure within each session, results do not change. In session 1 the average number of failures is 11.05 (73.67%), in session 2 is 10.5 (70%), and in session 3 is 10.55 (70.33%). We can safely conclude that hypothesis 1 is confirmed.

A central part of Simon' satisficing theory is that individual's ability to identify the optimum crucially depends on the environment in which she is asked to operate. The extent to which the environment is complex and new to the subject matters.

As a proxy for complexity we use the number of operators in the algebraic sum. That is, the longer the sum the more complex the environment. We think that this assumption is reasonable, because at longer algebraic sums correspond choice problems with greater cardinality and smaller probabilities of choosing the optimal alternative (see also figure 4.2). What we expect is that at more complex choice problems correspond more errors (i.e., number of choices different from the $\succeq$-maximal).

Hypothesis 2 *There are more errors at more complex problems, ceteris paribus.*

We called the algebraic sums with 1, 2, and 3 operators *algebraic sums (or choice problems) of complexity 1 (C1), 2 (C2), and 3 (C3)*, respectively. For each level of complexity, we computed the number of times each subject chose an alternative different from the $\succeq$-maximal and then averaged across subjects. Since each subject solved 5 choice problems of complexity i, where $i = 1, 2, 3$, then averages are numbers between 0 and 5. The average number of errors for choice problems of complexity 1 is 2.2, of complexity 2 is 4, and of complexity 3 is 4.5. In percentage, 44%, 80%, and 90%, respectively. We then compared the average number of errors per subject across complexity by employing a one-tailed T-test for dependent samples. Results are reported in figure 4.3.[2]

Variables	Mean Diff.	St.Err.Mean Diff.	t-stat.	Df	Sig.(1-tailed)
C2-C1	1.783	.193	9.229	59	.000
C3-C2	.500	.157	3.189	59	.001
C3-C1	2.283	.175	13.061	59	.000

Figure 4.3: T-test for Dependent Samples - Differences in Average Error Per Complexity

Differences in the average error are all positive and significant at the 1% level. Hypothesis 2 is, therefore, confirmed.

Since, according to Simon, not only complexity, but also the extent to which the environment is new to the subject affects performances, we expect

[2]We tested H_0: the average error at complexity i is equal to the average error at complexity j, against H_1: the average error at complexity i is greater than the average error at complexity j, where $(i, j) \in \{(2, 1), (3, 1), (3, 2)\}$.

that the more familiar is the environment, the less number of mistakes subjects commit. As a proxy for *familiarity of the environment* we employ the order with which subjects faced screen shots. We believe that it is natural to assume that the first screen shot that subjects faced constituted for them a new kind of environment. On the contrary, at the fourth or fifth screen shot, it seems plausible to assume that subjects learnt and found themselves in a more familiar situation than at the first or at second one.

Hypothesis 3 *Errors decrease with screen shot order, ceteris paribus.*

We counted the number of errors each subject committed at the ith screen shot (Si), with $i = 1, 2, 3, 4, 5$, and then averaged across subjects. Since each subject solved 3 problems for each screen shot, then averages are numbers between 0 and 3. Results are reported in figure 4.4.

	S1	S2	S3	S4	S5
# of Average Errors	2.43	2.18	2.08	1.95	2.05
% of Average Errors	81.11	73.78	69.44	65.00	68.33

Figure 4.4: Average Errors Per Screen shot Order

From figure 4.4 it seems that average errors decrease with screen shot order until the fourth screen shot and then increases again. A T-test for dependent samples reveals that only the differences in average error between S1 and S3, S1 and S4, and S1 and S5 are significant at the 1% level (see figure 4.5).[3] Moreover, the differences in average error between S1 and S2 and S2 and S4 are significant at the 5% level. However, it is important to note that even though there is an average increase in errors from S4 to S5, the average error at S5 is still less than the one at S1 and this difference is significant.

[3] We tested H_0: the average error at screen shot Si is equal to the average error at the screen shot Sj, against H_1: the average error at the screen shot Si is greater than the average error at the screen shot j, where $(i, j) \in \{(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)(3, 4), (3, 5), (4, 5)\}$.

Variables	Mean Diff.	St.Err.Mean Diff.	t-stat.	Df	Sig.(1-tailed)
S1-S2	.250	.118	2.120	59	.019
S1-S3	.350	.108	3.227	59	.001
S2-S3	.100	.125	.799	59	.214
S1-S4	.483	.097	5.007	59	.000
S2-S4	.233	.110	2.124	59	.019
S3-S4	.133	.120	1.112	59	.136
S1-S5	.383	.107	3.598	59	.000
S2-S5	.133	.140	.955	59	.172
S3-S5	.033	.152	.219	59	.414
S4-S5	-.100	.125	-.799	59	.207

Figure 4.5: T-test for Dependent Samples - Differences in Average Error Per Screen shot Order

An interpretation of this result could be that from the first to the fourth screen shot subjects learnt and progressively became more and more familiar with the environment. Consequently, they progressively committed less and less errors on average. Then, from the fourth screen shot they mastered the situation, but committed more mistakes because got tired of solving algebraic sums.

In any case the test seems to suggest that the variable *familiarity of the environment* does not have a strong effect on subjects performance. Hence, we conclude that hypothesis 3 is only partially confirmed.

4.4.2 Choices

Simon (1955) argues that individuals search through the choice set and stop searching as soon as they identify the first alternative that meets some threshold. The aspiration level that defines an alternative to be satisfactory again depends on the environment. If the environment is relatively new to subjects and relatively complex, then the threshold tends to decrease and invididuals are content with alternatives relatively far from the optimum. On the contrary, when the environment is relatively familiar to subjects and relatively simple, then the threshold tends to *appreciate* and inviduals are content with alternatives relatively close to the optimum. Therefore, we expect that at more complex problems solutions are more sub-optimal. In addition, chosen alternatives are closer to the optimum at subsequent screen shots.

Hypothesis 4

1. *Solutions are more sub-optimal at more complex problems, ceteris paribus.*

2. *Chosen alternatives are closer to the optimum at subsequent screen shots, ceteris paribus.*

In order to test these hypotheses we first classified alternatives in indifference classes according to the above preference relation $\succeq$, where '1' identifies the first class, '2' the second class, and so on for the remaining classes. In a given algebraic sum, the first class includes all ordered sequences of numbers such that the distance between the Intermediate Result and the Goal is zero when the time expires. In general the ith class contains all sequences such that, when the time expires, the distance between the Intermediate Result and the Goal is D_i, provided that $D_j \geq D_i \geq D_k$, for all $j \geq i \geq k$ and $i = 1, \ldots, N$, where N is the number of indifference classes.

The graph of figure 4.6 shows the distribution of chosen indifference classes per complexity.[4]

From the graph it seems that subjects actually chose more sub-optimal solutions at more complex problems. In order to get confirmation of this intuition, we calculated for each level of complexity the average chosen indifference class per subject and then performed a T-test (see figure 4.7).[5]

[4] Indifference classes are cut at the 18th class because of space needs. However, all visible distributions include more than the third quartile. Specifically, $P(Ind.Class \leq 18\text{th}) = 0.9567$ for C1, $P(Ind.Class \leq 18\text{th}) = 0.8933$ for C2, and $P(Ind.Class \leq 18\text{th}) = 0.7967$ for C3.

[5] We tested H_0: the average chosen class at complexity i is equal to the average chosen class at complexity j, against H_1: the average chosen class at complexity i is greater than the average chosen class at complexity j, where $(i, j) \in \{(2, 1), (3, 1), (3, 2)\}$.

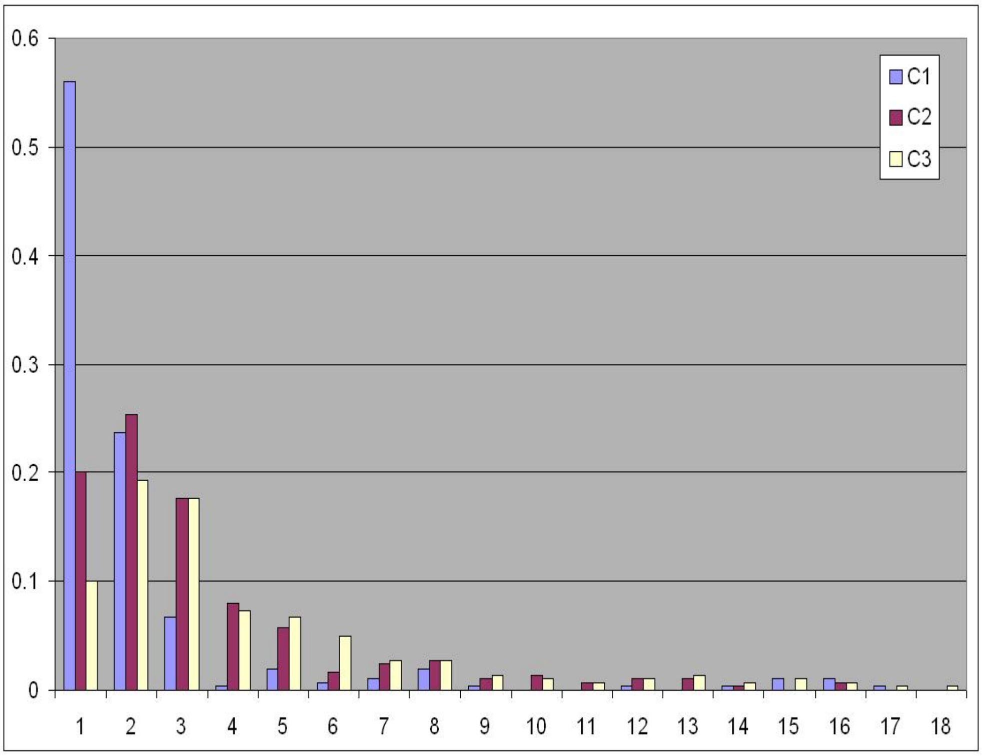

Figure 4.6: Distribution of Chosen Indifference Classes Per Complexity

The test reveals that all differences in the average chosen class are positive and significant at the 1% level. Hence we can conclude that hypothesis 4.1 is confirmed, that is, solutions are more sub-optimal at more complex problems.

We repeated the same exercise for testing hypothesis 4.2. The graph of figure 4.8 shows the distribution of chosen indifference classes per screen shot order.[6]

From the graph it seems that for the first two or three classes frequencies increase with screen shot order at least until the second/third screen shot. For the remaining classes there is not a clear effect and it is difficult to draw

[6]Indifference classes are cut at the 20th class because of space needs. However, all visible distributions include more than the third quartile. Specifically, $P(Ind.Class \leq 20\text{th}) = 0.7778$ for S1, $P(Ind.Class \leq 20\text{th}) = 0.8944$ for S2, $P(Ind.Class \leq 20\text{th}) = 0.9056$ for S3, $P(Ind.Class \leq 20\text{th}) = 0.9333$ for S4, and $P(Ind.Class \leq 20\text{th}) = 0.9611$ for S5.

Variables	Mean Diff.	St.Err.Mean Diff.	t-stat.	Df	Sig.(1-tailed)
C2-C1	3.3067	.5840	5.662	59	.000
C3-C2	5.2933	1.0916	4.849	59	.000
C3-C1	8.6000	1.2452	6.906	59	.000

Figure 4.7: T-test for Dependent Samples - Differences in Average Chosen Class Per Complexity

any kind of conclusion.

In order to shed light on this this relationship, we calculated the average chosen indifference class per subject for each screen shot and then performed a T-test (see figure 4.9).[7]

What we can infer from the test is that the average chosen class decreases with screen shot order, because mean differences are all positive (second column of figure 4.9). This implies that subjects got progressively closer to the optimum. However, only the differences in average chosen class between S1 and all the other screen shots, between S2 and S5, and between S3 and S5 are significant at the 1% level. The differences between S2 and S4 and S4 and S5 are significant at the 10% level. This implies that subject actually improved their performances over time, but this effect is not strong. Therefore, hypothesis 4.2 cannot be fully confirmed.

The results of this subsection suggest that subjects' behavior appear to be consistent with the satisficing heuristic. Moreover, it seems that *complexity* has a stronger impact on average performance than the variable *familiarity of the environment*.

[7] We tested H_0: the average chosen class at screen shot Si is equal to the average chosen class at the screen shot Sj, against H_1: the average chosen class at the screen shot Si is greater than the average chosen class at the screen shot j, where $(i,j) \in \{(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(2,5)(3,4),(3,5),(4,5)\}$.

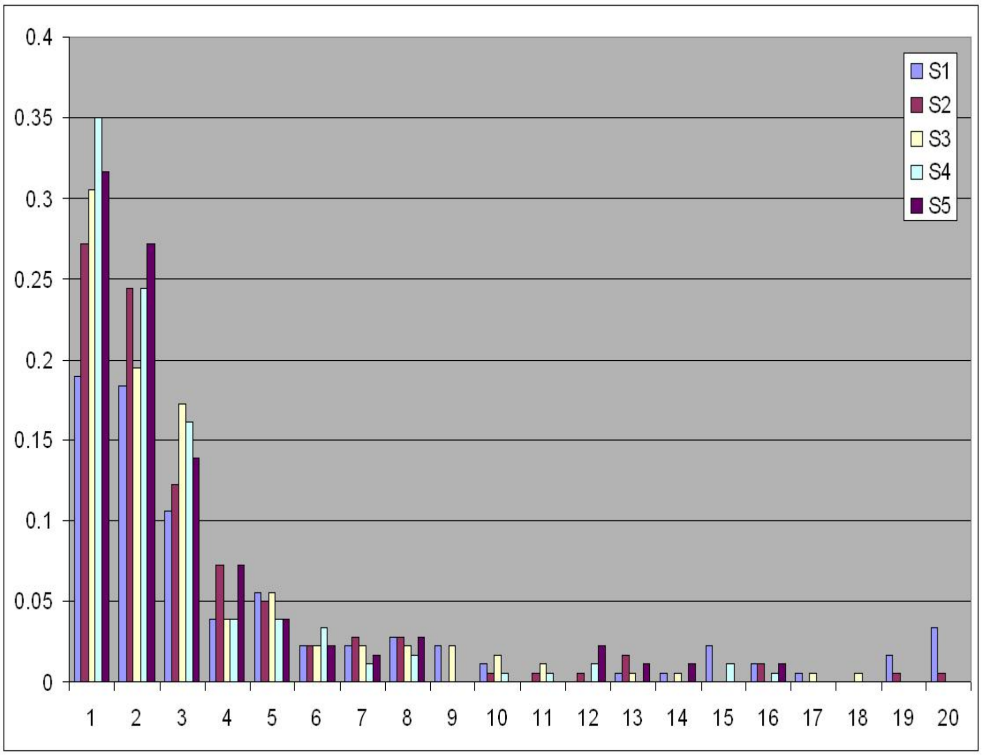

Figure 4.8: Distribution of Chosen Indifference Classes Per Screen Shot Order

4.4.3 Decision Time

Subjetc had 75 seconds for solving each screen shot. They were free to allocate the amount of available time among algebraic sums as they wanted. What we expect is that subjects spend more time on more complex problems than on simpler ones. This is also consistent with the bounded rationality assumption: subjects do not have infinite computational capabilities and, therefore, handle simple problems in relatively little time and concentrate most in solving more complex problems.

Hypothesis 5 *Decision time increases with complexity.*

We computed the average amount of time (in milliseconds) each subject spent on algebraic sums of different complexity and then averaged across

Variables	Mean Diff.	St.Err.Mean Diff.	t-stat.	Df	Sig.(1-tailed)
S1-S2	4.989	1.0480	4.761	59	.000
S1-S3	5.2278	1.047	4.991	59	.000
S2-S3	.2389	.9625	.2480	59	.402
S1-S4	6.3000	1.2130	5.193	59	.000
S2-S4	1.3110	.8880	1.4770	59	.072
S3-S4	1.0722	.8787	1.2200	59	.114
S1-S5	7.4500	1.082	6.887	59	.000
S2-S5	2.4610	.7680	3.204	59	.001
S3-S5	2.2222	.7874	2.8220	59	.003
S4-S5	1.150	.8690	1.3230	59	.095

Figure 4.9: T-test for Dependent Samples - Differences in Average Chosen Class per Screen shot Order

subjects. Let DT1, DT2, and DT3 denote the amount of time spent on algebraic sums of complexity 1, 2, and 3, respectively. On average subjects spent 16,281.97ms, 24,237.46ms, and 26,400.65ms on algebraic sums of complexity 1, 2, and 3, respectively. We then performed a T-test in order to check whether these differences are significant or not (see figure 4.10).[8]

Variables	Mean Diff.	St.Err.Mean Diff.	t-stat.	Df	Sig.(1-tailed)
DT2-DT1	7.95549E3	1.11612E3	7.128	59	.000
DT3-DT2	2.16318E3	1.55605E3	1.390	59	.085
DT3-DT1	1.01187E4	1.27629E3	7.928	59	.000

Figure 4.10: T-test for Dependent Samples - Differences in Average Decison Time Per Complexity

What emerges from the test is that subject clearly spent more time on algebraic sums of complexity 2 and 3 than on sums of complexity 1. In fact

[8]We tested H_0: the average amount of time spent on sums of complexity i is equal to the average amount of time spent on sums of complexity j, against H_1: the average amount of time spent on sums of complexity i is greater than the average amount of time spent on sums of complexity j, where $(i, j) \in \{(2, 1), (3, 1), (3, 2)\}$.

mean differences (DT2-DT1 and DT3 - DT1) are positive and statistically significant at the 1% level. However, even though on average subjects spent more time on algebraic sums of complexity 3 than on sums of complexity 2, this difference is significant only at the 10% level. An interpretation of this result might be that whereas subjects clearly found algebraic sums of complexity 2 and 3 more demanding than algebraic sums of complexity 1, they did not have the same feeling about sums of complexity 2 and 3. Even though subjects spent on average more time on the most complex sums, it seems that sums of higher complexity (C2 and C3) were perceived almost as equally demanding. Nevertheless, we can safely conclude that hypothesis 5 is confirmed.

4.4.4 Clicks

Subjects were told in the instructions that only completed algebraic sums (sums without empty spaces between operators) are taken into account for calculating the payoff. Clearly, in order to complete a sum of complexity 1 two insertions are necessary: one for the first and one for the second number. Similarly, for completing an algebraic sum of complexity 2 and 3 at least 3 and 4 inputs are needed, respectively. However, subjects were free to change the inserted numbers as many times as they wanted before the time expired. We believe that it is interesting to investigate the extent to which subjects exploited this opportunity.

We call *click* the action by which a number is inserted into an algebraic sum. We can interpret a click as an intermediate decision, because after a number is inserted in an algebraic sum, the correspondent Intermediate Result gets updated and by looking at it subjects can easily figure out how far they are from the Goal. As we have just seen, the minimum number of clicks for completing an algebraic sum of complexity 1 is 2, for a sum of complexity 2 is 3, and for a sum of complexity 3 is 4. We first checked whether subjects made extra clicks, i.e. more clicks than the minimum for each level of complexity.

Hypothesis 6 *Subjects made extra clicks.*

We computed the average number of extra clicks each subject made for every level of complexity and checked whether these averages are significantly greater than zero by employing a one-sample T-test (see figure 4.11).[9] Let EC1, EC2, and EC3 denote the average number of extra clicks at complexity 1, 2, and 3, respectively.

Variable	Mean Diff.	t-stat.	Df	Sig.(1-tailed)
EC1	1.025	9.068	59	.000
EC2	2.1844	9.189	59	.000
EC3	1.9106	10.125	59	.000

Figure 4.11: One-sample T-test - Average Number of Extra Clicks per Complexity

Mean differences are all positive and significant at the 1% level. This means that subject actually made extra clicks indipendently of the complexity of the problem.

According to Simon (1955), subjects do not know all alternatives in the choice set before deciding, but discover and analyze them sequentially. If at relatively simple problems, individuals easily figure out how the choice set looks like, at more complex one, they have difficulties. He argues that, for this reason, they employ a trials and errors kind of search procedure. Hence, we expect that extra clicks, if any, should increase with complexity, because, at more complex problems should correspond more trials to get close to the optimum.

Hypothesis 7 *Extra clicks increase with complexity.*

We used the average number of extra clicks each subject made for every level of complexity for performing a T-test (see figure 4.12).[10]

[9] We tested H_0: the average number of extra clicks at complexity i is equal zero,

Variables	Mean Diff.	St.Err.Mean Diff.	t-stat.	Df	Sig.(1-tailed)
EC2-EC1	1.1594	.2023	5.731	59	.000
EC3-EC2	-.2739	.2857	-.959	59	.171
EC3-EC1	.8856	.2209	4.009	59	.000

Figure 4.12: T-test for Dependent Samples - Differences in Average Number of Extra Clicks Per Complexity

From figure 4.12 we can infer that subjects made on average significantly more extra clicks at problems of complexity 2 and 3 than at problems of complexity 1. However, subjects made more extra clicks at problems of complexity 2 than at problems of complexity 3. Even though the difference in not significant, this result is surprising. An explanation could be that subjects judged choice problems of complexity 3 as too difficult and preferred on average to draw their attention to simpler problems. Alternatively, subjects may have judged problems of complexity 2 and 3 almost equally demanding and decided to focus more on problems of complexity 2 because there were more chances of getting close to the optimum. The second explanation would also be consistent with the results of hypothesis 5.

4.5 Order

Each screen shot contained a list of three algebraic sums of different complexity randomly ordered. The order with which sums had to be solved was completely up to subjects discretion. In this section we investigate the way in which subjects decided to approach this issue.

Following Caplin, Dean and Martin (2009), we identified five strate-

against H_1: the average number of extra clicks at complexity i is is greater than zero, where $i = 1, 2, 3$.

[10] We tested H_0: the average number of extra clicks at complexity i is equal to the average number of extra clicks at complexity j, against H_1: the average number of extra clicks at complexity i is greater than the average number of extra clicks at complexity j, where $(i, j) \in \{(2, 1), (3, 1), (3, 2)\}$.

gies: Top-Bottom (TB), Bottom-Top (BT), Simple-Complex (SC), Complex-Simplex (CS), and Uncategorized (U). The TB (BT) strategy implies that subject solve algebraic sums from the top (bottom) to the bottom (top) of the screen shot. The SC (CS) strategy, instead, implies that subjects solve sums from the simplest (more complex) to the more complex (simplest). That is, first sums of complexity 1 (3), then of complexity 2 (2), and finally of complexity 3 (1). The U strategy encompasses any strategy that does not fall in any of the previous categories.

We then categorized played screen shots according to these strategies. Notice that TBs and SCs kind of strategies are not mutually exclusive. For this reason we identified also the intersections. Moreover, since each of the 60 subjects went over 5 screen shots, then we collected 300 observations (1 for each played screen shot). Results are summarized in figure 4.13.

Strategy	#	%
TB	132	44.00
BT	38	12.67
SC	120	40.00
CS	13	4.33
TB∩SC	39	13.00
TB∩CS	10	3.33
BT∩SC	22	7.33
BT∩CS	0	0
U	68	22.67
Total	300	100.00

Figure 4.13: Frequencies of Chosen Strategies (TB, BT, SC, CS, U) and Intersections

The results suggest that the most frequent strategy is the TB one, followed by the SC one. Intersections seem to reveal that strategies BT and CS were chosen only when they coincided with strategies SC and TB, respectively. Only the 22.67% of the observations were not classifiable.

We also investigated how the choice of a particular strategy affected the performance. We concentrated only on strategies TB, SC, and U, because these were the most frequently used. This analysis turned out to be complicated because subjects were allowed to switch from one strategy to another one across the five screen shots. For this reason we proceeded in the following way. We first averaged chosen indifference classes within each screen shot, so that for each observation (played screen shot) we had a measure of performance. Then, we categorized played screen shots according 'pure' strategies. That is, once that we classified the screen shots according to TB, SC, or U, we eliminated all played screen shots that were consistent with more than one strategy. Subsequently, for every pair of strategies, we created two subgroups of screen shots. One group, called *dependent*, contained screen shots played by subjects who used both strategies. That is, for each subject we had two observations: one screen shot played with one strategy and another screen shot played with the other strategy. The second group, called *independent*, contained screen shots played by subjects who used either one or the other strategy. Figure 4.14 reports the number of observations per group for every pair of strategies.

Pairs	U vs SC		U vs TB		SC vs TB	
Strat.	U	SC	U	TB	SC	TB
Dep.	19	19	26	26	16	16
Indep.	22	12	15	14	15	24

Figure 4.14: No. of Observations per Group for Each Pair of Strategies

Our goal was to check, for every pair of strategies, whether the use of one strategy yields on average more material payoff than the use of another strategy. We performed, for each pair of strategies, a T-test for dependent samples for the observations that belong to the dependent group (figure 4.15) and a t-test for independent samples for the observations that belong to the independent group (figure 4.16).[11]

[11] In both t-test we tested H_0: the average chosen class at screen shots played according

Variables	Mean Diff.	St.Err.Mean Diff.	t-stat.	Df	Sig.(1-tailed)
U-SC	2.294	1.551	1.479	18	.076
U-TB	3.564	1.551	2.298	25	.015
SC-TB	1.500	.716	2.095	15	.027

Figure 4.15: T-test for Dependent Samples - Differences in Average Chosen Class Per Strategy (Dependent Group)

The first test reveals that all differences are positive and significant (U-SC at the 10% and U-TB and SC-TB at 5% level). This implies that the strategy that yielded on average the greatest payoff is TB, followed by SC. The worst strategy is U. This result is not surprising, because we think that in general subjects who behaves according to U do not approach screen shots in a systematic and organized way and, therefore, it seems reasonable to expect them to perform worse than those who use TB or SC.

	Eq. Variance		T-test for Equality of Means				
Pair	F	Sig.	Mean Diff.	St.Err.Mean Diff.	t-stat.	Df	Sig.(1-tailed)
U-SC (Eq.Var.)	15.797	.000	3.849	1.879	2.048	32	.025
U-SC (¬Eq. Var.)			3.849	1.394	2.762	22.327	.006
U-TB (Eq.Var.)	13.671	.001	2.932	1.619	1.811	27	.041
U-TB (¬Eq.Var.)			2.932	1.564	1.875	14.454	.041
SC-TB (Eq.Var.)	.354	.555	-.562	1.121	-.501	37	.310
SC-TB (¬Eq.Var.)			-.562	.976	-.576	35.940	.284

Figure 4.16: T-test for Independent Samples - Differences in Average Chosen Class Per Strategy (Independent Group)

The results of figure 4.16 only partially confirm the findings of the test for dependent samples. The t-test for independent samples suggests that we can reject the null hypothesis that the variables *average chosen class* at U and

to strategy i is equal to the average chosen class at screen shots played according to strategy j, against H_1: the average chosen class at screen shots played according to strategy i is greater than the average chosen class at screen shots played according to strategy j, where $(i, j) \in \{(U, SC), (U, TB), (SC, TB)\}$. The T-test for independent samples includes also a Levene's test for equality of variances.

at SC have equal variance at the 1% significance level. Similarly, we cannot at U and at TB. Differences in means (second and fourth row of figure 4.16) are positive and statistically significant at the 1% and 5% level, respectively. This means that SC and TB are again revealed to be superior to U. On the other hand, looking at the pair (SC,TB) we first notice that we cannot reject the null hypothesis that the variables *average chosen class* at SC and at TB have equal variance. Moreover, though not significant, the difference in means is negative (sixth row of figure 4.16). That is, SC seems to perform better than TB. This partially contradicts our previous findings.

4.6 Individual Features

Collected data include general information about subjects, such as age, sex, and time taken to solve the questionnaire. In this section we investigate whether these individual features affect the performance.

We set as dependent variable the *subject average chosen class*. Independent variables were sex, age, time taken to solve the questionnaire, number of experiments to which the subject participated in 2010, and enrollment year.[12] We run an ordered probit (see figure 4.17).[13]

Results suggest only the estimates of the variables *time taken to solve the questionnaire* and *age* are significant at 1% and 10% level, respectively. Since the estimate for *time taken to solve the questionnaire* is very close to zero, but positive, then this implies that the more time subjects spent on the questionnaire, the worse they performed. This is an intuitive result, because the variable *time taken to solve the questionnaire* appears to be a good proxy for subjects intelligence. The estimate of the variable *age* is also positive. This implies that relatively older subjects preformed worse.

[12] Sex is a dummy variable equal to 1 when the subject is a male. Enrollment year is defined as follows: 1 = first-year student, 2 = second-year student, 3 = third-year student, 4 = Postgraduate Student, 5 = out-of-course student, and 6 = graduate who works.

[13] In order to reduce the number of thresholds, we approximated the dependent variable to the closest inferior integer.

	Estimate	St. Error	Wald	Df	Sig.
Class= 2	1.777	1.155	2.368	1	.124
Class= 3	2.914	1.157	6.347	1	.012
Class= 4	3.531	1.168	9.136	1	.003
Class= 5	3.852	1.176	10.724	1	.001
Class= 6	3.976	1.180	11.361	1	.001
Class= 7	4.089	1.183	11.947	1	.001
Class= 8	4.458	1.195	13.907	1	.000
Class= 9	4.662	1.203	15.020	1	.000
Class= 11	5.044	1.220	17.106	1	.000
Class= 12	5.315	1.233	18.582	1	.000
Class= 13	5.527	1.245	19.719	1	.000
Class= 14	5.959	1.274	21.896	1	.000
Class= 16	6.162	1.290	22.829	1	.000
Class= 18	6.431	1.316	23.895	1	.000
Class= 21	6.813	1.366	24.866	1	.000
Sex	-.162	.298	.296	1	.587
Age	.110	.061	3.257	1	.071
Time Quest.	5.934E-6	1.101E-6	29.070	1	.000
No. Exp.	.060	.046	1.693	1	.193
En. Year	-.243	.162	2.258	1	.133

Figure 4.17: Ordered Probit of Subjetc Average Chosen Class on Sex, Age, Time Quest., No.Exp., and En. Year

Having said that, a reason for which the regressors do not explain much of the dependent variable might be that each observation of the dependent variable is an average of 15 choices (each subject faced 3 algebraic sums for each of the 5 screen shots). Hence, a lot of variability of the data is absorbed by and hidden behind the average. This might have prevented the dependent variables from exerting their explanatory power.

4.7 Concluding Remarks

We run an experiment to test whether subjects behavior is consistent with the Simon (1955)' satificing heuristic.

We found that often subjects fail to choose the alternative that yields the maximum payoff, revealing that they have limited computational capabilities and that their behavior is not consistent with the standard assumption. The analysis of choice data clearly suggests subjects massively selected suboptimal solutions. The average performance is negatively affected by the complexity of the environment and, even though the effect is not strong, increases as subjects become more familiar with the environment. Decision time and extra-clicks data confirm these findings. However, it seems that an increase in complexity does not necessarily cause an increase in decision time and extra-clicks. A decisive increase occurs only when complexity increases from its lowest level (1) to higher levels (2 and 3). However, among problems of higher complexity there is not a clear effect. This may be due to the fact that either subjects do not perceive complexity as it actually is or they do, but prefer to concentrate their attention on relatively simpler problems (of complexity 2) because there are more chances of reaching the optimum. Further investigation would be needed to disentangle this issue. Concerning the order with which subjects solve problems, the most frequent strategies are Top-Bottom and Simple-Complex, which are also the ones that guarantee the highest payoff. We conclude that in general this evidence supports Simon's idea of bounded rationality.

The results provide also some useful insights for modeling satisficing behavior. For instance, both Caplin and Dean (2011) and the model proposed in the second chapter of this thesis assume that the threshold that defines an alternative to be satisfactory is fixed. However, this evidence suggests that the threshold varies across problems and crucially depends on complexity and, to a lesser degree, on the extent to which the environment is familiar to subjects. A nice exercise would be to extend these models in order to account for this possibility.

Since collected data include also intermediate choice, it would be interesting to analyze the path that led subjects to their final choice. In Caplin, Dean and Martin (2009) the alternatives that constitute a choice set are listed and visible. According to choice process data, subjects are allowed to change the chosen alternative as many times as they want before the time expires. Caplin, Dean and Martin find that subjects decide to switch only when a superior alternative is identified and chosen. That is, subjects always choose the best alternative in the consideration set. However, this may be due to the fact that in their experiment search history is partially visible (the provisional choice at any point in time). We expect that in a context, such as in this experiment, in which alternatives are not listed and search history is not visible, subjects do not behave in the same way. Rather, we expect that, especially at more complex problems, subjects end up choosing a non-optimal alternative in the consideration set. There are two reasons behind this intuition. First, subjects might not remember how to reach the superior alternative. Second, subject might fail to choose the superior alternative because they are not able to figure out what is the path that leads to it. An analysis of intermediate choices could shed light on how subjects explored the choice set and in particular on this issue.

Finally, a notable amount of experiments has been run within 'natural' environments, such as marketplaces (see Harrison and List (2004) for a survey and Palacios-Huerta and Volj (2009) and Apesteguia and Palacios-Huerta (2010) as more recent studies). It would be interesting to investigate whether subjects still exhibit satisficing attitudes also outside the laboratory by running a field experiment.

Chapter 5

Conclusions

In this thesis we investigate what are the effects of assuming economic agents to be boundedly rational on various fields of microeconomics. In particular we define a boundedly rational DM as an individual who follows the satisficing heuristic proposed by Simon (1955; 1956). We first propose a theory of boundedly rational choice within the revealed preference approach. Then, we investigate whether the fact that there is uncertainty about consumers' rationality enhances their welfare in several markets. Finally, we propose an experiment aimed at testing the extent to which subjects' behavior is consistent with the satisficing heuristic.

The next section highlights what is our contribution relative to the existing literature and discusses limitation of the present work and possible future developments.

Contribution Relative to the Existing Literature and Future Work

Our major contribution to choice theory is to provide a formalization of the satisficing heuristic within the revealed preference approach that differs from the existing studies (Rubinstein and Salant, 2006; Caplin and Dean, 2011). On the one hand, Rubinstein and Salant (2006) propose a model in which DMs analyze alternatives sequentially. We generalize their framework by assuming that DMs do not necessarily examine alternatives one by one. On the other hand, unlike Caplin and Dean (2011) we do not make use of choice process data, but we consider only final choices. Moreover, we perform a rich and complete analysis by axiomatically characterizing our procedure and providing behavioral definitions of satisfaction, attention, and preference under three different domains. This study could be improved in two ways. First, the relationships between our model and Rubinstein and Salant (2006) could be further investigated. A nice exercise would be to extend their results to menu sequences. Second, the experiment proposed in the fourth chapter highlights the fact that the threshold that defines an alternative to be satisfactory vary across choice problems. In particular it is strongly influenced by complexity. It would be interesting to extend our model by allowing the threshold to vary depending on the extent to which the choice problem under consideration is more or less complex. This would make the model more realistic.

The third chapter investigates what is the impact of uncertainty about consumers' rationality on their welfare in a variety of markets. An extensive body of literature has been developed in analyzing the effects of bounded rationality on industrial organization (Spiegler, 2011) that suggests that boundedly rational individuals are typically subject to exploitation. We show that it is not obvious *a priori* whether uncertainty enhances consumers' welfare. In particular it depends on the assumptions that we make on the market model. For instance, in the quality-competition model uncertainty increases boundedly rational consumers' welfare. On the contrary, in the monopolistic screening model this does not happen. None of the existing studies of which we are aware assume consumers to behave according to the satisficing heuristic. In addition, the analysis of the results allows us to derive some suggestions for policy-makers. On the other hand, the major weakness of this study is that it provides a simple and preliminary analysis. It could be improved both by adding more complexity and realism to the model and by investigating other markets than those analyzed in the present work.

In the fourth chapter we propose an experiment aimed at testing the extent to which subjects' behavior is consistent with the satisficing heuristic. We show that in general subjects behavior is consistent with it and that complexity has a stronger impact on subjects' performance than the variable *familiarity of the environment*. In addition we derive some useful insights for modeling satisficing behavior, such as the one mentioned above. The closest study to our work is Caplin, Dean and Martin (2009). We depart from it for several reasons. First, we ask subjects to perform a different task. Second, unlike Caplin, Dean and Martin (2009), we do not use choice process data, but other data enrichment techniques, such as intermediate choices and decision time. Third, our design does not allow subjects to monitor part of the search history. This study could be improved by further analyzing intermediate choices. This would allow us to figure out what was the path that led subjects to their final choice and to shed further light on the decision-making process. Moreover, it would be interesting to verify whether also outside the laboratory subjects exhibit satisficing attitudes by running a field experiment.

Appendix A

Appendix

A.1 Independence of the Axioms

WARP does not imply MI. Let $X = \{x, y, z\}$. $\{x\} = C(\{x, y, z\})$, $\{x\} = C(\{x, y\})$, $\{x\} = C(\{x, z\})$, and $\{y, z\} = C(\{y, z\})$. This choice correspondence satisfies WARP, but violates MI.$\blacksquare$

MI does not imply WARP. Let $X = \{x, y, z\}$. $\{x\} = C(\{x, y, z\})$, $\{y\} = C(\{x, y\})$, $\{x\} = C(\{x, z\})$, and $\{y\} = C(\{y, z\})$. This choice correspondence satisfies MI, but violates WARP.$\blacksquare$

A.2 Proofs of Propositions

Proof of Proposition 1. Suppose that c is an SCF.

Assume that $x \in UC_{\succ}(X; x^s)$. Suppose, by contradiction, that there exists a $(B, \{B_j\}) \in \mathcal{D}_1$ such that $x \in B_{m-1}^{c(B,\{B_j\})}$. Then there are two cases. Assume first that $c(B, \{B_j\}) \in UC_{\succ}(X; x^s)$. In this case, $\Gamma_{(B,\{B_j\})} = B_{\bar{j}}$. However, since $x \in UC_{\succ}(X; x^s) \cap B_{m-1}^{c(B,\{B_j\})}$, then $\bar{j} > \min\{j | B_j \cap UC_{\succ}(X; x^s) \neq \emptyset\}$, which leads to a contradiction. Next, assume that $c(B, \{B_j\}) \notin UC_{\succ}(X; x^s)$.

In this case $\Gamma_{(B,\{B_j\})} = B$ and $\{c(B,\{B_j\})\} = \max(B;\succ)$. However, since $x \in UC_{\succ}(X;x^s)$, then $x \succ c(B,\{B_j\})$, which leads to a contradiction.

Conversely, assume that $x \notin UC_{\succ}(X;x^s)$. We want to show that there exists some $(B,\{B_j\}) \in \mathcal{D}_1$ such that $x \in B_{m-1}^{c(B,\{B_j\})}$. Let $\{y\} = \max(X;\succ)$. Take any $(A,\{A_j\}) \in \mathcal{D}_1$ such that $y \in A_j \setminus A_k$ with $k < j$ and $x \in A_k$. Since $x^s \in X$, then necessarily $y \in UC_{\succ}(X;x^s)$. This implies that $y = c(A,\{A_j\})$, which is the desired result.■

Proof of Proposition 2. Suppose that c is an SCF.

Suppose first that $y = c(A,\{A_j\})$ and $x \in A_m^{c(A,\{A_j\})}$. Independently of whether $UC_{\succ}(A;x^s) \neq \emptyset$ or not, $x \in \Gamma_{(A,\{A_j\})}$. Next, assume that $y = c(A,\{A_j\})$ and there exists a $(B,\{B_j\}) \in \mathcal{D}_1$ such that $y \in B_{m-1}^{c(B,\{B_j\})}$. By Proposition 1, y is unsatisfactory. Therefore, $\Gamma_{(A,\{A_j\})} = A$. Since $x \in A$, then $x \in \Gamma_{(A,\{A_j\})}$.

Conversely, assume that $x \in \Gamma_{(A,\{A_j\})}$. Assume first that $UC_{\succ}(A;x^s) \neq \emptyset$. In this case $c(A,\{A_j\}) \in UC_{\succ}(A;x^s)$. Since $\Gamma_{(A,\{A_j\})} = A_{\overline{j}}$, where $\overline{j} = \min\{j | A_j \cap UC_{\succ}(A;x^s) \neq \emptyset\}$, then $x \in A_m^{c(A,\{A_j\})}$, which means that x attracts attention at $(A,\{A_j\})$. Next, assume that $UC_{\succ}(A;x^s) = \emptyset$. By Proposition 1, $c(A,\{A_j\})$ is unsatisfactory. If this is the case, then there exists a $(B,\{B_j\}) \in \mathcal{D}_1$ such that $x \in B_{m-1}^{c(B,\{B_j\})}$, which is the desired result.■

Proof of Proposition 3. Suppose that c is an SCF.

Assume first that $x \succ y$. Take a $(A,\{A_j\}) \in \mathcal{D}_1$ such that $A = \{x,y\} = A_1$. Since $A_1 = A$, then necessarily x and y attract attention at $(A,\{A_j\})$ and $\Gamma(A,\{A_j\}) = A$. Since $c(A,\{A_j\}) = \max(A;\succ)$, then $x = c(A,\{A_j\})$, as desired.

Conversely, assume that there is some $(A,\{A_j\}) \in \mathcal{D}_1$ such that $x = c(A,\{A_j\})$ and y attracts attention at $(A,\{A_j\})$. By Proposition 2, $y \in \Gamma_{(A,\{A_j\})}$. Since $\{c(A,\{A_j\})\} = \max(\Gamma_{(A,\{A_j\})};\succ)$, then $x \succ y$, which is the desired result.■

Proof of Proposition 4. Suppose that c is an SCF$_2$.

Assume that $x \in UC_{\succ}(X; x^s)$. Suppose, by contradiction, that there exists a $(B, B_1) \in \mathcal{D}_2$ such that $x \in B_1$ and $c(B, B_1) \notin B_1$. Then there are two cases. Assume first that $c(B, B_1) \in UC_{\succ}(B; x^s)$. In this case, $\Gamma_{(B,B_1)} = \bar{B} \supset B_1$. However, since $x \in UC_{\succ}(X; x^s) \cap B_1$, then it is not true that $UC_{\succ}(B \setminus B_1; x^s) \neq \emptyset$, which leads to a contradiction. Next, assume that $c(B, \{B_j\}) \notin UC_{\succ}(X; x^s)$. In this case $\Gamma_{(B,B_1)} = B$ and $\{c(B, \{B_j\})\} = \max(B; \succ)$. However, since $x \in UC_{\succ}(X; x^s)$, then $x \succ c(B, B_1)$, which leads to a contradiction.

Conversely, assume that $x \notin UC_{\succ}(X; x^s)$. We want to show that there exists some $(B, B_1) \in \mathcal{D}_2$ such that $x \in B_1$ and $c(B, B_1) \notin B_1$. Let $\{y\} = \max(X; \succ)$. Take any $(A, A_1) \in \mathcal{D}_2$ such that $y \in A \setminus A_1$ and $x \in A_1$. Since $x^s \in X$, then necessarily $y \in UC_{\succ}(X; x^s)$. This implies that $y = c(A, \{A_j\})$, which is the desired result.■

Proof of Proposition 5. Suppose that c is an SCF$_2$.

Assume first that $x \succ y$. Take a $(A, A_1) \in \mathcal{D}_2$ such that $A = \{x, y\} = A_1$. Notice that necessarily $A_1 = A = \Gamma(A, A_1)$. Since $c(A, A_1) = \max(A; \succ)$, then $x = c(A, A_1)$, as desired.

Conversely, assume that there is some $(A, A_1) \in \mathcal{D}_2$ such that $x = c(A, A_1)$ and $y \in A_1$. Since $y \in \Gamma_{(A,A_1)}$ and $\{c(A, A_1)\} = \max(\Gamma_{(A,A_1)}; \succ)$, then $x \succ y$, which is the desired result.■

Proof of Proposition 6. Suppose that c is an SCF$_2$.

If $x \in A_1 \vee x = c(A, A_1)$, then obviously $x \in \Gamma_{(A,A_1)}$. Next, assume that $y = c(A, A_1)$ and there exists a $(B, B_1) \in \mathcal{D}_2$ such that $y \in B_1$ and $c(B, B_1) \notin B_1$. By Proposition 4, y is revealed to be unsatisfactory. Hence, $\Gamma_{(A,A_1)} = A$. This implies that $x \in \Gamma_{(A,A_1)}$, which is the desired result.■

Proof of Proposition 7. Suppose that c is an SCC.

Assume that $x \in UC_{\succ}(X; x^s)$. Since x is satisfactory, then for any $A \in \mathcal{P}(X)$ such that $x \in A$ there exist a $\{A_j\} \in \mathcal{A}$ for which $x = c(A, \{A_j\})$. This implies that $x = c(X, \{X_j\})$ for some $\{X_j\} \in \mathcal{X}$. Since $x \in X$, then there exists a sequence Since $x^s \in X$, then there exists an $(A, \{A_j\}) \in \mathcal{D}_1$ such that $x = c(A, \{A_j\})$. This implies that $x \in C(X)$.

Conversely, assume that $x \in C(X)$. Suppose, by contradiction, that $x \in UC_{\succ}(X; x^s)$. This implies that $x = c(X, \{X_j\})$ for all $\{X_j\} \in \mathcal{X}$ and $\{x\} = C(X)$. However, since $x^s \in X$, then there are some $\{X_j\} \in \mathcal{X}$ such that $x^s = c(X, \{X_j\})$. In this case $C(X)$ is not a singleton, which leads to a contradiction. Hence, $x \in UC_{\succ}(X; x^s)$, as desired.■

Proof of Proposition 8. Suppose that c is an SCC.

Assume first that $x \succ_{\neg S} y$. Since x is unsatisfactory, then there is some $B \in \mathcal{P}(X)$ such that $x^s \in B$. In this case, $x \neq c(B, \{B_j\})$ for all $\{B_j\} \in \mathcal{B}$, which implies that $x \notin C(B)$. Next, since x and y are unsatisfactory, then there exists some $A \in \mathcal{P}(X)$ such that $x, y \in A$ and $x = c(A, \{A_j\})$ for all $\{A_j\} \in \mathcal{A}$. Hence, $\{x\} = C(A)$, as desired.

Conversely, assume that there is some $A \in \mathcal{P}(X)$ such that $\{x\} = C(A)$ and $y \in A$ and some $B \in \mathcal{P}(X)$ such that $x \notin C(B)$. Since $x \notin C(B)$, then this implies that x is unsatisfactory. Next, since there is some $A \in \mathcal{P}(X)$ such that $\{x\} = C(A)$, then also y is unsatisfactory. In addition, this implies that $x \succ_{\neg S} y$.■

Proof of Proposition 9. **Necessity**. Assume that $C(A) = \bigcup_{\{A_j\} \in \mathcal{A}} c(A, \{A_j\})$, where $c(A, \{A_j\})$ is an SCF. We can rewrite C as:

$$C(A) = \max(A; R)$$

where R is a weak order on X and the indifference classes apart from the maximal one are singletons.

It is immediate to see that C satisfies WARP. Moreover, since all indifference classes different from the maximal one are singletons, then also MI is satisfied.

Sufficiency. Assume that C satisfies WARP and MI. We have to show that there exist a weak order on X such that $C(A) = \max(A; R)$ and all indifference classes apart from the maximal one are singletons.

It is well-known that if C satisfies WARP, then there exist a weak order R such that $C(A) = \max(A; R)$ for any $A \in \mathcal{P}(X)$. Next, suppose, by contradiction, that $x, y \in C(A)$ and there is some $B \in \mathcal{P}(X)$ such that $x \in B$ and $x \notin C(B)$, so that x is not R-maximal in X. However, this immediately contradicts MI. Hence, there exist no such B and all indifference classes apart from the maximal one are singletons, which is the desired result.■

Proof of Proposition 10. It is immediate to prove existence. Assume that firms play $\mathbf{x}^{\max} = (x_i^{\max})_{i \in N}$. In this case firm i's profits are $\frac{1}{n} - c(x^{\max})$. We show that there are not profitable deviations. Suppose that firm i deviates by playing any x' such that $x^{\max} \succ x'$. This implies that its profits are $-c(x')$ and the deviation is not profitable.

Now we prove uniqueness. We first show that any equilibrium has to be symmetric and than that any symmetric strategy profile different from $\mathbf{x}^{\max}$ is not an equilibrium.

Assume that firms play $\mathbf{x}$, where $x_i \succ x_j$ for some $i \in N$ and for all $j \neq i$, so that $\mathbf{x}$ is not symmetric. Since the FRC always compares the products that firms produce in advance, then $\pi_i(\mathbf{x}) = \frac{1}{n(i,\mathbf{x})} - c(x_i)$ for all i and $\pi_j(\mathbf{x}) = -c(x_j)$ for all $j \neq i$. Each firm j can profitably deviate by supplying an alternative of the same quality as firm i's. Therefore, a necessary condition for a strategy profile to be an equilibrium is that it has to be symmetric.

Finally, suppose, by contradiction, that there is another symmetric Nash equilibrium $\mathbf{x}'$ different from $\mathbf{x}^{\max}$. Firm i's profits are $\frac{1}{n} - c(x')$. Assume that one firm deviates by playing x'', where $x'' \succ x'$. The firm that deviates gets a profit equal to $1 - c(x'')$. We have to show that $1 - c(x'') > \frac{1}{n} - c(x')$. Since

$\sup\{c(x)|x \in X\} = \frac{1}{n}$ and $c(x'') > c(x')$, then we can rewrite the previous inequality as $1 - \frac{1}{n} > \frac{1}{n} - c(x')$, which implies that $\frac{n-2}{n} > -c(x')$. Since $n \geq 2$, then the deviation is profitable, but this leads to a contradiction. Therefore, $\mathbf{x}^{\max}$ is unique.■

Proof of Proposition 11. We first prove existence. Suppose that firms play $\mathbf{x}^s = (x_i^s)_{i \in N}$. In this case firm i's expected payoff is $\pi_i(\mathbf{x}^s) = \frac{1}{n}(1 - c(x^s)) - \frac{n-1}{n}c(x^s) = \frac{1}{n} - c(x^s)$.

Suppose by contradiction that $\mathbf{x}^s$ is not a Nash equilibrium. If this is the case, then there must be at least one profitable deviation.

Assume first that firm i deviates, by playing some pure strategy x', where $x^s \succ x'$. In this case, firm i's expected payoff is equal to $\pi_i(x_i', x_{-i}^s) = -c(x')$, which means that the deviation is not profitable.

Next, suppose that firm i deviates, by playing some pure strategy x'', where $x'' \succ x^s$. In this case, firm i's expected payoff is equal to $\pi_i(x_i'', x_{-i}^s) = \frac{1}{n}(1 - c(x'')) - \frac{n-1}{n}c(x'') = \frac{1}{n} - c(x'')$. Since $x'' \succ x^s$, then $c(x'') > c(x^s)$. Hence, the deviation is not profitable.

Since there are no profitable deviations, then a contradiction takes place and $\mathbf{x}^s$ is a Nash equilibrium.

Now we prove uniqueness. As in proof of Proposition 10 any equilibrium has to be symmetric. Suppose, by contradiction, that there is another symmetric pure-strategy Nash equilibrium $\mathbf{x}^* = (x_i^*)_{i \in N}$ different from $\mathbf{x}^s$. Assume that firms play $\mathbf{x}^*$. Hence, each firm i's profit is $\frac{1}{n} - c(x^*)$. Suppose first that $x^s \succ x^*$. Applying the reasoning of the proof of Proposition 10 we conclude that $\mathbf{x}^*$ cannot a Nash equilibrium, because each firm i could profitably deviate by playing $x^{**} \succ x^*$. Next, suppose that $x^* \succ x^s$. Notice that any deviation to x''' such that $x^s \preceq x''' \prec x^*$ is profitable, because $c(x^*) > c(x''')$. Therefore, $\mathbf{x}^*$ is not an equilibrium, which leads to a contradiction. Hence, $\mathbf{x}^s$ is unique.■

Proof of Proposition 12. Given a strategy profile $\mathbf{x}$, we can rewrite firm i's payoff function $\pi_i^u(\mathbf{x}) = \rho\pi_i^{FRC}(\mathbf{x}) + (1-\rho)\pi_i^{BRC}(\mathbf{x})$ as follows.

$$\pi_i^u(\mathbf{x}) = \begin{cases} \rho\left(\frac{1}{n(i,\mathbf{x})} - c(x_i)\right) + (1-\rho)[q\,(1-c(x_i)) - (1-q)\,(c(x_i))] & \text{if } (x_i \succeq x^s) \text{ and} \\ & (x_i \succeq x_j \forall j \in N) \\ \rho(-c(x_i)) + (1-\rho)[q\,(1-c(x_i)) - (1-q)\,(c(x_i))] & \text{if } (x_i \succeq x^s) \text{ and} \\ & (x_j \succ x_i \text{ for some } j \in N) \\ \frac{1}{n(i,\mathbf{x})} - c(x_i) & \text{if } (x_i, x_j \notin UC(\succeq; x^s)) \\ & \text{and } (x_i \succeq x_j\ )\forall j \in N \\ -c(x_i) & \text{otherwise} \end{cases}$$

Assume that $\rho \geq \bar{\rho} = n(c(x^{\max}) - c(x^s))$. Notice that $\bar{\rho} \in (0,1)$. Suppose not and assume first that $\bar{\rho} \leq 0$. This implies that $c(x^{\max}) \leq c(x^s)$, which leads to a contradiction. Next, assume that $\bar{\rho} \geq 1$. This implies that $\frac{1}{n} - c(x^{\max}) \leq -c(x^s)$, which leads to a contradiction. Hence, $\bar{\rho} \in (0,1)$.

We first show existence. Suppose that firms play $\mathbf{x}^{\max} = (x_i^{\max})_{i\in N}$. Since $x_i \succeq x^s$ and $x_i \succeq x_j\ \forall j \in N$, then firm i's expected profits are $\pi_i^u(\mathbf{x}^{\max}) = \frac{1}{n} - c(x^{\max})$.

Suppose by contradiction that $\mathbf{x}^{\max}$ is not a Nash equilibrium. If this is the case, then there must be at least one profitable deviation.

Assume that firm i deviates, by playing some pure strategy x', where necessarily $x^{\max} \succ x'$. Clearly, if $x' \prec x^s$, then the deviation is not profitable, because, independently of whether the consumer is an FRC or a BRC, firm i's profits would be equal to $-c(x')$. Hence, assume that $x' \succeq x^s$. Since $x_i' \succeq x^s$ and $x_j \succ x_i'$ for some $j \in N$, then in this case firm i's profits are $\pi_i^u(x_i', x_{-i}^{\max}) = \rho(-c(x')) + (1-\rho)(\frac{1}{n}(1-c(x_i')) - \frac{n-1}{n}c(x_i')) = \frac{1}{n} - c(x') - \frac{\rho}{n}$. The deviation is profitable only if $\frac{1}{n} - c(x') - \frac{\rho}{n} > \frac{1}{n} - c(x^{\max})$ or, equivalently, when $\rho < n(c(x^{\max}) - c(x'))$. Since $\rho \geq \bar{\rho} = n(c(x^{\max}) - c(x^s))$ and $c(x') \geq c(x^s)$, then the deviation is never profitable and a contradiction takes place. Hence, $\mathbf{x}^{\max}$ is a Nash equilibrium.

Now we prove uniqueness. As in proof of Proposition 10 and 11 any equilibrium has to be symmetric. Suppose, by contradiction, that there is another symmetric pure-strategy Nash equilibrium $\mathbf{x}^* = (x_i^*)_{i\in N}$ different from $\mathbf{x}^{\max}$. Assume that firms play $\mathbf{x}^*$. Hence, each firm i's profit are $\frac{1}{n} - c(x^*)$. Suppose first that $x^s \succ x^*$. Applying the reasoning of the proof of Proposition 10 we conclude that $\mathbf{x}^*$ cannot a Nash equilibrium, because each firm i could profitably deviate by playing $x' \succ x^*$. Next, suppose that $x^* \succeq x^s$ and assume that firm i deviates by playing x'', where $x^{\max} \succeq x'' \succ x^*$. Firm i's profits are then $\pi_i^u(x_i'', x_{-i}^*) = \rho(1-c(x''))+(1-\rho)(\frac{1}{n}(1-c(x_i''))-\frac{n-1}{n}c(x_i''))$ $= \rho + \frac{1}{n} - c(x'') - \frac{\rho}{n}$. The deviation is profitable only if $\rho + \frac{1}{n} - c(x'') - \frac{\rho}{n} > \frac{1}{n} - c(x^*)$ or, equivalently, when $\rho > \frac{n}{n-1}(c(x'') - c(x^*))$. Since $\rho \geq \bar{\rho} = n(c(x^{\max}) - c(x^s))$, $c(x^{\max}) \geq c(x''') > c(x^*) \geq c(x^s)$, then the deviation is profitable and a contradiction takes place. Hence, $\mathbf{x}^{\max}$ is unique.

Assume now that $\rho < \bar{\rho} = \frac{n}{n-1}\left(c(x^{\max}) - c(x^s)\right)$. It is clear that in this case $\mathbf{x}^{\max}$ is not a Nash equilibrium.

Finally, we show that there are no symmetric Nash equilibria $\mathbf{x}^* = (x_i^*)_{i \in N}$, where $x^s \succ x^*$. Suppose that $\mathbf{x}^*$ is a Nash equilibrium. Suppose, by contradiction, that $x^s \succ x^*$. In this case, each firm i's profits are $\frac{1}{n} - c(x^*)$. Assume that firm i deviates by playing x^s. Firm i's profits are then $\pi_i^u(x_i^s, x_{-i}^*) = 1 - c(x^s)$. Since $1 - c(x^s) > \frac{1}{n} - c(x^*)$ is always true, then the deviation is profitable and a contradiction takes place. Hence, $\mathbf{x}^*$ is not a Nash equilibrium.■

Proof of Proposition 13. Suppose first that the firm plays $x^{\max}$. If this is the case, the only candidate to be an equilibrium is strategy profile $(E|FRC, E|BRC, x^{\max})$, because the strategy of not entering is dominated for both types. Let us check whether this is a PBE. Let $p(FRC) = 1 - p(BRC)$ be the prior probability that the consumer is FRC. In pooling PBE the firm does not update its beliefs. So, let $p(FRC) = p(FRC|E) = p$.

The firm supplies $x^{\max}$ if and only if

$$
\begin{aligned}
p(1 - c(x^{\max})) + (1-p)(1 - c(x^{\max})) &> p(-c(x^s)) + (1-p)(1 - c(x^s)) \\
1 - c(x^{\max}) &> -pc(x^s) + 1 - p - c(x^s) + pc(x^s)
\end{aligned}
$$

$$p > c(x^{\max}) - c(x^s)$$

Since $1 > c(x^{\max}) > c(x^s) > 0$, then this implies that the choice of $x^{\max}$ over x^s is feasible. Next, the consumer has not incentives to deviate because, independently of which type she is, entering is better than not entering. Indeed, $u(x^{\max}) > \bar{u}$ and $v(x^{\max}) > \bar{v}$. Finally, the posterior probability $p(FRC|NE) \in [0, 1]$, because the strategy NE is always strictly dominated.

Next, assume that the firm plays x^s. In this case the only candidate to be an equilibrium is the strategy profile $(NE|FRC, E|BRC, x^s)$. The reason is that if the firm plays x^s, the FRC prefers to stay out and the BRC prefers to enter. Using the Bayes' rule we get

$$\begin{aligned}
p(FRC|E) &= \frac{p(FRC)p(E|FRC)}{p(FRC)p(E|FRC) + p(BRC)p(E|BRC)} \\
&= \frac{p(FRC)(0)}{p(FRC)(0) + p(BRC)(1)} \\
&= 0
\end{aligned}$$

and

$$\begin{aligned}
p(FRC|NE) &= \frac{p(FRC)p(NE|FRC)}{p(FRC)p(NE|FRC) + p(BRC)p(NE|BRC)} \\
&= \frac{p(FRC)(1)}{p(FRC)(1) + p(BRC)(0)} \\
&= 1
\end{aligned}$$

Let $q = p(FRC|E)$. The firm supplies x^s if and only if

$$\begin{aligned}
q(1 - c(x^{\max})) + (1 - q)(1 - c(x^{\max})) &< q(-c(x^s)) + (1 - q)(1 - c(x^s)) \\
1 - c(x^{\max}) &< 1 - c(x^s) \\
c(x^s) &< c(x^{\max})
\end{aligned}$$

Since this inequality is always true, then, given these posterior beliefs, x^s is always preferred to $x^{\max}$. Next, the FRC and the BRC have no incentives to deviates, because $\bar{u} > u(x^s)$ and $\bar{v} < v(x^s)$, respectively.■

Proof of Proposition 14. Problem 1 can be rewritten as follows.

$\max_{p_x,q_x} p_x - \alpha q_x$

s.t. (i) $p_x - \ln(q_x + 1) \leq -u^{-1}(\bar{u})$

We first compute the Lagrangian.

$\mathcal{L}(p_x, q_x, \lambda) = p_x - \alpha q_x - \lambda(p_x - \ln(q_x + 1) + u^{-1}(\bar{u}))$

Now, let us write down the Kuhn-Tucker conditions.

$$\frac{\partial \mathcal{L}}{\partial p_x} = 1 - \lambda = 0 \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial q_x} = -\alpha + \frac{\lambda}{q_x + 1} \leq 0 \qquad q_x\left(\frac{\partial \mathcal{L}}{\partial q_x}\right) = 0 \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \ln(q_x + 1) - p_x - u^{-1}(\bar{u}) \geq 0 \qquad \lambda\left(\frac{\partial \mathcal{L}}{\partial \lambda}\right) = 0 \tag{A.3}$$

together with $p_x > 0$ and $q_x, \lambda \geq 0$.

By condition 1, $\lambda^* = 1$. Therefore, constraint (i) binds (holds with equality). Next, suppose, by contradiction, that $q_x = 0$. This implies that $p_x = -u^{-1}(\bar{u}) < 0$, which leads to a contradiction. Therefore, $q_x > 0$ and condition 2 must bind. Plugging $\lambda^* = 1$ into condition 2 and solving for q_x, we get $q_x^* = \frac{1}{\alpha} - 1$. Next, plugging q_x^* into condition 3 and solving for p_x, we get $p_x^* = \ln(\frac{1}{\alpha}) - u^{-1}(\bar{u})$. Let this solution be denoted by x^c.■

Proof of Proposition 15. Problem 2 is the following.

$\max_{p_y,q_y} p_y - \alpha q_y$

s.t. (i) $p_y - \bar{p} \leq 0$

(ii) $\bar{q} - q_y \leq q$

This problem can easily be solved graphically (see figure A.1).

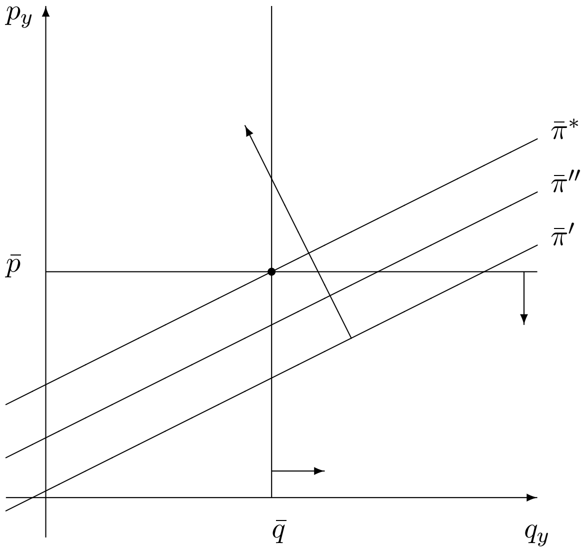

Figure A.1: Problem 2

Fixing $\pi = \bar{\pi}$, let $p_y = \alpha q_y + \bar{\pi}$ be the straight line that represents an isoprofit. Since the level of profits equals the intercept on the y-axes, then the higher profits the greater intercept. Next, since $\alpha \in (0, 1)$, then the gradient vector points to North-West. Finally, since $p_y \leq \bar{p}$ and $q_y \geq \bar{q}$, then the optimal solution is $q_y^* = \bar{q}$ and $p_y^* = \bar{p}$. Let this solution be denoted by y^c.■

Proof of Proposition 16. Problem 3 can be rewritten as follows.

$\max_{p_x,p_y,q_x,q_y} \rho\,(p_x - \alpha q_x) + (1-\rho)\,(p_y - \alpha q_y)$

s.t. (i) $p_x - \ln(q_1 + 1) \leq -u^{-1}(\bar{u})$

(ii) $p_y - \bar{p} \leq 0$

(iii) $\bar{q} - q_y \leq 0$

(iv) $p_x - \ln(q_x + 1) - p_y + \ln(q_y + 1) \leq 0$

(v) $\bar{p} - p_x \leq 0$

We first compute the Lagrangian.

$\mathcal{L}(p_x, p_y, q_x, q_y, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6) = \rho\,(p_x - \alpha q_x) + (1-\rho)\,(p_y - \alpha q_y) - \lambda_1(p_x - \ln(q_x+1) + u^{-1}(\bar{u})) - \lambda_2(p_y - \bar{p}) - \lambda_3(\bar{q} - q_y) - \lambda_4(p_x - \ln(q_x+1) - p_y + \ln(q_y+1)) - \lambda_5(\bar{q} - p_x)$

Now, let us write down the Kuhn-Tucker conditions.

$$\frac{\partial \mathcal{L}}{\partial p_x} = \rho - \lambda_1 - \lambda_4 + \lambda_5 = 0 \tag{A.4}$$
$$\frac{\partial \mathcal{L}}{\partial p_y} = (1-\rho) - \lambda_2 + \lambda_4 = 0 \tag{A.5}$$
$$\frac{\partial \mathcal{L}}{\partial q_x} = -\rho\alpha + \frac{\lambda_1}{q_x+1} + \frac{\lambda_4}{q_x+1} \leq 0 \tag{A.6}$$
$$\frac{\partial \mathcal{L}}{\partial q_y} = -(1-\rho)\alpha + \lambda_3 - \frac{\lambda_4}{q_y+1} \leq 0 \tag{A.7}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = \ln(q_x+1) - p_x - u^{-1}(\bar{u}) \geq 0 \tag{A.8}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = \bar{p} - p_y \geq 0 \tag{A.9}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda_3} = q_y - \bar{q} \geq 0 \tag{A.10}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda_4} = \ln(q_x+1) - p_x - \ln(q_y+1) + p_y \geq 0 \tag{A.11}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda_5} = p_x - \bar{p} \geq 0 \tag{A.12}$$

together with $p_x, p_y > 0$, $q_x, q_y, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \geq 0$, $q_x\left(\frac{\partial \mathcal{L}}{\partial q_x}\right) = 0$, $q_y\left(\frac{\partial \mathcal{L}}{\partial q_y}\right) = 0$, $\lambda_1\left(\frac{\partial \mathcal{L}}{\partial \lambda_1}\right) = 0$, $\lambda_2\left(\frac{\partial \mathcal{L}}{\partial \lambda_2}\right) = 0$, $\lambda_3\left(\frac{\partial \mathcal{L}}{\partial \lambda_4}\right) = 0$, and $\lambda_5\left(\frac{\partial \mathcal{L}}{\partial \lambda_5}\right) = 0$.

Notice that since $\bar{q} > 0$, then, by condition 10, condition 7 binds. Therefore, it must be that $\lambda_3 > 0$. This implies that $q_y^* = \bar{q}$. Next, condition 5 implies that $\lambda_2 > 0$. Hence, $p_y = \bar{p}$. Further, assume by contradiction that $q_x = 0$. Then, by condition 8, $p_x \leq -u^{-1}(\bar{u})$. Since $u^{-1}(\bar{u}) > 0$, then $p_x < 0$, which leads to a contradiction. Therefore, $q_x > 0$ and condition 6 binds.

Finally, notice that it cannot be that $\lambda_1 = \lambda_4 = 0$, because otherwise, by condition 4, $\lambda_5 = -\rho$, which leads to a contradiction.

- Assume first that $\lambda_5 = 0$. This implies that $p_x > \bar{p}$.

  - Suppose that $\lambda_1, \lambda_4 > 0$. Hence, constraints (i) and (iv) bind. Moreover, by condition 4, $\lambda_1 + \lambda_4 = 1$. Using this result in condition 6 and solving for q_x, we get $q_x^* = \frac{1}{\alpha} - 1$. Next, plugging q_x^* into condition 8 and solving for p_x, we get $p_x^* = \ln\left(\frac{1}{\alpha}\right) - u^{-1}(\bar{u})$. By condition 11, it must be that

$$\begin{aligned} \ln\left(\frac{1}{\alpha}\right) - \ln\left(\frac{1}{\alpha}\right) + u^{-1}(\bar{u}) &= \ln(\bar{q}+1) - \bar{p} \\ u^{-1}(\bar{u}) &= \ln(\bar{q}+1) - \bar{p} \\ \bar{u} = u(\bar{p}, \bar{q}) & \end{aligned}$$

  Moreover, since $p_x > \bar{p}$ and $\bar{p} = \ln(\bar{q}+1) - u^{-1}(\bar{u})$, then

$$\begin{aligned} p_x &> \bar{p} \\ \ln\left(\frac{1}{\alpha}\right) - u^{-1}(\bar{u}) &> \ln(\bar{q}+1) - u^{-1}(\bar{u}) \\ \frac{1}{\alpha} - 1 &> \bar{q} \end{aligned}$$

  - Assume $\lambda_1 > 0$ and $\lambda_4 = 0$. This implies that constraint (i) binds and constraint (iv) holds with strict inequality. Moreover, by condition 4, $\lambda_1 = \rho$. Using this result in condition 6 and solving for q_x, we get $q_x^* = \frac{1}{\alpha} - 1$. Next, plugging q_x^* into condition 8 and solving for p_x, we get $p_x^* = \ln\left(\frac{1}{\alpha}\right) - u^{-1}(\bar{u})$. By condition 11, it must be that

$$\begin{aligned} \ln\left(\frac{1}{\alpha}\right) - \ln\left(\frac{1}{\alpha}\right) + u^{-1}(\bar{u}) &> \ln(\bar{q}+1) - \bar{p} \\ u^{-1}(\bar{u}) &> \ln(\bar{q}+1) - \bar{p} \\ \bar{u} &> u(\bar{p}, \bar{q}) \end{aligned}$$

Moreover, since $p_x > \bar{p}$ and $\bar{p} > \ln(\bar{q}+1) - u^{-1}(\bar{u})$, then

$$\begin{aligned} p_x &> \bar{p} \\ \ln\left(\frac{1}{\alpha}\right) - u^{-1}(\bar{u}) &> \ln(\bar{q}+1) - u^{-1}(\bar{u}) \\ \frac{1}{\alpha} - 1 &> \bar{q} \end{aligned}$$

– Assume that $\lambda_1 = 0$ and $\lambda_4 > 0$. This implies that constraint (i) holds with strict inequality and constraint (iv) binds. Moreover, by condition 4, $\lambda_4 = \rho$. Using this result in condition 6 and solving for q_x, we get $q_x^* = \frac{1}{\alpha} - 1$. Next, plugging q_x^* into condition 11 and solving for p_x, we get $p_x^* = \ln\left(\frac{1}{\alpha}\right) - \ln(\bar{q}+1) + \bar{p}$. By condition 8, it must be that

$$\begin{aligned} \ln\left(\frac{1}{\alpha}\right) - \ln\left(\frac{1}{\alpha}\right) + \ln(\bar{q}+1) - \bar{p} - u^{-1}(\bar{u}) &> 0 \\ u^{-1}(\bar{u}) &< \ln(\bar{q}+1) - \bar{p} \\ \bar{u} &< u(\bar{p}, \bar{q}) \end{aligned}$$

Moreover, since $p_x > \bar{p}$, then

$$\begin{aligned} p_x &> \bar{p} \\ \ln\left(\frac{1}{\alpha}\right) - \ln(\bar{q}+1) + \bar{p} &> \bar{p} \\ \frac{1}{\alpha} - 1 &> \bar{q} \end{aligned}$$

- Suppose that $\lambda_5 > 0$. This implies that $p_x^* = \bar{p}$.

  – Suppose that $\lambda_1, \lambda_4 > 0$. This implies that constraints (i) and (iv) bind. Plugging p_x^* into condition 8 and solving for q_x, we get $q_x^* = e^{\bar{p}+u^{-1}\bar{u}} - 1$. Next, by condition 11

$$\begin{aligned}\bar{p}+u^{-1}(\bar{u})-\bar{p} &= \ln(\bar{q}+1)-\bar{p}\\ u^{-1}(\bar{u}) &= \ln(\bar{q}+1)-\bar{p}\\ \bar{u} &= u(\bar{p},\bar{q})\end{aligned}$$

- Suppose that $\lambda_1 > 0$ and $\lambda_4 = 0$. This implies that constraint (i) binds and constraint (iv) holds with strict inequality. Plugging p_x^* into condition 8 and solving for q_x, we get $q_x^* = e^{\bar{p}+u^{-1}\bar{u}} - 1$. Next, by condition 11

$$\begin{aligned}\bar{p}+u^{-1}(\bar{u})-\bar{p} &> \ln(\bar{q}+1)-\bar{p}\\ u^{-1}(\bar{u}) &> \ln(\bar{q}+1)-\bar{p}\\ \bar{u} &> u(\bar{p},\bar{q})\end{aligned}$$

- Suppose that $\lambda_1 = 0$ and $\lambda_4 > 0$. This implies that constraint (i) holds with strict inequality and constraint (iv) binds. Plugging p_x^* into condition 11 and solving for q_x, we get $q_x^* = \bar{q}$. Next, by condition 8

$$\begin{aligned}\ln(\bar{q}+1)-\bar{p}-u^{-1}(\bar{u}) &> 0\\ u^{-1}(\bar{u}) &< \ln(\bar{q}+1)-\bar{p}\\ \bar{u} &< u(\bar{p},\bar{q})\end{aligned}$$

This concludes the proof.■

A.3 Proofs of Theorems

Proof of Theorem 1. It is easy to check necessity.

Sufficiency. Assume that c satisfies FAWARP.

STEP 1: There exists a strict linear order $\succ$ on X, where $x \succ y$ whenever $x = c(A, \{A_j\})$, where $A = A_1 = \{x, y\}$.

Proof. Let $x \succ y$ whenever $x = c(A, \{A_j\})$, where $A = A_1 = \{x, y\}$. Notice that y attracts attention at $(A, \{A_j\})$. It is immediate to see that $\succ$ is asymmetric, irreflexive, and complete. Next, we show that $\succ$ is transitive. Assume that $x \succ y$ and $y \succ z$. Consider a $(C, \{C_j\}) \in \mathcal{D}_1$ such that $C = C_1 = \{x, y, z\}$. Since $C_1 = C$, then necessarily x, y, and z attract attention at $(C, \{C_j\})$. Notice that if $z = c(C, \{C_j\})$, then, by FAWARP, $z \succ y$, which leads to a contradiction. Similarly, it cannot be that $y = c(C, \{C_j\})$. Therefore, $x = c(C, \{C_j\})$. Then, by FAWARP, $x \succ z$. Hence, $\succ$ is transitive. Therefore, we conclude that $\succ$ is a strict linear order.$\square$

STEP 2: Let $(A, \{A_j\}) \in \mathcal{D}_1$ and assume that $\Gamma_{(A,\{A_j\})} \subseteq A$ is the set of alternatives that attract attention at $(A, \{A_j\})$. Then, $\{c(A, \{A_j\})\} = \max(\Gamma_{(A,\{A_j\})}; \succ)$.

Proof. Let $(A, \{A_j\}) \in \mathcal{D}$ and assume that $\Gamma_{(A,\{A_j\})} \subseteq A$ is the set of alternatives that attract attention at $(A, \{A_j\})$. Let $x = c(A, \{A_j\})$. Suppose, by contradiction, that $y \succ x$ for some $y \in \Gamma_{(A,\{A_j\})} \setminus \{x\}$. In this case, $y = c(B, \{B_j\})$, where $B = B_1 = \{x, y\}$. Since x and y attract attention at $(B, \{B_j\})$, then, by FAWARP, $x \neq c(A, \{A_j\})$, which leads to a contradiction. Therefore, there are no $y \in \Gamma_{(A,\{A_j\})} \setminus \{x\}$ such that $y \succ x$. This implies that $\{c(A, \{A_j\})\} = \max(\Gamma_{(A,\{A_j\})}; \succ)$, which is the desired result.$\square$

STEP 3: There exists a partition $(S, X \setminus S)$ of the grand set X such that $c(A, \{A_j\}) \in S$ for any $(A, \{A_j\})$ with $A \cap S \neq \emptyset$.

Proof. Define $S = \{x \in X | \nexists (B, \{B_j\}) \in \mathcal{D}_1 \text{ such that } x \in B_{m-1}^{c(B,\{B_j\})}\}$. Consider any $(A, \{A_j\}) \in \mathcal{D}_1$ such that $x \in A \cup S \neq \emptyset$. Suppose, by contradiction, that $y = c(A, \{A_j\}) \notin S$. Since $y \notin S$, then all alternatives in A attract attention at $(A, \{A_j\})$. By STEP 2, $y \succ x$. Next, since $y \notin S$, then there exists a $(B, \{B_j\}) \in \mathcal{D}_1$ such that $y \in B_{m-1}^{c(B,\{B_j\})}$. Let $z = c(B, \{B_j\})$. Notice that y attracts attention at $(B, \{B_j\})$ and, therefore, by STEP 2, $z \succ y$. Let $(C, \{C_j\}) \in \mathcal{D}_1$ such that $C = \{x, y, z\}$ and $C_1 = \{x, y\}$. Notice that $(C, \{C_j\})$ exists. Suppose not. Assume first that $z \in A$. Since all alternatives in A attract attention at $(A, \{A_j\})$, then, by STEP 2, $y \succ z$, a contradiction. Hence, $(C, \{C_j\}) \neq (A, \{A_j\})$. Next, suppose that $(B, \{B_j\}) = (C, \{C_j\})$. This would imply that $x \notin S$, a contradiction. Therefore, $(C, \{C_j\})$ exists.

We want to show that any choice from $(C, \{C_j\})$ leads to a contradiction. Assume first that $x = c(C, \{C_j\})$. In this case, by STEP 2, $x \succ y$, which leads to a contradiction. Next, assume $y = c(C, \{C_j\})$. This implies that all alternatives in C attract attention at $(C, \{C_j\})$ and, therefore, by STEP 2, $y \succ z$, which leads to a contradiction. Assume then that $z = c(C, \{C_j\})$. However this contradicts the fact that $x \in S$. Since $\{c()\}$ cannot be empty, then $c(A, \{A_j\}) \in S$, which is the desired result.$\square$

STEP 4: For any $(A, \{A_j\}) \in \mathcal{D}_1$,

$$\Gamma_{(A,\{A_j\})} = \begin{cases} A_{\min\{j|A_j \cap S \neq \emptyset\}} & \text{if } A \cap S \neq \emptyset \\ A & \text{otherwise} \end{cases}$$

Proof. By STEP 2, $\{c(A, \{A_j\})\} = \max(\Gamma_{(A,\{A_j\})}; \succ)$, where $\Gamma_{(A,\{A_j\})}$ is the set of alternatives that attract attention at $(A, \{A_j\})$. Suppose first that $A \cap S \neq \emptyset$. We have to show that $\Gamma_{(A,\{A_j\})} = A_1$. Assume first that $x \in \Gamma_{(A,\{A_j\})}$. Since $A \cap S \neq \emptyset$, then, by STEP 3, it must be that $c(A, \{A_j\}) \in S$. If this is the case, then $x \in \Gamma_{(A,\{A_j\})}$ implies that $x \in A_m^{c(A,\{A_j\})}$ and $A_m^{c(A,\{A_j\})} = A_{\min\{j|A_j \cap S \neq \emptyset\}}$. Hence, $x \in A_{\min\{j|A_j \cap S \neq \emptyset\}}$. Conversely, assume that $x \in A_{\min\{j|A_j \cap S \neq \emptyset\}}$. This immediately implies that $x \in \Gamma_{(A,\{A_j\})}$.

On the other hand, if $A \cap S = \emptyset$, it is immediate to see that $\Gamma_{(A,\{A_j\})} = A$.$\square$

Therefore, we conclude that c is an SCF.$\blacksquare$

Proof of Theorem 2. It is easy to check necessity.

Sufficiency. Assume that c satisfies FAWARP$_2$. STEP 1, STEP 2, and STEP 3 are analogous to those of Theorem 1. For this reason we omit the proof.

STEP 1: There exists a strict linear order $\succ$ on X, where $x \succ y$ whenever $x = c(A, A_1)$, where $A = A_1 = \{x, y\}$.

STEP 2: Let $(A, A_1) \in \mathcal{D}_2$ and assume that $\Gamma_{(A,A_1)} \subseteq A$ is the set of alternatives that attract attention at (A, A_1). Then, $\{c(A, A_1)\} = \max(\Gamma_{(A,A_1)}; \succ)$.

STEP 3: There exists a partition $(S, X \setminus S)$ of the grand set X such that $c(A, A_1) \in S$ for any (A, A_1) with $A \cap S \neq \emptyset$, where $S = \{x \in X | \nexists (A, A_1) \in \mathcal{D}_2$ such that $x \in A_1$ and $c(A, A_1) \notin A_1\}$.

STEP 4: For any $(A, A_1) \in \mathcal{D}_2$,

$$\Gamma_{(A,\{A_j\})} = \begin{cases} A_1 & \text{if } A_1 \cap S \neq \emptyset \\ \bar{A} & \text{if } (A \setminus A_1) \cap S \neq \emptyset = A_1 \cap S \\ A & \text{otherwise} \end{cases}$$

where $\bar{A} = A_1 \cup \{x \in A \setminus A_1 | c(A, A_1) \succ x\} \cup \{c(A, A_1)\}$.

Proof. By STEP 2, $\{c(A, A_1)\} = \max(\Gamma_{(A,A_1)}; \succ)$, where $\Gamma_{(A,A_1)}$ is the set of alternatives that attract attention at (A, A_1). Suppose first that $A_1 \cap S \neq \emptyset$. We have to show that $\Gamma_{(A,A_1)} = A_1$. Assume first that $x \in \Gamma_{(A,A_1)}$. Since $A_1 \cap S \neq \emptyset$, then, by STEP 3, it must be that $c(A, A_1) \in S$. This implies that case 2 and case 3 of Definition 7 cannot occur. Hence, only case 1 of Definition 7 can occur, that is, $x \in A_1$, as desired. Conversely, assume that $x \in A_1$. This immediately implies that $x \in \Gamma_{(A,A_1)}$.

Assume that $(A \setminus A_1) \cap S \neq \emptyset = A_1 \cap S$. We have to show that $\Gamma_{(A,A_1)} = \bar{A}$, where $\bar{A} = A_1 \cup \{x \in A \setminus A_1 | c(A, A_1) \succ x\} \cup \{c(A, A_1)\}$. Assume first that $x \in \Gamma_{(A,A_1)}$. Since $(A \setminus A_1) \cap S \neq \emptyset = A_1 \cap S$, then, by STEP 2, $c(A, A_1) \in S$, which rules out case 2 of Definition 7. This implies that $x \in A_1$ (case 1) and $x \in A \setminus A_1$ only if $c(A, A_1) \succ x$ (case 3) or $x = c(A, A_1)$. Hence, $x \in \bar{A}$. Conversely, assume that $x \in \bar{A}$. Since, by STEP 3, it must be that $c(A, A_1) \in S$, then $c(A, A_1) \notin A_1$, which rules out only case 2 of Definition 7. Hence, $x \in \Gamma_{(A,A_1)}$, as desired.

Finally, assume that $A \cap S = \emptyset$. We have to show that $\Gamma_{(A,A_1)} = A$. Clearly, $x \in \Gamma(A, A_1)$ implies $x \in A$. Conversely, assume that $x \in A$. Since $A \cap S = \emptyset$, then there is some $(B, B_1) \in \mathcal{D}_2$ such that $c(A, A_1) \in B_1$ and $c(B, B_1) \notin B_1$. This implies that $x \in \Gamma(A, A_1)$ as desired.$\square$

Therefore, we conclude that c is an SCF$_2$.$\blacksquare$

Proof of Theorem 3. We first prove that if the extended choice function c is an SCF, then the restriction of c to $\mathcal{D}_L$ is a $D_{R,\delta}$.

Assume that c is an SCF. Define the relation R in the following way: $\max(X; R) \equiv UC_{\succ}(X; x^s)$ and R ranks the unsatisfactory alternatives as $\succ$ does. It is easy to see that $c(A, \{A_j\}) = D_{R,\delta}$ for any $(A, \{A_j\}) \in \mathcal{D}_L$ and $\delta(x^s) = 1$.

Next, we show that a $D_{R,\delta}$ can be extended to an SCF if and only if $\delta = 1$ and the sets in $\mathcal{I}_{(R;\setminus \max)}$ are singletons.

We first prove that if a $D_{R,\delta}$ can be extended to an SCF, then $\delta = 1$ and the sets in $\mathcal{I}_{(R;\setminus \max)}$ are singletons. Assume that $D_{R,\delta}$ can be extended to an SCF. This means that c is a SCF and $c(A, \{A_j\}) = D_{R,\delta}$ for all $(A, \{A_j\}) \in \mathcal{D}_L$. Suppose, by contradiction, that $\delta = 1$ and the sets in $\mathcal{I}_{(R;\setminus \max)}$ are singletons. Then there are three cases.

Case (i): $\delta = 2$ and the sets in $\mathcal{I}_{(R;\setminus \max)}$ are singletons. If this is the case, then $c(A, \{A_j\}) \in UC_{\succ}(X; x^s)$, but there exists some $(A, \{A_j\}) \in \mathcal{D}_L$ such that $A_{i-1}^{c(A,\{A_j\})} \cap UC_{\succ}(X; x^s) \neq \emptyset$. However, this contradicts the fact that whenever $UC_{\succ}(X; x^s) \cap A \neq \emptyset$, then $\Gamma_{(A,\{A_j\})} = A_{\bar{j}}$, where $\bar{j} = \min\{j | A_j \cap UC_{\succ}(A; x^s) \neq \emptyset\}$, which leads to a contradiction. Hence, $\delta = 1$.

Case (ii): $\delta = 1$ and there is a non-singleton set in $\mathcal{I}_{(R;\setminus \max)}$. Assume that

$y, z \in X$ belong to that set. Therefore, $y, z \notin UC_{\succ}(X; x^s)$. Assume WLOG that $y \succ z$. Let $(A, \{A_j\}), (A, \{A'_j\}) \in \mathcal{D}_L$, where $A_1 = \{z\}$, $A_2 = \{y, z\}$, $A'_1 = \{y\}$, and $A'_2 = \{y, z\}$. Since $\delta = 1$, then yI_1z, that is, $z = c(A, \{A_j)$ and $y = c(A, \{A'_j\})$. However, this contradicts FAWARP and, therefore, c is not an SCF, which leads to a contradiction. Hence, the sets in $\mathcal{I}_{(R;\setminus\max)}$ are singletons.

Case (iii): $\delta = 2$ and there is a non-singleton set in $\mathcal{I}_{(R;\setminus\max)}$. A contradiction takes place from what is proved in cases (i) and (ii), which is the desired result.

Conversely, assume that the restriction of c to $\mathcal{D}_L$ is a $D_{R,\delta}$, where $\delta = 1$ and the sets in $\mathcal{I}_{(R;\setminus\max)}$ are singletons. We want to show that c can be extended to an SCF. We first show that the restriction of c to $\mathcal{D}_L$ is an SCF. Define $UC_{\succ}(X; x^s) \equiv \max(X; R)$. Moreover, define $\succ$ in the following way: $x \succ y$ for all $x \in \max(X; R)$ and $y \notin \max(X; R)$ and $\succ$ ranks all alternatives $y \notin \max(X; R)$ as R does. Since $\delta = 1$, then c always picks from $(A, \{A_j\}) \in \mathcal{D}_L$ the first $x \in UC_{\succ}(X; x^s)$ that appears in the menu sequence. Next, the fact that all sets in $\mathcal{I}_{(R;\setminus\max)}$ are singletons rules out the possibility that FAWARP is violated in the way it is explained in case (ii). Therefore, the restriction of c to $\mathcal{D}_L$ is an SCF. Next, complete the relation $\succ$ by imposing the satisfactory alternatives to be ranked according to any order. Moreover, for the remaining problems $(A, \{A_j\}) \in \mathcal{D}_1 \setminus \mathcal{D}_L$ impose c to satisfy FAWARP. Hence, c is an SCF.■

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