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Abstract

Managing disequilibrium and its consequent traverse is one of the important topics in economic theory. During such traverse, the evolving economic system faces various imbalances and coordination problems that arise within and outside the system. In order to traverse the disequilibrium, the economic system should innovate, create, and manage its resources in a viable manner. This dissertation reports studies in modelling ‘disequilibrium traverse economies’, in the sense of Hicksian Neo-Austrian, Time-to-Build, theory. In the first chapter, we discuss the origins of traverse analysis and the role of ‘time-to-build’ framework in modelling macro-dynamics. We also provide a review of the literature concerning ‘time-to-build’ tradition in modelling traverse and business cycles. Then, we discuss in detail how this framework actually enters into these models and the assumptions that underpin them. In the second chapter, we provide an overview of different notions of viability, which are widely used in economic literature, and formulate viability conditions for Amendola and Gaffard’s Neo-Austrian model. We then simulate the model, with and without the viability conditions, to study the dynamics of the evolving system. The main aim of this chapter is to explore and analyse the importance of viability creating mechanisms during dynamic traverse.

In chapter three, we analyse some of the classics in business cycle theory and how the ‘time-to-build’ framework plays an important role in modelling economic fluctuations. The contribution of this chapter is to provide insights on various assumptions and the choice of mathematical formalisms that underpin these business cycle models. In conclusion, we also suggest that the Neo-Austrian tradition should be placed as a part of the rich tradition of business cycle theory. In the fourth chapter, we take the ‘time-to-build’ model and improve the framework by endogenizing one of the main engines of growth, i.e., innovations, in a non-stochastic, non ad-hoc, manner. We model innovations using Turing Machine metaphor, so that we can encapsulate the intrinsic uncertainties of Research and Development processes in an insightful way. The enhanced ‘time-to-build’ model thus developed is then simulated for various policy parameters, such as - R&D policies, interest rates, and bankruptcy policies, and the resulting traverses are studied. In the last chapter, we provide a comprehensive summary of the novel contributions of the dissertation and list some of the future research paths that can be traversed.

Keywords: Disequilibrium Modelling, Traverse Analysis, Business Cycle Theory,
Neo-Austrian Model, Time-to-Build

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It goes without saying that, even though this dissertation is a product of the accumulation of ideas and interactions I had with my mentors, any remaining infelicities or errors should be taken as mine.

¹ I will pay my gratitude not just in this section but also in all the coming sections of my life.

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Chapter 1

Introduction

Traditionally, theoretical studies of growth dynamics focus on the steady state equilibria. Here, the entire economy is considered to be in the steady state situation and, for a given set of initial conditions, the dynamics of the system are analyzed. The main drawback with the orthodox, steady-state, approach is that qualitative changes cannot be deduced from a comparison of alternative states of an economy essentially defined within its basic structures. Moreover, in such models, as Day (1993: 21) observes, equilibrium is characterized by “optimal equilibrium strategies” that perfectly account for all the interactions within the economy and its environment. Therefore, these static, or dynamically static, models are ineffective in explaining structural and disequilibrium dynamics between different steady states arising due to qualitative and global changes, such as technology, market structure, and evolution of preferences. Their limitation is largely due to the way in which the intrinsic economic forces that cause the qualitative and global changes are modelled. They are often presented in a very restrictive manner, or as exogenous variables, thus making the above models ineffective in explaining the rich structure of the evolutionary growth dynamics. In fact, the economic forces that cause the qualitative changes emerge within the system, thus “*changing sets of utilized activities and tight constraints. When these latter sets switch, the variables and equations governing the evolution of the system switch, in effect bringing in a different set of causal structures.*” (Day, 1993: 38; *italics in original*)

In such economic systems, evolution disequilibrates the existing economic structures and, consequently, new economic structures have to be build and managed in such a way that the system *traverses* from its present position to another predetermined position or desired equilibrium. During such transitions, production, la-

bour, and money imbalances and coordination problems arise within the economic system and hinder the sustainability of the system. In a dynamic environment, economies evolve by creating and adapting to new demand and supply conditions. This feature emphasizes the need for a closer understanding of the nature and patterns of the changes in core economic structures, in order to manage the disequilibrium situation effectively.

Investigating the causes of dynamicity of economic growth, Lowe (1976: 9; *italics* in original) points out that “the root of all these difficulties is *technological*. Obstruction of resource shifts, bottlenecks in production, inelasticity of supply owing to the *longue durée* of capital formation and even more to the large costs of sunk capital”. Therefore, in order to achieve growth, the resources are to be allocated in such a way that the system adapts to the necessary changes and evolves over time. But the process of viably achieving economic growth along a traverse is not an *automatic* process, because the new economic structures emerge, within the system, and evolve in *undecidable* ways. Hence, during disequilibrium, the viability of the traversing economic system becomes *the crucial analytical problem* (AG, 1998). Hence, it is very important to systematically delineate the processes through which these intrinsic forces emerge and evolve within the economy, in order to understand the emergence of growth and its traverses.

Although much of the research on studying the dynamics of structural change and economic growth has been carried out, very less research has been done on understanding and analyzing the problem of *traverse*

1.1 Traverse analysis

Traverse analysis, which views the economic system to be in a state of perpetual transition, has a rich tradition in the history of economic thought¹. Even though, problems of traverse have been implicitly addressed in the works of Michal Kalecki, Joan Robinson, and Adolph Lowe, it was only in the late 1960s that the notion of traverse was first formalized, by J.R. Hicks, and, further, pioneered by John Hicks. The traverse is a path between two steady states or equilibria. Explaining the notion of traverse and its importance, Hicks (1965: 184; *italics added*) writes,

Suppose that we have an economy which has in the past been in equilibrium in one set of conditions; and that then, at time 0, a new set of conditions is imposed; is it possible (or how is it possible) for the economy to get into the new equilibrium, which is appropriate to the new conditions? We do not greatly diminish the generality of *our study of disequilibrium* if we regard it in this way, *as a Traverse from one path to another. And there is some advantage to be gained from greater specification of the initial position from which the Traverse takes off (that is what the point really is)*. Chiefly, it enables us to split up the kinds of adjustment that have to be made, so as to take different kinds separately.

Hicks was interested in understanding how an economic system could create and manage its resources, in order to evolve and reach the desired state. The dominant theory for modelling and analysing the optimal path of the system, between two steady states, originated from von Neumann's ([1938] 1945) seminal paper. In this paper, von Neumann built a formal model on economic growth and the balanced

¹ For example, Keynes while stressing the importance of analysis of transition regimes writes “that it is in the transition that we actually have our being:...” (Keynes, 1936: 343, footnote 3; In Velupillai 2011)

growth paths that the economies may adopt in order to reach the desired capital state or desired equilibrium. This is applicable if the economic system wants to move from one steady state to another and if the initial and final states are not close enough, then, even if the economic system incurs extra cost, the system can adopt von Neumann's balanced growth paths, or later known as 'Turnpike' paths, to quickly reach the desired final state. But the main limitation of the 'Turnpike' theory, according to Hicks (1965), is that it was a case of an optimization problem focusing on the maximization of terminal capital and *not* on the flow of consumption outputs. Hence, "[e]xcess production of consumption goods (in excess of the prescribed minimum) during the period of plan is treated as valueless; nothing (it is reckoned) is gained from it." (Hicks, 1965: 206) But when the economic system is in disequilibrium, finding the new prices that would ensure a viable traverse to the desired equilibrium is not an easy matter. "If unsuitable prices are adopted, and adhered to for long, unsuitable techniques will be adopted; the problem of getting into equilibrium will be further complicated, and the approach to equilibrium will be retarded" (Hicks, 1985: 143) Therefore, it is necessary to understand the intertemporal complementarity characteristics of production process and the factors that influence it, in order to study the future traverses. Moreover, "[i]n an actual economic situation, ... (because of the advances in technology) the equilibrium at which the economy is aiming is continually shifting." (Hicks, 1985: 143) In such dynamic economic environments, where the desired equilibrium is continuously shifting, the flow theory (Hicks, 1965) offers a better tool for transitional analysis. Therefore, Hicks adopted the flow theory in order to analyse the impacts of qualitative changes in the economy

on the capital value² of the production processes, and the resulting *truncation* of production flows (see Hicks, 1970, 1973; Nuti, 1973).

Moreover, Hicks was interested in analyzing the structural changes and traverses of an evolving economy but the *time less* property of the neoclassical production function³ limited J. R. Hicks studying the transitional economics in an insightful manner. In order to carry out traverse analysis, Hicks needed a framework that would enable him to model and analyse the production in its *essential* nature - production as a process in time. That is, the construction of productive structures precedes utilization; in other words, the process of production takes time to build and to utilize.

The ‘Time-to-build’ framework plays an important role in modelling the macro-dynamics because it encapsulates the evolving processes in a more tractable manner. During disequilibrium, decisions are taken at every point of time will in turn

² It should be noted that

[a]s a consequence of taking into account durable capital goods the existence of ‘*jointness of production*’ arises in a twofold sense, firstly in the von Neumann-Sraffa sense whereby that part of a fixed capital good not used up in production at the end of a period can be regarded as a component of the output of that period...; secondly in the sense of ‘*intertemporal joint production of final output at different dates*’.... Whilst the second aspect is paid much attention in Hicks’ analysis, the first one simply does not enter into consideration because of the assumption of complete vertical integration. (Hagemann and Kurz, 1976: 681)

³ As J.R. Hicks (1965: 293) points out,

[t]here is no ‘production function’ in Jevons or Marshall, Walras or Pareto, Menger or Böhm-Bawerk. There is in Wicksell, but he is careful to confine it to his model of ‘production without capital’... The originators of the ‘production function’ theory of distribution (in the static sense, where I still think that it should be taken fairly seriously) were Wicksteed, Edgeworth and Pigou.

In another seminal work debunking the neoclassical production function, Shaikh (1974) explains the limitations of *aggregate production function*, from both theoretical and empirical bases, using a ‘Humbug economy’.

decide the future traverses of the system, and its sustainability over time. Therefore, in order to study such dynamic economic systems, one needs to incorporate the time profile of the economic activities into the model. This is because the history of the economic system⁴ plays an important role in explaining why and how the system had reached its current position/state. On emphasizing the same point, Hicks writes: “there is some advantage to be gained from greater specification of the initial position from which the Traverse takes off (*that is what the point really is*).” (1965: 184; *italics added*) The future possible traverses are decided by the both past and current economic decisions, along with the available resources at any given point in time. This is the reason why the ‘time-to-build’ framework becomes an essential ingredient in most of the disequilibrium macrodynamic models. The ‘time-to-build’ framework captures the essential characteristics of the production technology and provides much richer information regarding the existing economic structures and the possible future states, thus making the macrodynamic models more tractable and more suitable for traverse analysis.

Furthermore, traverse analysis can adopt two methods of analysis, depending on the particular structural specification and the focus of investigation. “[A] traditional distinction stresses the difference between ‘horizontal’ and ‘vertical’ representations of the economic structure.” (Silva and Teixeira, 2008: 283) “Horizontal representation describe the economic system as a circular structure, with economic

⁴ As Schumpeter writes,

Economic development ... is merely a part of universal history, only separated from the rest for purposes of exposition. Because of this fundamental dependence of the economic aspect of things on everything else, it is impossible to explain *economic* change by previous *economic* conditions alone. For the economic state of a people does not emerge simply from the preceding economic conditions, but only from the preceding total situation. ([1911], 1934: 58; *italics in original*)

activities being clustered into mutually dependent classes” – i.e., a substitution perspective (Lowe, 1976). In contrast, vertical representation takes the complementarity perspective, excluding the consideration of substitutive interdependencies by stressing the unidirectional relationships and the asymmetric dependence in the clustering process (Hicks, 1970, 1973). The substitution view of resources is tricky because, instead of a single variable or a structure, a collection of key variables and structures with different causal links and varying complexities has to be changed. Thus, the vertical integration technique is more advantageous than the horizontal one⁵. Hence, Hicks adopted the Austrian, vertically integrated, ‘time-to-build’, framework in which the production processes are essentially conceptualized as a number of separable elementary processes being performed over time. Hicks revived the Austrian theory of capital by extending the original single-input and single-output schema into a system with multiple stream of inputs and a stream of outputs. This enabled Hicks to analyse the intertemporal complementarity of the production factors within an evolving economic system in a more effective way.

Hicks’ Neo-Austrian approach provided a valuable tool to study the dynamic traverse of an economic system and to analyse the factors and conditions that would enable the system to embark on the desired growth paths or hinder it from doing so. But, when the economic system is not in equilibrium production, human resources and financial resources can no longer be optimally managed and the system faces various imbalances and co-ordination problems. Emphasizing the dynamic relationship between the production factors, Amendola and Gaffard (1998:66 ; henceforth AG), write,

⁵ Sraffa (1960: App. A), in his *magnum opus*, took an insightful standpoint and reduced the *horizontal* structure to *vertical* structures by using *Sub-Systems*, thus making the structural analysis more *tractable*.

[In] [o]ut of equilibrium ... there is a sequence of determination moments [for determining prices, quantities of labour]; and this ... implies the existence of reaction lags and constraints which may have different implications and bring about different developments. The situations to which prices and quantities have to adapt may not only differ greatly but are likely to change over time, and these changes are themselves in part the result of what has been happening to prices and quantities along the way.

If unsuitable prices are adopted then the money proceeds will not be synchronized with the money required for carrying out production so the new and on-going projects will be jeopardized.

In such out-of-equilibrium situations, the role of money and credit takes a central stage. In fact, credit can be instrumental for both carrying out production processes and for fostering research and development activities (Schumpeter, [1911] 1934), hence more emphasis has to be paid to study the interactions between production and money. Moreover, if the new technology that is being adopted requires a new set of skills then the economic system cannot viably traverse unless the human resources are trained to carry out the new production technology. Therefore understanding the intertemporal complementarity between production, labour, and money, along with other factors, and developing relevant viability creating mechanisms would ensure the economic system to reach its desired state/equilibrium.

Following Hicks, AG (1998, 2006) further developed the Neo-Austrian model, by modelling the dynamic relations between production, labour, and money, in a more comprehensive way, suitable for simulational studies. The AG model provides a decisive tool for the policy-maker to understand the dynamic interactions between production, labour and money, and also to develop policies that would ensure that

the economic system traverses to a desired growth path. AG's model is insightful in exploring and explaining the intertemporal complementarities of the productional and innovational processes. However, it limits itself from explaining the viability conditions through which the economic system could sustain and evolve over time.

Why are the viability conditions more important, especially, during the structural change? The reasons are the following: during disequilibrium, the investments and consumption are not *harmonized* and, as a consequence, due to the various imbalances and coordination problems, the ability of the economic system to evolve over time may be hindered. In such circumstances, the viability of the system will be at stake and underlying viability conditions and mechanisms to the centre stage. In fact, these conditions become a prerequisite for economic system's sustainability.

[In order t]o restore viability a mechanism must be introduced by the modeller that allows for the formation of new "model agents" to whom the resources associated with disappearing agents are transferred. Alternatively, a new organizational form can be installed that will transfer resources so as to maintain individual agent feasibility and overall systems viability...

Such *viability creating mechanisms* are the analog of equilibrium "existence" proofs, but in the out-of-equilibrium setting. They are required to guarantee the existence of a continuing "solution" to the system in terms of feasible actions for all of its constituent model components.

The study of such adaptive economic systems

might provide better "engines for discovering truth," and they might help formulate more effective mechanisms for steering the economy away from precipitous hazards and along less bumpy paths.

(Day, 1993: 38-9; *italics* in original)

Thus, for analysing traverse, it is important that we model the economic activities

with its time profile, because that would provide a richer history of the initial conditions of the economic system and its future possible growth paths. Moreover, the policy maker can use simulational models/tools to analyse, various scenarios and viability creating mechanisms, and devise policies to viably traverse the dynamic disequilibrium trajectory.

1.2 Time-to-Build framework and Macro-dynamic models

The Time-to-Build framework has a rich tradition in economic analysis (Böhm-Bawerk, 1890), and has been used as a crucial structure in modelling economic fluctuations and macro-dynamics, in general. The *classic* by Frisch (1933), titled “On propagation and impulse problems”, can be seen as the fountainhead of business cycle theory and macro-dynamics modelling tradition. Of Frisch’s many contributions, one was to identify and distinguish the two *fundamental* problems that an oscillating economic system faces, i.e., the *propagation* problem and the *impulse* problem. “The propagation problem is the problem of explaining by the structural properties of the swinging system, what the character of the swings would be in case the system was started in some initial situation. *This must be done by an essentially dynamic theory, that is to say, by a theory that explains how one situation grows out of the foregoing.*” (Frisch, 1933: 1; *italics added*) Frisch’s work was very influential because it provided insights on economic fluctuations and showed the usefulness of mathematical tools in economic theory. More importantly, Frisch’s work became a benchmark as how one ought to model the macroeconomic dynamics. Kalecki in his seminal paper titled “A Macrodynamic Theory of Business Cycles”, at the Econometric Society meeting at Leyden, contributed to the business cycle theory by ap-

proaching the problem of economic fluctuations from a different perspective, i.e., from a Marxian perspective.

It is also very interesting to note that in both the models, and the models that followed [Goodwin (1951a), Kydland and Prescott (1982)], had ‘time-to-build’ characteristics of the capital production, as a crucial ingredient in order to generate business cycles. For example, in Frisch’s model, the technology took ε time periods to build the capital so at each given point of time I/ε of the investment gets accumulated and the investment made at time t will be only realized at time $t+\varepsilon$. Like Frisch, Kalecki also introduces the ‘time-to-build’ framework for the capital orders or investments and it is this time lag between orders (capital investment) and deliveries (capital realization) that causes the business cycles. Even though these models have the ‘time-to-build’ framework as a tool to generate business cycles, these models are different in their conceptualizations. While Kalecki viewed the system to be always in *transition* due to the capitalistic nature of economic system, Frisch viewed the economic system to be tending towards equilibrium in the absence of impulses.

The business cycle models can be broadly categorized into two groups based on the equilibrium positions. For example, the business cycles can be seen as a cyclical movement around equilibrium and the system not reaching it. In the other case, one can view the business cycle process as a movement between two equilibriums. The first case is not a disequilibrium process but the second case is. The business cycle, viewed as a movement from one equilibrium position to another, is a disequilibrium process. It is characterized as a creative destruction process, where the existing capital structures become obsolete and new structures are to be built, in order to reach a new equilibrium. This creative destruction process is nothing but a business cycle. The business cycle models of Tinbergen, Frisch, Kalecki, Goodwin, Hicks,

and Kydland and Prescott (henceforth, K&P) belong to the first category and the Neo-Austrian tradition belongs to the second category. Moreover, the mathematical tools used for modelling business cycles are different in these seminal works and so it would be interesting to investigate how the ‘time-to-build’ framework enters in their model. Furthermore, it would be insightful to undertake a detailed investigation of the mathematical formulations, along with its underlying assumptions.

The present frontier topic in business cycle theory and macro dynamics modelling is K&P’s Real Business Cycle theory. Their model aims to integrate growth and business cycle theory by incorporating the ‘time-to-build’ framework within a standard growth model. In order to generate business cycle, they utilize the ‘time-to-build’ framework for capital production and assume that the impulses that affect the production technology and the information flow within the system take the form of stochastic shocks. The RBC theorists claim their theoretical technology, to conceptualize the impulses as erratic shocks, is inspired by Frisch’s 1933 paper. But it is interesting to note that Frisch, after addressing the propagation problems and illustrating a case of modelling the impulses as erratic shocks, knowing the limitation of assuming the impulses as erratic shocks, goes on further and explains that

[t]he idea of erratic shocks represents one very essential aspect of the impulse problem in economic cycle analysis, but probably it does not contain the whole explanation. There is also present another source of energy operating in a more continuous fashion and being more intimately connected with the permanent evolution in human societies. The nature of this influence may perhaps be best exhibited by interpreting it in the light of Schumpeter's theory of the innovations and their role in the cyclical movement of economic life.

(Frisch 1933: 33; *italics added*)

After hinting at the different ways in which the impulses can be modelled, Frisch, in the last section, uses Schumpeter's clock metaphor in order to explain the endogenous generation of innovations, as impulses, in the economic system. These impulses arise within the system and move the system away from ever reaching any equilibrium and consequently, hinting the use of non-stochastic tools for modelling economic fluctuations.

Why RBC theorists did not further develop their theoretic technology, than assuming the impulses as erratic shocks, is unclear and also their methodological framework does not allow the model to explain how these impulses actually emerge in the system. And, it should be noted that technology plays an important role in the 'time-to-build' characteristics of capital production, so any innovation in technology would emerge as an impulse within the system. Therefore, the question is - how to model these innovations in a non-stochastic way that is not *ad hoc*?

1.3 Engines of economic growth

Knowledge is our most powerful engine of production; it enables us to subdue Nature and forces her to satisfy our wants. Organization aids knowledge...
Marshall (1920: 115)

One of the earliest work on formal economic growth models can be traced back to the work of John von Neumann ([1938] 1945) and since then there has been an increasing amount of research in this area which attempts to study and model growth. The main thrust in the analysis of structural changes, which are caused by the economic forces that arise intrinsically⁶ within the economic system, can be at-

⁶ Schumpeter quite crisply captures the disequilibrium process by describing that "what we are about to consider is that kind of change arising from within the system *which so displaces its equilibrium point that the new one cannot be reached from the old one by infinitesimal steps.*" ([1911], 1934: 64; fn1; italics in the original.)

tributed to Schumpeter. In one of his classics, Schumpeter ([1911], 1934: 63-4; *italics added*) writes

[b]y “development,” .. we shall understand only such changes in economic life as are not forced upon it from without but arise *by its own initiative, from within*. [T]he mere growth of the economy, as shown by the growth of population and wealth, [will not] be designated here as a process of development. Development in our sense is a distinct phenomenon, entirely foreign to what may be observed in the circular flow or in the tendency towards equilibrium. *It is spontaneous and discontinuous change in the channels of the flow, disturbance of equilibrium, which forever alters and displaces the equilibrium state previously existing.*

Schumpeter, being a *socio-economist*, elegantly explains the growth process as a “*creative destruction*” process. The term *creative destruction* encapsulates the creation of new production and market structures, while the old techniques/structures are destroyed, thus hinting at creative destruction as one of the possible causes of *business cycles*. Schumpeter, in Austrian vein, viewed the economic system to be in a vertically integrated form, with production as its *core*. Furthermore, his analysis was focused on studying how competitive environments foster innovation processes, leading to new products and processes, and thus economic growth. The orthodox endogenous growth models (Romer, 1986; 1990; 1993) that aim to explain the dynamics of growth limit themselves from providing any valuable insights on how the phenomena actually takes place. Furthermore, the timeless property of the production function puts the endogenous growth models into a straitjacket and limits their efficacy in explaining the dynamic economic traverse. Moreover, technology appears as a pre-condition and not as a result of the process of innovation (AG, 2006). The

emergence of innovations and their dynamic interactions with the production processes are captured in a very limited manner by these models. In order to understand the structural change, due to a technological change, we need to understand the underlying economic factors in the first place. So the relevant questions here are the following: How do innovations emerge? How do the innovational processes evolve over time? How do these processes affect the production process? How can we model this phenomenon in a comprehensive and tractable way?

A possible answer to these questions is in two parts: first, to model production as a process over time. This would enable us to encapsulate and enlist the economic dynamics in a more tractable manner. Second, we need to model innovations, with their intrinsic uncertainties, in non-*ad hoc*, Schumpeterian way by which they *emerge* within the economic system. The first part of the problem can be dealt with by adopting the time profile of the production process, i.e., ‘time-to-build’ framework. For the second, there is the challenge of modelling innovations in their natural form. The prevailing attempts that aim to model innovations define the creation and accumulation processes of knowledge/ideas either as a *deterministic* or *stochastic* function. By doing so, they fail to capture the intrinsic uncertainties of the innovational processes in an insightful manner. The deterministic way of modelling innovations have been criticised by Zambelli (2004; 2005), who points out, that unlike production, innovations occur in an *undecidable* manner. The other, *not-so*-dominant models like evolutionary growth model, multisectoral and Neo-Austrian models acknowledge the indeterminacy of the occurrence of innovations. However, as they are unable to model the dynamic phenomena, they tend to assume the technological changes as either exogenous or stochastic factor. Adopting the Schumpeterian vision of regarding the intrinsic forces to be arising intrinsically within the system naturally

enforces the need to conceptualize innovational processes in a more realistic, non-exogenous, non-stochastic way. If so, how innovations can be modelled in a more tractable way?

Innovations, by their very nature, are *undecidable* phenomena that emerge in indeterminate ways. They cannot be forecasted *a priori*, and its complexity varies in uncomputable degrees. Therefore, one of the promising ventures is to bring together the fields of computability and algorithmic information theory and harness its richness to model and to tame innovations.

1.3.1 Economic systems as computing systems

Innovation is a systemic phenomenon... - AG, (2006: 22)

The economic problems, whether formalized by mathematical tools or not, are essentially computational problems which the economic agents solve to optimize, or satisfice, under a set of constraints. Even though the economic problems are computational problems “there is no explicit model of computation underpinning its optimization problem formulation...” Velupillai (2010: 75). One way to formalize the economic problems, with an underlying model of computation, is to view the economic problems as *decision problems*⁷ and view the economic agents as *problem solvers*⁸ trying to solve the *economic decision problems*. This conceptualization of economic agents as problem solvers/computing machines, and the formulation of

⁷ A *decision problem* is a problem that asks “whether there exists an *algorithm* to *decide* whether a mathematical assertion does or does not have a proof; or a formal problem does or does not have a solution.” (Velupillai, 2010:75)

⁸ As Newell and Simon’s encapsulated (1972: 9-12) a “man to be an information processing system, at least when he is solving problems.” See also Velupillai 2010: part IV.

economic problems as decision problems, would naturally underpin our theories with computability and algorithmic information theory.

Can we also regard an economy as a computing machine? The answer for this question is affirmative as well. Hicks viewed the economic system as one in which the inputs are processed to produce the outputs over *time*. This process of transformation is quintessentially a *computational process*. Therefore, it is possible that an economic system can be viewed as a computing device, which is trying to compute the solution over time. Emphasizing this point, Goodwin (1951b: 1-2), in one of his seminal paper, describes that

[t]he existing economic relations determine the method by which the economy makes its repeated tries and a solution is a stationary value of a variable which satisfies these relations in the sense that the same value will be repeated and no further adaptation is necessary. If the relations are such as to yield a stable system, then an undisturbed mechanism will find its stationary solution, otherwise it will depart from it or forever oscillate about it.

...
Such a machine is an exact analogue of a continuous dynamic process. Therefore it seems entirely permissible to regard the motion of an economy as a process of computing the answers to the problems posed to it.

But during disequilibrium, the destination of the economic system is mostly *unknown*. So, the system attempts to employ certain viability mechanisms in order to create and manage its resources. These, in turn, would ensure that the system traverses in the desired growth paths and reaches the destination state, if there is such a state. This is analogous to saying that for a computing system

[t]he desired value is not known, but rather the difference between successive trials is taken as the error and this is successively reduced

to zero – hence the name zeroing servo. At this point we have found the answer, our process repeats itself until disturbed. Thus the economy may be regarded as slowly computing the answer to an ever-changing problem. (Goodwin, 1951b:3)

Therefore, the economic system can then be represented in form of a computing device, as a dynamical system, governed by set of equations/rules/algorithms. By *Church-Turing thesis*, for any computable process there exists a Turing Machine that would perform the same operation. As the economic processes are *essentially* computing processes, the economic phenomena can be modelled as Turing Machines. Hence, an economic system can be modelled as a Turing Machines that is trying to compute answers for the problems posed to it. Will this approach enable us to model the intrinsic uncertainties of innovational processes too? Yes, the famous *non-halting problem of the Turing Machines*⁹ would encapsulate the intrinsic uncertainties of the innovational processes thus enabling us to model innovations in an insightful way.

The rigorous use of computability theory in discussing and demonstrating the computability aspects of the models in micro and macroeconomics can be found in the works of Velupillai (2000; 2005; 2010). In trying to tame the uncomputabilities, like that of the innovational processes, Zambelli explores the possibilities of modelling innovational processes using Busy Beavers¹⁰ (see Zambelli, 2004; 2005), in explaining the dynamic innovational phenomena. Why is the Busy Beaver metaphor relevant in economic analysis? The reason is that even though the Busy Beavers have

⁹ The *halting problem of Turing Machines* is that for a given algorithm and input, when fed into a Turing Machine, it is impossible to predict *a priori* whether the Turing Machine will halt by answering to the decision problem or not.

¹⁰ *Busy Beaver* was first introduced and defined by Tibor Radó (1962) in his seminal paper, “On non-computable numbers”, demonstrated a computable function that grows faster than many computable functions and thus uncomputable.

a deterministic and well-defined algorithm/structure, they produce unpredictable and indeterminate behaviour, structures, and dynamics. The Turing Machine metaphor captures the intrinsic uncertainties of the R&D processes. The famous *Unsolvability of the Halting problem of the Turing Machines* captures the indeterminacy of the innovational processes thus enabling us to model innovations in a new insightful way. Zambelli (2004) uses the *Busy Beaver* metaphor in the context of endogenous growth models to model the processes of creation of ideas and captures the intrinsic uncertainty underlying the innovational process. Moreover, the richness of computability and algorithmic complexity theories would enable us to model the dynamic economic phenomena in a more insightful way and, as Velupillai (2010: 53) envisages, “[t]his way the *ad hoc*ery of stochastic assumptions can be reduced or even eliminated in economics.”

1.3.2 Algorithmic complexity of innovations

Occam’s razor: “If presented with a choice between indifferent alternatives, then one ought to select the simplest one”.

Modelling innovations using Turing Machine metaphor will incorporate the intrinsic uncertainties of finding new ideas, or knowledge or innovations, but the important question is - how does one know whether the idea/ knowledge found is really an innovation or not? Algorithmic complexity theory provides us with tools to define the notion of an innovation in this context. One can measure innovations according to their computational and algorithmic complexity of the idea/knowledge. Computational complexity theory “tries to identify problems that are feasibly computable.” (Li and Vityani, 1993: 35) The number of steps and the time required for computing a function or desired output string can be used as a measure of the computational complexity of the object. On the other hand, algorithmic complexity is the measure

of the complexity of the algorithm/rules/instructions that when fed into a computing machine will produce the desired object/output. Li and Vityani (1993: v; *italics* in original) explain algorithmic complexity of an object as follows:

the amount of information in a finite string is the size (number of binary digits or *bits*) of the shortest program that, without additional data, computes the string and terminates.

The works of Velupillai (2010) and Zambelli (2004; 2005) show that ideas, innovation, commodities can be encoded as information bit string that has been processed by a Turing Machine. That is, when an algorithm is fed into a Turing Machine that will compute/produce the desired output. For example, let the production blueprint (i.e., production algorithm) of a product is of size n and, if we assign one labour for carrying out one task, then n number of labour will be used to produce the product. Therefore, a process innovation is an innovation that reduces the number of labour required for the production of the output. So the R&D team searches new Turing Machines that would reduce the algorithmic complexity of the production blueprint. The lower the algorithmic complexity of the production blueprint lower the production labour required for the production of output. Moreover, due to the halting problem for the Turing Machines, it would be impossible for the R&D labour to predict beforehand if the R&D activities they are carrying out will end discovering new Turing Machines with lower algorithmic complexity or not. By using the Turing Machine metaphor we can encapsulate the intrinsic uncertainties of innovational processes in a tractable manner. And by harnessing the richness of algorithmic information theory and computability theory we can model the innovation in a more insightful and Schumpeterian way.

1.4 Structure of the Dissertation

The dissertation is divided into three main chapters. In the second chapter, we focus on the problems of structural change due to innovational processes, and consequent traverse from one steady state to another, along the disequilibrium dynamic paths of AG's Neo-Austrian model. The aim of the chapter is to analyse AG model and develop viability measures and mechanisms that ensues a self-replacing state of the disequilibrated system. We will trace back the origins of the notion of viability and illustrate its significance in disequilibrium analysis. Later, we emphasize the importance of viable management of resources, viz., production, human and monetary resources, and analyse the significance of implementing viability conditions and mechanisms necessary for the system to traverse in a sustainable way. We then simulate the model to demonstrate the possible traverses of the economic system, with and without the viability conditions.

In the third chapter, we discuss some of the classics in business cycle theory in the context of the role played by the 'time-to-build' framework in modelling business cycles. The discussion will also focus on how these classic business cycle models differ from each other, from a methodological point of view. Some interesting insights between linear and non-linear models will be discussed. In conclusion, we will show how the Neo-Austrian approach forms a part of the business cycle theory. As an exercise, illustrating the richness of non-linear business cycle models for modelling economic fluctuations, we take Goodwin's business cycle model and simulate the model for higher orders and various initial conditions.

In the fourth chapter, we aim to harness the intrinsic uncertainties of a Turing Machine (TM) and use the TM metaphor to model process innovation within a '*time to build*' framework. This enhanced model is used to analyse the dynamic interac-

tions of innovation and production processes and study its possible traverses and *viable* paths. In particular, we illustrate the dynamics of the economic system depending upon different policy parameters – such as R&D policy, taxation policies and bankruptcy policies. The ultimate goal of this chapter is to provide strategic tools and forecasts for the policy makers for effective change and resources management in a volatile environment.

References

- AMENDOLA, M. AND GAFFARD, J. (1998): *Out of Equilibrium*, Oxford University Press, Oxford.
- (2006): *The Market Way to Riches: Behind the Myth*, Edward Elgar, Cheltenham, UK ; Northampton, MA.
- BÖHM-BAWERK, E. (1890): *Capital and Interest*, London: Macmillan. Translated by William Smart. Reprint, 1959.
- DAY, R. (1993): "Nonlinear Dynamics and Evolutionary Economics" in *Nonlinear Dynamics and Evolutionary Economics*. DAY, R. AND CHEN, P., Oxford University Press, Oxford.
- FRISCH, R. (1933): "Propagation and Impulse Problems in Dynamic Economics", pp. 171-205, in: *Economic Essays in honour of Gustav Cassel*, George Allen & Unwin Ltd., Museum Street, London.
- GOODWIN, R. (1951a): The Nonlinear Accelerator and the Persistence of Business Cycles, *Econometrica*, Vol. 19, #1, January, pp. 1-17.
- (1951b): "Iteration, Automatic Computers and Economic Dynamics", *Metroeconomica*, Vol. 3, (1): 17.
- HAGEMANN, H. AND KURZ, H. D. (1976): 'The Return of the Same Truncation Period and Reswitching of Techniques in Neo-Austrian and more General Models', *Kyklos*, Vol. 29, pp. 678-708.
- HICKS, J. R. (1965): *Capital and Growth*. Oxford: Clarendon Press.
- (1970): "A Neo-Austrian Growth Theory", *Economic Journal*, Royal Economic Society, Vol. 80, (318): 257-281.
- HICKS, J. (1973): *Capital and Time: A Neo-Austrian Theory*, Clarendon Press, Oxford.
- (1985): *Methods of Dynamic Economics*, Oxford University Press, Oxford.
- KALECKI, M. (1935): A Macrodynamical Theory of Business Cycles, *Econometrica*, Vol. 3, #3, July, pp.327-344.
- KEYNES, J.M. (1936): *The General Theory of Employment, Interest and Money*. Macmillan and Co. Limited: London.
- KYDLAND, F. AND PRESCOTT, E. (1982): Time to Build and Aggregate Fluctuations, *Econometrica*, Vol. 50, #6, November, pp.1345-1370.

- LI, M., AND VITANYI, P., (1993): *An Introduction to Kolmogorov Complexity and its Applications*, Springer-Verlag, Berlin and Heidelberg.
- LOWE, A. (1976): *The Path of Economic Growth*. Cambridge University Press, New York.
- MARSHALL, A. (1920): *Principles of Economics* (Revised Edition ed.). London: Macmillan
- NEWELL, A. & H.A. SIMON (1972): *Human Problem Solving*, Prentice-Hall Inc., Englewood Cliffs, NJ.
- NUTI, D.M. (1973): On the Truncation of Production Flows, in: *Kyklos*, 26(3), pp.485-496
- RADÓ T. (1962): ‘On non-computable functions’, *Bell System Technical Journal*, 41 (May), pp. 877–84.
- ROMER P. (1986): ‘Increasing Returns and Long Run Growth’, *Journal of Political Economy*, 94, pp. 1002–37.
- (1990): ‘Endogenous Technological Change’, *Journal of Political Economy*, 98, pp. S71–S102.
- (1993): ‘Two strategies for economic development: using ideas and producing ideas’, in *Proceedings of the World Bank Annual Conference on Development Economics*, IBRD, World Bank, Washington, DC.
- SILVA, E. AND TEIXEIRA, A. (2008): “Surveying Structural Change: Seminal Contributions and a Bibliometric Account ,” *Structural Change and Economic Dynamics*, vol. 19, no. 4, December, pp. 273-300.
- SCHUMPETER, J. (1934): *The Theory of Economic Development*. Cambridge: Harvard University Press. (New York: Oxford University Press, 1961.) First published in German, 1911.
- SHAIKH, A. (1974): “Laws of Production and Laws of Algebra: The Humbug Production Function.” *The Review of Economics and Statistics* LVI(61, February):115-2
- SRAFFA, P. (1960): *Production of Commodities by Means of Commodities*, Cambridge University Press, Cambridge.
- VELUPILLAI K. VELA. (2000): *Computable Economics*, Oxford University Press.
- (2005): *Computability, Complexity and Constructivity in Economic Analysis*,

Blackwell, Oxford.

--- (2010): *Computable Foundations for Economics*, Routledge, Oxon.

--- (2011): Non-Linear Dynamics, Complexity And Randomness: Algorithmic Foundations. *Journal of Economic Surveys*, 25: no. doi: 10.1111/j.1467-6419.2010.00668.x

VON NEUMANN, J. ([1938] 1945-6): A Model of General Economic Equilibrium, *Review of Economic Studies*, Vol. XIII, No. 1, pp. 1-9.

ZAMBELLI, S. (2004): 'Production of Ideas by means of Ideas: a Turing Machine metaphor', *Metroeconomica*, 55 (2-3), pp. 155-79.

--- (2005): 'Computable Knowledge and Undecidability: A Turing Machine Metaphor', in Velupillai K. Vela (ed.) *Computability, Complexity and Constructivity in Economic Analysis*, Blackwell, pp. 233-63.

ZVONKIN, A.K AND LEVIN, L.A. (1970): The Complexity of Finite Objects and the Development of the Concepts of Information and Randomness by Means of the Theory of Algorithms. *Russian Mathematical Surveys*. Vol.25, # 6: 83-124.

Chapter 2

Viably Traversing the *Dynamic* Disequilibrium Trajectory?

Abstract

Evolutionary structural change, influenced by innovations, through creative destruction is one of the important driving forces of economic growth. During the dynamic disequilibrium of an evolutionary economic growth, the economic system is subjected to various endogenous imbalances. Consequently, the ability of the economic system to re-construct itself to grow, by creating and accommodating the necessary changes, could be jeopardized. In such *indeterminate* periods, the economic systems should co-ordinate, create, and manage resources in order to traverse through trajectories of dynamic disequilibria. The present models provide a limited insight on how the system, under various constraints, evolves over time along paths of dynamic disequilibria. This chapter explores the different factors of disequilibrium dynamics and tries to explain some of the viability creating mechanisms that accompany structural changes within a vertically integrated, Neo-Austrian dynamic sectoral model. The results reinforce the need for a deeper understanding of the dynamically complex interactions of the production, labour and financial variables in order, for the economic system, to realize and sustain growth.

Keywords: Traverse Analysis, Neo-Austrian Model, Viability, Near-Decomposability.

2.1. Introduction

"Eventually in the process of simulating such a model, some of the feasibility conditions for a given "model agent" will be violated. To restore viability a mechanism must be introduced by the modeler that allows for the formation of new "model agents" to whom the resources associated with disappearing agents are transferred. Alternatively, a new organizational form can be installed that will transfer resources so as to maintain individual agent feasibility and overall systems viability as an analog of "Chapter Eleven" proceedings [which is on 'Self-Organization as a Process in Evolution of Economic Systems'], unemployment insurance, and welfare payments.

Such *viability creating mechanisms* are the analog of equilibrium "existence" proofs, but in the out-of-equilibrium setting. They are required to guarantee the existence of a continuing "solution" to the system in terms of feasible actions for all of its constituent model components. When they are explicitly represented, then not only the population of production processes evolve, *but also the population of agents, organizations and institutions.* "

(Day, 1993: 38-9; *italics* in original)

An evolutionary economic system can be characterized, *partially*, by qualitative changes, such as innovations and technology, and the resulting structural changes, which in turn lead to dynamic adjustments and adaptations. In a resulting disequilibrium, it is imperative to understand and model the possibly time-varying production coefficients, with a focus on short-term goals, and also focus on long-term horizons. The asynchronicity of demand and supply, consumption and investment, can cause ripple effects not only in the current production line, but also in the future production processes.

In disequilibrium, the economy may embark on various traverses: which may lead to a steady state or move the system further away from it, *even if a steady state can be shown to exist.* In this context, the viability of the economy is *at stake*; and a systematic study to understand the viability conditions, which would enable the sys-

tem to achieve the desired state or a steady state, is a difficult task. As Hicks (1985: 156) points out "the Classics [referring to the classical economists] were mainly concerned [with]: how does economic growth come about? how, when started, can it be maintained?" They were mainly interested in exploring "the big problems" (Hicks 1985: 156), which were: to comprehend the dynamics of economic growth and the *viability* conditions that are necessary to achieve growth during evolutionary change.

Economic growth is *multiphase* in nature (Day, 1992), understanding this *systemic* (Amendola and Gaffard, 2006) yet dynamic phenomenon will help in managing the disequilibrium processes. Any failure to do so will have serious effects on both existing and future disequilibrating forces thus jeopardizing the processes undertaken to bring the economic system to equilibrium. In such dynamic disequilibria, the viability of the structural change becomes crucial for the firms and the market governing bodies, like financial institutions. If the disequilibrium forces are *not adequately managed*, it may lead to the collapse of the market and even the economy as a whole.

Why is viability important in disequilibrium? How do we initiate and maintain the viability processes? In a structural disequilibrium,

the adjustment proceeds entirely on this [synchronizing the supply to the demand] plan, there are limits to what can be done. Though there is an incentive, even in a Fixprice system, to bring into production unused capacity (not only fixed capital capacity, but surplus stocks of materials) when it can be utilized by its present proprietor; and though resources that are wholly unused may be transferred, ... the mechanism of transference from those whose need is less to those whose need is more urgent must unquestionably be clogged. ... [T]he urgency in question is urgency in the breaking of bottlenecks. *Flexibility* along the Traverse is of major importance. (Hicks, 1985: 141-42; *italics* added)

The bottlenecks arising along the traverse are not caused by *exogenous* factors

alone, as emphasized in orthodox macroeconomic theories, but also because of the *endogenous* factors and the result of the actions and developments in the past. Economic growth is *a recurrent and long-lasting phenomenon* (Lowe, 1976: 8), which requires systemic analysis of sequences of states and processes that differ in the quality and quantity of the resources utilized. Equilibrium theory, based on optimality (static or dynamic), fails to provide a tractable or reasonable solution for this dynamic problem (Kaldor 1972; Lowe 1976; Hicks 1973, 1985). Kaldor observes that "[a]ttempts have been made to graft growth and development to equilibrium theory, but they have not succeeded in transforming it into a sequence analysis in which the course of development is dependent on the path of evolution" (1975: 348)¹. The key for analysing the evolutionary problem is to focus the attention on secular analysis, not in logical time but in *real time*, by adopting the time profile of the processes to unravel the mysterious nature of growth.

Production as a *process* is the blueprint of the Austrian tradition and, what followed, the Neo-Austrian approach. Hicks (1970) in his seminal work presented a tractable version of the Austrian model, on the lines of Böhm-Bawerk, by developing the original single input - single output schema into a multi input - multi output schema (Hicks, 1973). The focus of this approach was not restricted to productive processes alone, but extended its scope to analyse the disequilibrium phenomenon in an ensuing traverse to a new equilibrium. Unlike the formalization of production and technology in neoclassical theory, here, the technology is not assumed to be *given* at a particular time. Therefore the economic system should construct the productive ca-

¹ One such attempt can be traced to Stigler's (1951) *theory of the firm*. Stigler in his model conceptualized production as a set of activities in time to study the dynamics of innovations within a firm, so he adopted the vertical integration schema. Having started with the right schema, he failed to model and explain the complex innovational processes as he confined his analysis by assuming a simple life cycle.

capacity in order to carry out production. Any failure to synchronize construction with utilization would shift the system away from its goals and might even restrain it from self-evolving. Thus, the quest to reach new steady state equilibrium depends crucially on the *viability creating mechanisms* (Day, 1993: 38-39) that drive the economy forward. These mechanisms have to be carefully formulated to establish and perform the targeted task, but it is tricky as the tasks and dynamics towards the production frontiers are *hazy* (Winter, 2005: 235-37). Moreover, even if it is clear, the *relative inflexibility* of economic structures makes it challenging to operationalize the mechanisms consistently, over time. And the institutions and organizations, that emerge out of disequilibrium, "in reality are, after all, most often invented in response to economic pressures caused by unemployment, bankruptcy, poverty, and other *problems of inviability* experienced by individuals and organizations" (Day, 1993: 39; *italics added*).

Therefore, it is necessary to develop an *evolutionary tool* which can enable us to model the productive process over time, along with various *causal* factors, and provide *insights* on the future trends and viability mechanisms to *achieve* the desired goals. The time profile of the productive structures in the Neo-Austrian approach provides us with a possible operational, and theoretically consistent, scheme to model and analyse the dynamics of disequilibrium growth. The Neo-Austrian approach has been imaginatively developed by Amendola and Gaffard (1998, 2006) - (henceforth AG 98, AG 06) for simulational applications. AG brings together various factors into a comprehensive sectoral model to study the disequilibrium dynamics of innovations and its impact on economic growth. However, the analysis is limited because it leaves open the questions related to viability preconditions of both production and innovational processes. This chapter takes up such a task to explore and

explain the viability preconditions of innovational *disequilibrium* dynamics within the Neo-Austrian context.

2.2. Economics of Viability

What is viability? What is its role *in a disequilibrium context*? Viability can be defined as the fundamental economic prerequisite for the processes undertaken by the economic system to consistently achieve evolutionary structural change along its traverse over real time. Moreover, process of viability is *an algorithmic process* by which the system satisfies or moves toward the viability ensuing conditions. In a nutshell, we can describe viability as a means by which *the desired states* can be reached by utilizing the available production, labour, and financial resources.

Lets take a simple 2-sector economy; one sector produces wheat and the other produces iron. Both of these commodities, wheat and iron, measured in tons, are used as inputs for production by the two sectors. That is, sector 1 uses 20 tons of wheat and 3 tons of iron to produce 25 tons of wheat and the sector 2 uses 10 tons of wheat and 7 tons of iron to produce 15 tons of iron. The above 2-sector economy can be represented as below:

Input			Output
Wheat	Iron		
Sector 1: 20 tons Wheat	+ 3 tons Iron	⇒	25 tons Wheat
Sector 2: 10 tons Wheat	+ 7 tons Iron	⇒	15 tons Iron
-----	-----		
30 tons Wheat	10 tons Iron		

(2.1)

The economy is said to be in self-replacing state if the output produced by the sectors is greater than or equal to the total quantity used for production by the economic system, only then the system will be able to self-reproduce itself over time. In the above example (2.1), we can see that the sectors uses 30 tons of wheat while the

sector 1 produces only 25 tons of wheat, thus the economic system is in a non-self-replacing state. In order to make the system viable and bring it to a self-replacing state we have to find the right *multipliers*² for each sector. Let y_w and y_I be the multipliers of the sectors 1 and 2 respectively.

$$\begin{aligned} 20 y_w + 3 y_w &\Rightarrow 25 y_w \\ 10 y_I + 7 y_I &\Rightarrow 15 y_I \end{aligned} \quad (2.2)$$

Rearranging the equations (2.2) in such a way that the products used for production and what is being produced, we get the equation below:

$$\begin{aligned} 20 y_w + 10 y_I + 0 y_S &= 25 y_w \\ 3 y_w + 7 y_I + 5 y_S &= 15 y_I \end{aligned} \quad (2.3)$$

Where, y_S denotes the surplus of commodities.

A system is *non-viable* if there are some sectors that use more for the means of production than what is being produced, while the other sectors do not produce surpluses. A non-self-replacing economic system can be brought to a self-replacing state only if surpluses exist in some of the sectors. In our system, sector 2 produces a surplus of 5 tons of iron, therefore we can attempt to reallocate the available resources in such a way that the system becomes self-replacing. In our example, there are two equations and three unknowns, so we fix $y_w=1$.

$$\begin{aligned} 5 y_w - 10 y_I + 0 y_S &= 0 \\ - 3 y_w + 8 y_I - 5 y_S &= 0 \end{aligned} \quad (2.4)$$

Solving the equation above, we get $y_I = 1/2$ and $y_S = 1/5$. Multiplying the sectors with appropriate coefficients, the economy would be as follows:

² The multipliers are unknowns used as "reference" and their fractions are calculated in such a way that the system's deficit becomes null (Chiodi, 1992).

$$\begin{array}{rcl}
 20 \text{ tons Wheat} + 3 \text{ tons Iron} & \Rightarrow & 25 \text{ tons Wheat} \\
 5 \text{ tons Wheat} + 3.5 \text{ tons Iron} & \Rightarrow & 7.5 \text{ tons Iron} \\
 \text{-----} & & \text{-----} \\
 25 \text{ tons Wheat} & & 6.5 \text{ tons Iron}
 \end{array} \tag{2.5}$$

We observe that in the newly rearranged economic system (2.5), the total quantity of products used for production is greater than or equal to the total production, therefore making the system viable and self-reproduce itself overtime.

Generalizing the above example, let A, B, C, \dots, K be the sectors in an economy producing commodities a, b, c, \dots, k , respectively, which are then used as inputs by the sectors for production. The economic system can be represented as:

$$\begin{array}{rcl}
 a_A + b_A + \dots + k_A & \Rightarrow & a \\
 a_B + b_B + \dots + k_B & \Rightarrow & b \\
 \dots & & \dots \\
 a_K + b_K + \dots + k_K & \Rightarrow & k
 \end{array} \tag{2.6}$$

let, s_A, s_B, \dots, s_K , be the surpluses and $y_A, y_B, y_C, \dots, y_K$ be the multipliers for the sectors A, B, C, \dots, K , respectively. y_s denotes the multiplier for the surpluses in every sector.

$$\begin{array}{rcl}
 a_A y_A + a_B y_B + \dots + a_K y_K + s_A y_S & = & a y_A \\
 b_A y_A + b_B y_B + \dots + b_K y_K + s_B y_S & = & b y_B \\
 \dots & & \dots \\
 k_A y_A + k_B y_B + \dots + k_K y_K + s_K y_S & = & k y_K
 \end{array} \tag{2.7}$$

The surpluses ($s_{A,B,C,\dots,K}$) in every sector should be greater than or equal to zero, i.e., when there is a deficit in a sector then the surplus for that sector will take the value zero. Rearranging the expressions we get:

$$\begin{aligned}
 (a - a_A)y_A - a_B y_B - \dots - a_K y_K &= s_A y_S \\
 -b_A y_A + (b - b_B)y_B - \dots - b_K y_K &= s_B y_S \\
 &\dots \dots \dots \dots \dots \dots \dots \dots \dots \\
 -k_A y_A - k_B y_B - \dots + (k - k_K)y_K &= s_K y_S
 \end{aligned} \tag{2.8}$$

For a system to be viable, the multipliers $y_A > 0, y_B > 0, \dots, y_B > 0$ and $y_S \geq 0$. Only then the non-self-replacing system can be brought to a self-replacing state.

The pioneering attempts to characterize and formalize the notion of *viability* in economic analysis can be traced back to Hawkins and Simon's paper (1949)³ [Henceforth, H-S] and Sraffa's *Production of Commodities by Means of Commodities* (1960). H-S's theorem on characterization of viability (1949: 245-47) is a special case of *viability*; in fact it is *solvability*, which is one dimension of the notion of viability. H-S (1949: 245-47) analysed the ‘open’ Leontief model and formulated a necessary and sufficient condition and showed that if all the principal minors of the matrix $(I-A)$ are positive then the equation $(I-A)x=y$ will have a positive solution; Where, I=Identity matrix, A= $(n-1) \times (n-1)$ technical production coefficients matrix, $x= (n-1) \times 1$ output vector and $y= (n-1) \times 1$ demand vector. The theorem states that the economic system will "always have a unique, non-negative solution for any non-negative final demand, provided that the system has a non-negative solution for a positive bill of goods in a certain year" (Nikaido, 1970: 18).

The Matrix 'A', used by H-S, is also known as the technical coefficient matrix of a production technology. Therefore, H-S's notion of viability is "exclusively confined to the *technology* of the economy, with the social and political aspects put out

³ See also Gale, 1960 (Chapter: 9). Gale developed the notion of viability for a simple Leontief model by explicitly considering labour in his analysis, but, like Hawkins and Simon, confined his notion of viability *exclusively* to the technical coefficients matrix of production.

of the picture, and the 'viability' of the economy manifests itself in the potential existence of a *surplus* of commodities on which consumption goods can find their place *as a residue*, heavily depending upon the tenet of the technology." (Chiodi, 2010: 324; *italics* in original) So their analysis provides a limited insight into the viability creating mechanisms that drive the economy forward along a disequilibrium path.

Sraffa's notion of viability, on the other hand, is "synonymous of *survival* of the system as a whole" (Chiodi, 2010: 324). Sraffa explained the importance of viability of the system in the Chapter 1.

This formulation [i.e., the model that is being built in the book] presupposes the system's being in a self-replacing state; but every system of the type under consideration is capable of being brought to such a state merely by changing the proportions in which the individual equations enter it. (Systems which do so with a surplus are discussed in §4ff. *Systems which are incapable of doing so under any proportions and show a deficit in the production of some commodities over their consumption even if none has a surplus do not represent viable economic systems and are not considered.*) (Sraffa, 1960: 5, fn. 1; *italics* added)

Sraffa's notion of viability is not just about solvability but also about co-ordinating, creating, and managing resources within and between the sectors during the indeterminate periods of growth. To explore the notion of viability, Chiodi (1992, 1998) took a non-self-replacing Sraffian Multisectoral model to study its dynamic behaviour by developing a viability precondition for the system to become self-replacing. If the viability condition is not satisfied, then the system will become – or will continue to be – non-self-replacing and jeopardizing the viability of production, and hence the survivability of the system over time in a self-replacing mode. The system should find the *right* multiplier for each input, such that the inputs and the outputs, within and between the sectors, are synchronized to make the system viable. Therefore, the economic system can be interpreted as *a computing device* that attempts to

calculate the appropriate multiplier values in order to synchronize its resources in such a way that it would enable the system to self-replace itself and sustain over time. The Multisectoral model gives rich dynamics of the adaptive adjustment of the inputs between various sectors in order for the system to *reproduce itself*. However, the model does not explain the dynamic adjustments of production factors and its traverses within sectors, nor does it account for the feasibility of such multipliers.

The Neo-Austrian model gives the appropriate schema for exploring the dynamic complementarity of the resources and how the resources are to be managed in order to ensure that the economy is self-replacing. AG's (1998, 2006) vertically integrated, Neo-Austrian model⁴ conceptualizes and analyses the dynamics of production, human resources, financial resources, markets and firms, over time. In the following sections, I will list the three important structures - production, labour, and financial structures - by which the viability of the system can be ensured during disequilibrium.

2.2.1 Production Dynamics

When an economy is out-of-equilibrium, production, which is *a core* aspect of the economic system, should be successfully co-ordinated within the system's dynamic constraints. Any excessive imbalances in the production process may move the system beyond the technically feasible range of viable operations and bring the whole system to a halt. For a discussion of the viability conditions in the Neo-Austrian context, let us consider the AG model (1998, 2006). Let I be the number of firms, producing m different products, operating in an economy. The total output (B) produced is the total input (A) used for production processes times the productive capacity (x) that has been started

⁴ See Appendix A for more details of the model.

at time (1 to $n^c + n^u$). Where, n^c = Construction period and n^u = Utilization period.

$$\sum_{T=t-(n^c+1)}^{n^c+n^u} B(T) = \sum_{T=t-(n^c+n^u)}^{n^c+n^u} A(T).x(T) \quad (2.9)$$

The input vector (A) is given as,

$$[A] = [a_j^i(t)]; \forall t = 1, 2, \dots, n^c + n^u; \forall i = 1, 2, \dots, I; \forall j = 1, 2, \dots, m; \quad (2.10)$$

$$a_j^i(t) = [a_j^{ic}(t), a_j^{iu}(t)]; \quad (2.11)$$

$$a_j^i(t) = a_j^{ic}(t); \forall t = 1, 2, \dots, n^c; \quad (2.12)$$

$$a_j^i(t) = a_j^{iu}(t); \forall t = n^c + 1, \dots, n^c + n^u; \quad (2.13)$$

where,

a = Inputs (*in real terms*);

a_j^{ic} = Input used for construction (c) by firm (i) for product (j);

a_j^{iu} = Input used for utilization (u) by firm (i) for product (j);

i = Index of the firms;

j = Index of the products;

t = Time;

n^c = Construction period;

n^u = Utilization period.

The output vector (B) is given as (*in real terms*),

$$[B] = [b_j^i(t)]; \forall t = 1, 2, \dots, n^c + n^u; \forall i = 1, 2, \dots, I; \forall j = 1, 2, \dots, m; \quad (2.14)$$

$$b_j^i(t) = 0; \forall t = 1, 2, \dots, n^c; \quad (2.15)$$

$$b_j^i(t) = b_j^{iu}(t); \forall t = n^c + 1, \dots, n^c + n^u; \quad (2.16)$$

where, $b_j^i(t)$ = Output of the commodity j of the firm (i) at each given time (t), and

the total productive capacity (x) vector is given as

$$[x] = [x_j^{ic}(t), x_j^{iu}(t)]; \forall t = 1, 2, \dots, n^c + n^u; \forall i = 1, 2, \dots, I; \forall j = 1, 2, \dots, m; \quad (2.17)$$

where,

$[x_j^{ic}] = x_j^{ic}(t); \forall t = 1, 2, \dots, n^c$ is the productive capacity during the construction phase

(c);

$[x_j^{iu}] = x_j^{iu}(t); \forall t = n^c + 1, \dots, n^c + n^u$ is the productive capacity for the utilization (u).

The notion of viability in the Neo-Austrian model is similar to that of Chi-odi's notion of viability in a Sraffian Multisectoral model. The system is self-replacing, thus viable, if the output from the production is greater than or equal to the inputs required for production.

$$\sum_{t=n^c+1}^{n^c+n^u} b_j^i(t) \geq \sum_{t=1}^{n^c+n^u} (a_j^{ic}(t) \cdot x_j^{ic}(t) + a_j^{iu}(t) \cdot x_j^{iu}(t)); \forall i = 1, 2, \dots, I; \forall j = 1, 2, \dots, m. \quad (2.18)$$

Moreover, for the system to survive and viably continue the production processes, the money proceeds (see appendix 2 A) from the final products should be equal to the costs of inputs and labour (see equation 2.19). Therefore, the warranted rate of growth⁵ of the productive system should be equal to the growth rate of income. If the desired and realized costs and proceeds are different, then the disequilibrating forces will hamper the viability procedures and move the system further away from the *natural* rate of system. *Natural rate of the system can be defined as the rate that would ensure the viability of the economic system.* If the warranted rate is not brought to the level of its natural rate, then the system will, over time, become non-self-replacing. In order to bring the economic system closer or equal to the natural rate of growth the viability creating production, labour, and monetary mechanisms should be employed to systematically synchronize the disequilibrium.

$$\sum_{t=n^c+1}^{n^c+n^u} (p_j(t) b_j^i(t)) = \sum_{t=1}^{n^c+n^u} W^i(t); \forall i = 1, 2, \dots, I; \forall j = 1, 2, \dots, m; \quad (2.19)$$

⁵ The warranted rate is the same as the *required rate of growth*, as in Harrod's growth model (1939, 1948). "When the income is growing at the rate required to make entrepreneurs desire to invest just the amount that is being invested, we say income is growing at the warranted rate" (Baumol, 1951: 42).

where, p_j^i is the price of the output commodities b_j^i , and W^i is the minimum desired Wage Fund for the system to be self-replacing (*in monetary values*),

$$W^i(t) = \sum_{j=1}^m w_j^i(t) \cdot (a_j^{ic}(t) \cdot x_j^{ic}(t)) + (a_j^{iu}(t) \cdot x_j^{iu}(t)) \quad \forall j = 1, 2, \dots, m; \quad (2.20)$$

the minimum desired Wage Fund, at any given time, should be sufficient for production to be carried on. W can be determined by the product of the wage rate (w) and the total input ($a \cdot x$).

The productive system is viable if there exists a *feasible utilization period* (\hat{n}^u) for the chosen technique, along with the set of *prices* (\hat{p}_j^i) for the *produced commodities* (\hat{b}_j^i) and *wage rate* (\hat{w}_j^i) for the total *input* ($a_j^i \cdot x_j^i$) that would ensure the self-replacing state of the production process. Then the viability condition can be stated as (*in monetary values*):

$$\sum_{t=n^c+1}^{n^c+\hat{n}^u} (\hat{p}_j^i(t) \hat{b}_j^i(t)) - \sum_{t=1}^{n^c+\hat{n}^u} (\hat{w}_j^i(t) (a_j^{ic}(t) \cdot x_j^{ic}(t)) + (a_j^{iu}(t) \cdot x_j^{iu}(t))) \geq 0. \quad (2.21)$$

Whenever a new technology or technique is introduced, the sunk costs rise during the construction phase so the profits realized in the utilization phase should be synchronized with the construction phase to ensure the viability of the self-replacing system. It is also necessary to handle the production *capital value* (K) carefully, at any given time, especially while *traversing*. According to the *truncation theorem*,

[t]he maximized present value of a (production) process is a monotonically decreasing function of the rate of interest; and, thus, a corollary is: Equating the present value to zero, ensures the uniqueness of the internal rate of return. (Velupillai 2000: 6)

When the capital value (K) of any productive process reaches zero or negative val-

ues, the productive system becomes non-self-replacing. So, as a general rule, the capital value should always be positive (Hicks 1973: 39), in order for the system to be viable. The viability condition that the economic system has to satisfy is the following:

$$K_j^i = \sum_{t=1}^{n^c+n^u} (q_j^i(t)R^{-t}) \geq k^*; k^*(t) \geq 0; \quad (2.22)$$

where,

$$q_j^i(t) = (p_j^i(t)b_j^i(t)) - W_j^i(t) = \text{Value realized by the production at each time } (t);$$

Where, $W_j^i(t)$ = Wage fund of the firm (i) for production of product (j) at time (t).

R^{-t} = Discount rate;

Where, $R = 1 + r$;

r = Rate of interest at each given time (t).

k^* = Threshold value of capital (K); Preferably, $k^*(t) \geq c(t)$;

The efficiency of a technique (Hicks 1973: 42), *in terms of capital value*, can be calculated by,

$$K_j^i = -\left(W_j^{ic}\right) \sum_{t=1}^{n^c-1} R^{-t} + (1 - W_j^{iu}) R^{-n^c} \sum_{t=1}^{n^u-1} R^{-t} = 0 \quad (2.23)$$

where,

W_j^{ic} = Wage fund of the firm (i) for construction workers used for production j^{th} product.

W_j^{iu} = Wage fund of the firm (i) for utilization workers used for production j^{th} product.

In disequilibrium, firms should accumulate surplus resources to manage the viability of the processes undertaken along the traverse. As the capital value of the existing process tends to zero, new production processes, with positive capital value,

should be introduced in order for the system to *reproduce* itself. The rate at which the new productive processes start depends not only on the innovational techniques, but more importantly on the availability of the capital resources (Day 1993: 37). Therefore, the ability of the firm to choose and adopt the *appropriate* technique will, in turn, determine the firm's consequent traverses. The efficiency curve⁶ gives the viability condition for the profitability of the technique and this, in turn, guides the system through a viable traverse.

The above Neo-Austrian model with viability conditions was simulated⁷ to study the dynamics of capital value. The result shows that at time 10, the system starts to oscillate over time and brings the capital value zero and then negative at time 150 (see Figure 2.1). Any use of the existing technology after time 150 would hamper the production processes and moves the system away or even out of *the viability corridors*, thus hindering the growth process. In a dynamic disequilibrium, the truncation

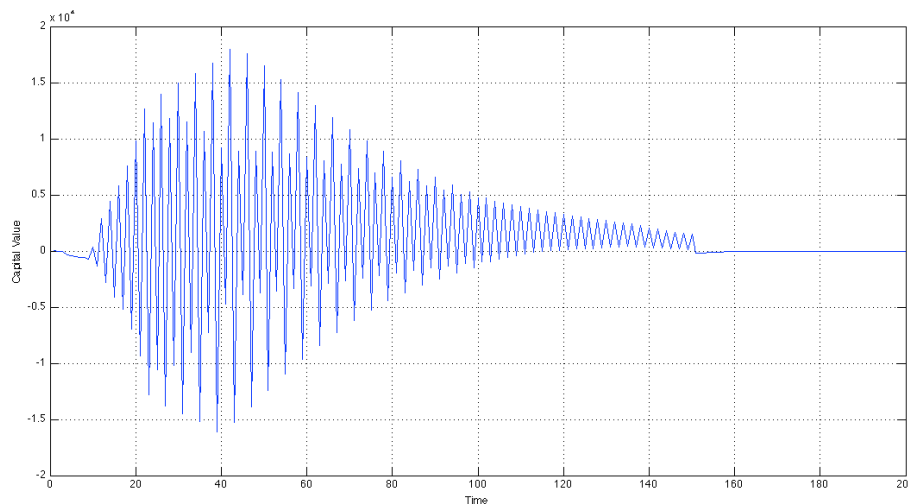


Figure 2.1: The dynamics of capital value

⁶ The *efficiency curve of the technology* not only indicates the maximum rate of return available at each value of W , but also shows what technique must be adopted to get the maximum rate of return (Hicks, 1973: 40).

⁷ All the simulations have been carried out by using Matlab. (See Appendices 2A & 2B)

of production processes may happen frequently. Therefore, it is necessary for the system to learn and to accumulate essential skill levels and physical stocks in the form of savings and credit, which can be used for further unexpected imbalances that may arise in the future.

2.2.2. Labour Dynamics

Managing production is just one of the many variables that determine viability and *labour* is another important one. In AG's model, labour plays a pivotal role in the production process and any technological change, in turn, would require *specific* knowledge to carry out the production viably. More importantly, the labour market should not just meet the quantitative needs of the production processes, but also be qualitatively skilled enough to carry out innovational processes. Let the labour demand (L^d) at any given time be denoted (*in real terms*) as

$$L^d(t) = \sum_{j=1}^m \sum_{h=1}^l (a_{h,j}^c(t) \cdot x_j^c(t)) + (a_{h,j}^u(t) \cdot x_j^u(t)); \forall t = 1, 2, \dots, n^c + n^u; \forall h = 1, 2, \dots, l; \quad (2.24)$$

Where, h is the skill level of the labour L required to carry out production of j^{th} commodity.

The labour supply (L^s) depends on the natural growth rate of population and on the elasticity of the wage rate

$$L^s(t) = (1 + g)^t L^s(0) \cdot w(t)^v \quad (2.25)$$

where,

g = Natural growth rate⁸;

w = Average wage rate;

v = Wage elasticity of the labour supply.

⁸ The natural growth rate of the population is assumed exogenous in the model. Nonetheless, the model can be simulated and studied for various sets of growth rate to understand its complex dynamics.

For the system to operate viably, it is necessary that the ratio of labour supply to labour demand be greater than or equal to, or *at least tend* towards β^* (see equation 2.26). Importantly, the skill level required for production should be matched with the available labour in the market, since otherwise the production will be hindered:

$$(L^s(t) / L^d(t)) \geq \beta^*; \beta^* \geq 1; \forall t = 1, 2, \dots, n^c + n^u; \quad (2.26)$$

Hicks (1973: 47-62) analyzed the dynamics of the traverse in an evolving system by considering the full employment case, and pointed out that the system is *adaptively* stable. The limitation in the full employment case is that, Hicks implicitly assumed, the knowledge or skill level required for the new and old technology to be the same while the system traverses. If this rather strong assumption is relaxed, then the production processes cannot be carried out, unless workers are given the required training. In out-of-equilibrium, the *competence loss* threatens the functionality of the system. Figure 2.2 depicts the adaptive adjustment process of the demand and supply of labour. The system should reinforce the viability conditions by attracting a new workforce, and by providing training and incentives in such a way that the production is synchronized. Any lag in the adjustment of labour supply to labour demand

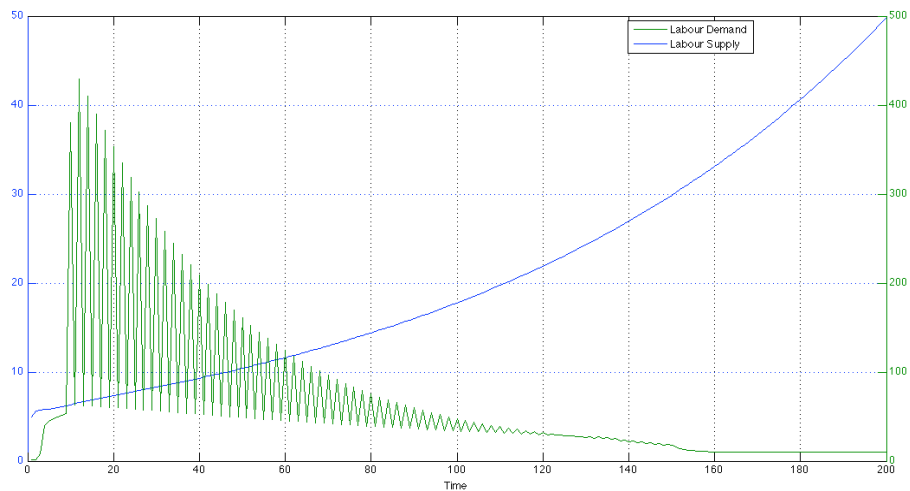


Figure 2.2: The dynamics of labour demand and supply

would have an effect on the present and future processes making the self-replacing system *unviable* over time.

2.2.3 Monetary Dynamics

The third important factor in the viability process is *money*. Under ideal conditions, the money supply should be synchronized to the money demand over time, but it is hard to do so since the value of innovations and technology – more importantly of disequilibria – cannot be predicted *a priori*. Given this, the economic system calls for an adaptive *viability creating monetary policy*. Financial resources are one of the main instruments by which the disequilibrated system can be studied and managed, and any policy that does not help in traversing the disequilibrium will eventually lead to the undermining of the viability of the economy. It is important that the system attempts to synchronize investments, revenues, savings, and costs by employing viability creating mechanisms, over time. Hicks, in *Capital and Time* (1973: 99), has emphasized a similar point: "To industrialize, without the savings to support your industrialization, is to ask for trouble." That is, the construction of new techniques and the utilization of present techniques should be in such a way that the total expenditure incurred is less than or equal to the financial resources available. The productive processes can only be realized if the system satisfies, or at least, *moves towards* the viability conditions. The available financial resources F^i of firm (i) at any given time (t) is calculated by

$$F^i(t) = m^i(t-1) + h^i(t-1) + f_s^i(t) - c^i(t) \quad (2.27)$$

where,

m = Money proceeds from sales at time (t);

h = Idle money balance from the previous time period. If the human resources constraint is more stringent then the financial constraint money balances are

involuntarily accumulated;

f_s = Available external financial resources;

c = Take out.

The economic system is self-replacing if it satisfies the following financial viability conditions, along with the production and labour viability conditions:

$$\left(\left(\sum_{t=1}^T F^i(T) - \sum_{t=1}^T W^i(T-1) \right) - (W^i(T) + c^i(T)) \right) \geq 0; \forall T \in t = 1, 2, \dots, T, \dots, n^c + n^u; \quad (2.28)$$

$$\left(\left(\sum_{t=1}^T F^i(T) - \sum_{t=1}^T W^i(T-1) \right) / f_d^i(T) \right) \geq \alpha^*; \alpha^* \geq 0; \forall T \in t = 1, 2, \dots, T, \dots, n^c + n^u; \quad (2.29)$$

where, f_d^i is the amount of financial resources demanded for carrying out the productive processes of firm (i).

We can say that the financial resource of the firm is *healthy*, if the ratio of money finance available to the demand for external financing is greater than or equal to α^* .

During disequilibrium, the money supply (f^s) and money demand (f^d) are not synchronized, but the productive and viability processes must necessarily be carried on.

In fact, in the early stages of economic growth it is necessary for the system to create an environment for the firms to research and develop innovations and new technologies. These can only be realized if there is an adequate amount of money to carry out the production activities; if not, the system cannot *embark* on a growth process. In this situation, α^* will not only be an indicator of the financial power of the firm but also help to monitor the capital-debt ratio of the firms. The monetary authorities can use α as one of the measures to analyse the potential risks of the economic system and devise various viability mechanisms to bring α to a desired level.

During out-of-equilibrium situations, *Credit* plays a vital *dual* role as a substitute, until the money supply is synchronized with money demand, and as a means

to create a favourable environment to foster new innovational activities. Credit also cushions the rippling imbalances and brings the system to the desired viable traverse. Apart from credit, the lending interest rate, in general, plays a very critical part in the disequilibrium situation. It should be noted that the rate of interest⁹ are endogenously generated and so is the market rate of interest, at least *partially*. When situations like inflation, low demand, and high supply arise, the capital value of the production tends to zero and then negative. According to *the fundamental theorem* (Hicks, 1973: 39), it would be non-profitable and unviable for the system to be self-replacing. Through the efficiency curve, if we know the natural rate of interest and the efficiency, in real and capital value terms, we can find the *feasible* length of the production process. But if the characteristics of the technology or technique undertaken are evolving, it would be almost impossible to predict the length of the productive processes. For viability of the evolving system it is important to know the length of production and its capital value as they will determine the future traverse profiles.

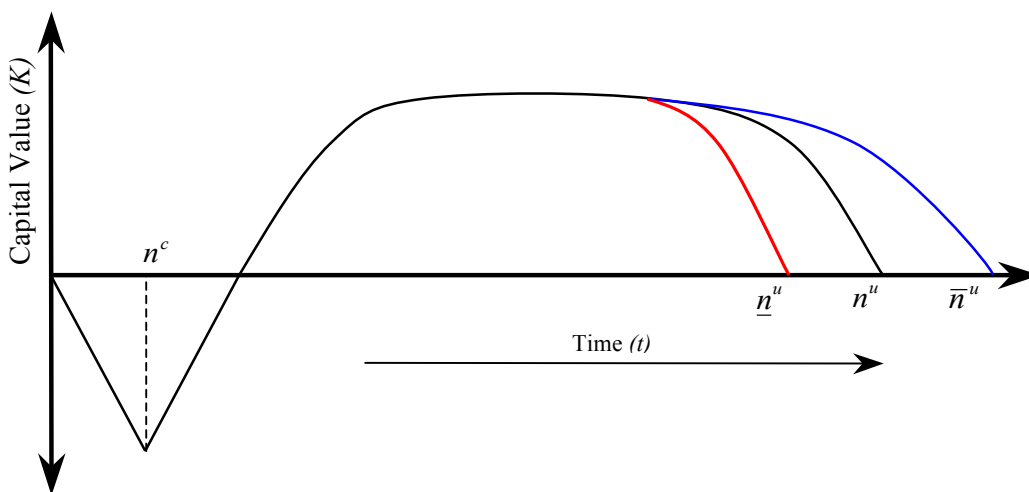


Figure 2.3: The possible traverses of the economic system

⁹ The rate of interest, referred here, is the same as the *yield* of the process, or the *internal rate of return* (Hicks, 1973: 22).

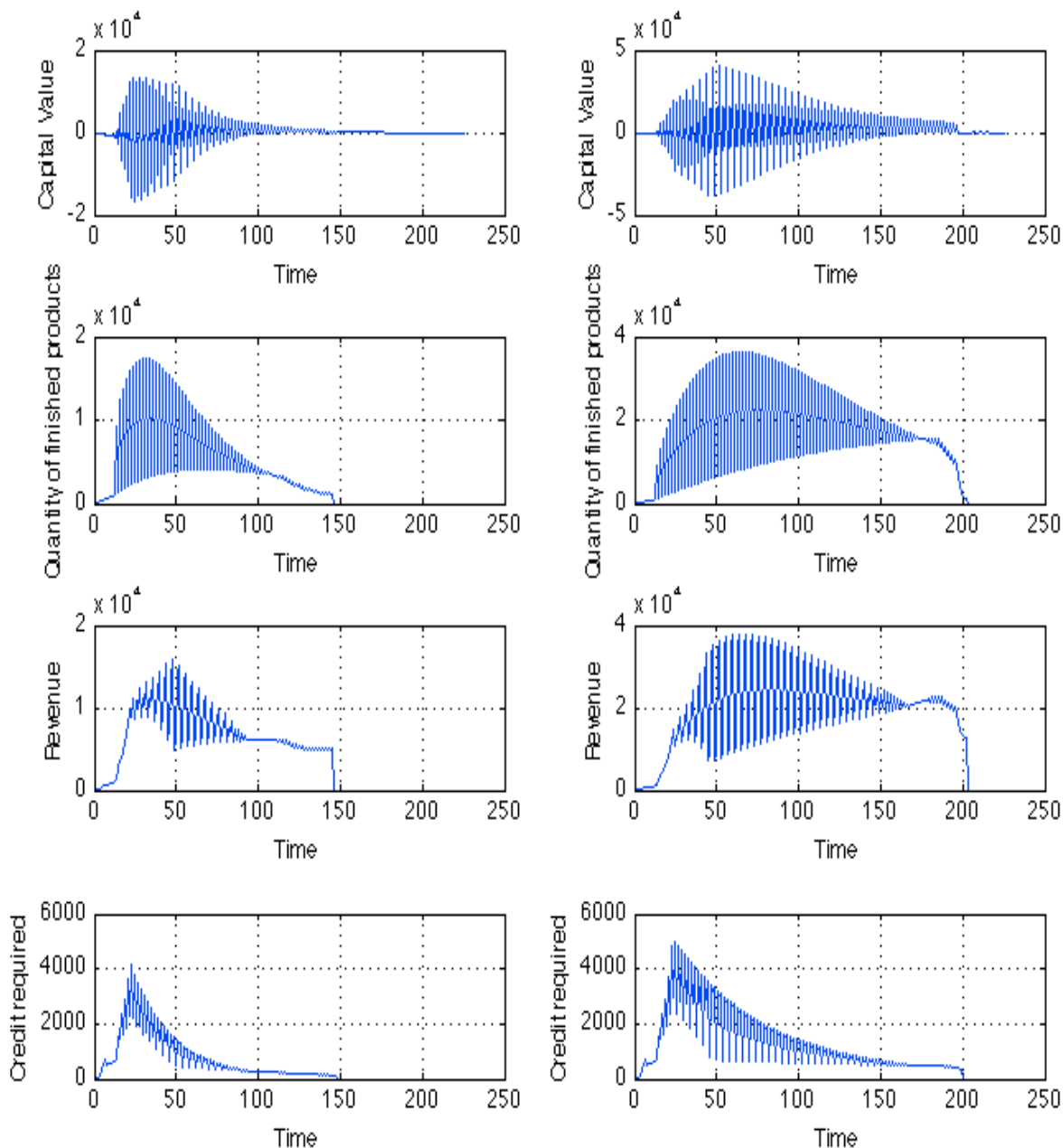
In a dynamic environment, if the capital value of production is zero, even if there exists *effective* demand for the products, the productive activities will be truncated, i.e., $\underline{n}'' \geq n''$ (desired utilization period is greater than actual). In this situation the market rate of interest should be *matched* with the natural rate of interest, by doing so the utilization will be brought to its original utilization period n'' or can even be increased to \bar{n}'' to fulfil the demand (see Figure 2.3). Therefore the key is to manage construction (n^c) and utilization (n'') phases and to keep the debt *low*, for sustaining and paving way for future *evolutions* to be viable. Irving Fisher (1907: 353) observes

[t]hat long processes (assuming their length to be measurable) are more productive than short processes is, as Böhm-Bawerk says, a general fact not a necessary truth. The reason lies in *selection*. It is not true that, of all *possible* productive processes, the longest are the most productive; but it is true that, of all productive processes *actually employed*, the longest are also the most productive. No one will select a long way unless it is at the same time a better way. All the long but unproductive processes are *weeded out*.

(In Velupillai, 1995: 559; *italics* added)

It is true that the unproductive processes are *weeded out* over time, but it should be noted that in disequilibrium, any failure to create viability mechanisms would result in the truncation of the *efficient* productive processes as well. Moreover, the competitive environment forces the firms to choose, construct and utilize new technologies even if it is in the evolutionary phase. During the process, there may be high chances of failures. To make things worse, if there is an increase in lending or market interest rates, it would even make it harder for organisations to cope with the change and to experiment with new productive technologies to move the technology

frontier outwards. This can only be realised if the market interest rate is lowered and other additional financial resources are fed into the economy. Therefore, in such situation, whenever necessary, the institutions should intervene and bring down the market rate of interest to its *natural rate* so as to make the capital value positive and the system viable. To illustrate the importance of viability conditions, and the sensi-



a) Dynamics without viability conditions

b) Dynamics with viability conditions

Figure 2.4: The dynamics of the capital value of the production processes, quantity of output produced, revenue generated and credit required.

tivity to initial conditions, we have modelled and simulated AG model to study the dynamics of the system over time. In the absence of viability conditions, the capital value of the production process becomes negative at time 147 (see 2.4 (a)) and so the production process dies off. But when these conditions are introduced, due to introduction of credit and for different reaction co-efficient market-determined wage rates and price rates, and human resource learning rate, the capital value of the production process is positive till time 205 (see figure 2.4 (b)). Moreover, the model is sensitive to initial conditions so the policy maker should develop policies in such a way that the system embarks the desired growth path and traverse viably.

2.3 Near-Decomposability and Approximate Viability

In an ideal situation, all the sectors operating in an economy would satisfy the viability conditions, thus ensuring its sustainability, but what if an economy could not satisfy the viability conditions? How could a policy maker manage such economic situations? The key lies in the comment made by Sraffa during a discussion of Hicks's paper, at Corfu conference on "The Theory of Capital", 1958, where he explains that

one should emphasize the distinction between two types of measurement. First, there was the one in which the statisticians were mainly interested. Second there was the measurement in theory. The statisticians' measures were only approximate... The theoretical measures required absolute precision. Any imperfections in these theoretical measures were not merely upsetting, but knocked down the whole theoretical basis....

(Sraffa, 1961, In Lutz and Hague, pp.305-6)

The distinction between theoretical and actual measurement, i.e., precise measurement and approximate measurement, provides us with an idea that if an economy is in a non-viable state, as a practitioner, a policy maker can approximate

the non-viable system to check if the system can be brought to a nearly viable state or not. How can we bring a non-self-replacing system to a nearly self-replacing state? To illustrate a case, let us take an economy (2.30) that consists of 3 sectors producing commodities such as wheat, iron and pigs – wheat and iron are denoted in terms of tons and pigs in gross. The commodity thus produced by the sectors enters as input for its own production and also for the production in other sectors.

	Input				Output
	Wheat	Iron	Pigs		
Sector 1:	2000 tons Wheat	+ 400 tons Iron	+ 1 Pig	⇒	6000 tons Wheat
Sector 2:	1000 tons Wheat	+ 600 tons Iron	+ 0 Pig	⇒	1300 tons Iron
Sector 3:	2000 tons Wheat	+ 300 tons Iron	+ 5000 Pigs	⇒	6000 Pigs

	5000 tons Wheat	1300 tons Iron	5001 Pigs		

The above system is said to be in a self-replacing state if the total production of the commodities is greater than or equal to the total quantity of the commodities used for production by the economy, hence the system is viable. Let us take a similar economy (2.31) in which the total number of pigs produced by sector 3 is 5000 instead of 6000.

	Input				Output
	Wheat	Iron	Pigs		
Sector 1:	2000 tons Wheat	+ 400 tons Iron	+ 1 Pig	⇒	6000 tons Wheat
Sector 2:	1000 tons Wheat	+ 600 tons Iron	+ 0 Pig	⇒	1300 tons Iron
Sector 3:	2000 tons Wheat	+ 300 tons Iron	+ 5000 Pigs	⇒	5000 Pigs

	5000 tons Wheat	1300 tons Iron	5001 Pigs		

The above system of equations (2.31) can be represented in a matrix form as below,

$$A = \begin{bmatrix} (6000 - 2000) & -400 & -1 \\ -1000 & (1300 - 600) & 0 \\ -2000 & -300 & (5000 - 5000) \end{bmatrix} \quad (2.32)$$

$$A = \begin{bmatrix} +4000 & -400 & -1 \\ -1000 & +700 & 0 \\ -2000 & -300 & 0 \end{bmatrix} \quad (2.33)$$

The system of equations represented by matrix A is *non-viable* because not all of its principle minors are positive (H-S condition), this is because the total number of pigs used as input by the economy is more than the total pigs number of produced, so the system is in a non-self-replacing state. Moreover, the multiplier for every sector was calculated and it was found that some of the multipliers were negative; therefore the system, as a whole, is non-viable. As an alternative, one can use the concept of *decomposability* in order to find a permutation matrix through which the matrix A can be checked as to whether the economic system can be decomposable or not. If it is decomposable, the economic system can be decomposed into smaller subsystems. This would imply that even if the economy as a whole cannot sustain over time, at least those subsystems that can self-replace itself could sustain over time while the others die off.

In our example, wheat and iron enter as inputs for production of pigs, while the pigs enter the system as an input for its own production and only a very small fraction (ε) of the pigs enters as input to sector 1. A square matrix (H) is said to be decomposable if there exists a permutation matrix (P) such that PHP^T takes the form (2.34); otherwise it is indecomposable.

$$PHP^T = \begin{bmatrix} B & 0 \\ C & D \end{bmatrix} \tag{2.34}$$

The matrix A (2.33), in the above example, cannot be decomposed as a very small fraction (ϵ) of the output of sector 3 enters as input for the production of iron, so by using the concept of near-decomposability, we can rearrange matrix A to A* in such a way that A* is nearly decomposable.

$$PAP^T = A^* = \begin{bmatrix} + 4000 & - 400 & - \epsilon \\ - 1000 & + 700 & 0 \\ - 2000 & - 300 & 0 \end{bmatrix} \tag{2.35}$$

The nearly decomposable matrix A* can be represented as

	Input		Output
	Wheat	Iron	Pigs
Sector 1 : 2000 tons Wheat	+ 400 tons Iron	+ ϵ Pigs	\Rightarrow 6000 tons Wheat
Sector 2 : 1000 tons Wheat	+ 600 tons Iron	+ 0 Pigs	\Rightarrow 1300 tons Iron
Sector 3 : 2000 tons Wheat	+ 300 tons Iron	+ 5000 Pigs	\Rightarrow 5000 Pigs

	5000 tons Wheat	1300 tons Iron	(5000 + ϵ) Pigs

In this case, as ϵ is a very small value we can neglect it so that the value (5000 + ϵ) in (2.36) can be considered as 5000. By doing so, the system is *nearly* viable in a short run but cannot sustain, i.e., self-replace itself, in a long run, unless the viability creating mechanisms are introduced into the system to make it viable in a long run.

Moreover, when the economy is composed of a very large number of sectors, it would be easier to decompose the economy, if possible, into various smaller groups of sectors so that a policy maker can focus on the core sectors that are non-viable and trying to reduce its influences on the other sectors and vice versa. It should be noted that by decomposing or nearly-decomposing a mathematical repre-

sentation of an economic system, i.e., in terms of a matrix, we lose a lot of information about the system, while the classification of commodities into basics and non-basics uses more of the information available (Bharadwaj, 1970). How much of the ‘additional information’ that is lost and its consequences, while one attempts to nearly-decompose the economic system in making the system approximately viable, is an open question that needs to be further analysed in detail.

2.4 Future Traverses

The analysis of traverse has deep roots in the history of economic thought, but was only *formalized* in the 1960s and 1970s by Hicks. The earliest versions of the model had very strong assumptions - such as the system is in a state of equilibrium, and when the qualitative shocks are introduced the system, it is explicitly assumed that the system reaches, or converges, to a new steady state equilibrium. Following Hicks, AG’s Neo-Austrian model gave a rich and a systemic outlook regarding the process of interaction within and between production, labour, financial and social variables. Even though their model provided new insights on the dynamics of co-ordination it should be noted that the model limits its focus to the analysis of the conditions under which these interactions *work or fail*.

Traverse is by definition along the disequilibrium path, so it is useful to know the path and the causes of these imbalances to understand the real structures of the evolving economic system. The strategy is to control the immediate imbalances and to create and monitor the viability inducing mechanisms, by working along the dynamic disequilibrium, in order to realize economic growth. Goodwin (1951: 3) ar-

gues, "the economy may be regarded as [a system] slowly computing the answer to an ever-changing problem". In that case, the system has to evolve by recognizing the changing problem and computing the answers through viability prerequisites. In the same way, for studying the dynamics of traverse, the disequilibriumed economic system can be conceptualized and constructed as *a computing device* which is trying to compute and find viable traverses in an attempt to reach an equilibrium, *if one exists*.

"[B]uilding reality-oriented models that reveal the *means suitable for the attainment of stipulated goals* of secular evolution" (Lowe, 1976: 8) becomes crucial for understanding the dynamic behaviour of the traverse and for viably managing it. However, the model should not only forecast the future trends and traverses but also should give guidelines on *the viability creating corridors* through which the economic system can traverse to reach the new equilibria. In this chapter, I have investigated the viability conditions in the Neo-Austrian approach and illustrated its importance while traversing the *dynamic* disequilibrium.

References

- AMENDOLA, M. AND GAFFARD, J. (1998): *Out of Equilibrium*, Oxford University Press, Oxford.
- (2006): *The Market Way to Riches: Behind the Myth*, Edward Elgar, Cheltenham, UK ; Northampton, MA.
- BAUMOL, W. (1951): *Economic Dynamics: An Introduction* / with a contribution by Turvey, R., Macmillan, New York.
- BHARADWAJ, K. (1970): "On the Maximum Number of Switches Between Two Production Systems", *Schweizerische Zeitschrift für Volkswirtschaft und Statistik*, Nr.4, Vol. 106: 409-429
- CHIODI, G. (1992): "On Sraffa's Notion of Viability, *Studi Economici*, Fascicolo 46.
- (1998): "On Non-Self-Replacing States, *Metroeconomica*, 49, (1): 97107.
- (2010): "The Means of Subsistence and the Notion of 'Viability' in Sraffa's Surplus Approach", in *Computable, Constructive and Behavioural Economic Dynamics. Essays in Honour of Kumaraswamy (Vela) Velupillai*, - Zambelli, S., (eds.), Routledge, London.
- DAY, R. (1992): "Chaos and Evolution in Economic Processes", in *Nonlinearities, Disequilibria and Simulation* - Velupillai, K., (eds.), Macmillan, Basingstoke.
- (1993): "Nonlinear Dynamics and Evolutionary Economics" in *Nonlinear Dynamics and Evolutionary Economics*. DAY, R. AND CHEN, P., Oxford University Press, Oxford.
- FISHER, I. (1907): *The Rate of Interest*, Macmillan, New York.
- GALE, D. (1960): *The Theory of Linear Economic Models*, McGraw-Hill, New York.
- GOODWIN, R. (1951): "Iteration, Automatic Computers and Economic Dynamics", *Metroeconomica*, Vol. 3, (1): 17.
- HARROD, R. F. (1939): "An Essay in Dynamic Theory", *The Economic Journal*, Vol. 49, (193): 14-33
- (1948): "Towards a Dynamic Economics: Some Recent Developments of Economic Theory and Their Application to Policy". Macmillan, London.
- HAWKINS, D. AND SIMON, H. (1949): "Note: Some Conditions of Macroeconomic

- stability", *Econometrica*, 3/4, (17): 245-248.
- HICKS, J.R. (1970): "A Neo-Austrian Growth Theory", *Economic Journal*, Royal Economic Society, Vol. 80, (318): 257-281.
- HICKS, J. (1973): *Capital and Time: A Neo-Austrian Theory*, Clarendon Press, Oxford.
- (1985): *Methods of Dynamic Economics*, Oxford University Press, Oxford.
- KALDOR, N. (1972): "The Irrelevance of Equilibrium Economics", *The Economic Journal*, Vol. 82, (328): 1237-1255.
- (1975): "What is Wrong with Economic Theory", *The Quarterly Journal of Economics*, Vol. 89, (3): 347-357.
- LOWE, A. (1976): *The Path of Economic Growth*. Cambridge University Press, New York.
- NIKAIDO, H. (1970): *Introduction to Sets and Mappings in Modern Economics*, North-Holland/American Elsevier, New York.
- SRAFFA, P. (1960): *Production of Commodities by Means of Commodities*, Cambridge University Press, Cambridge.
- (1961), Comments in the Discussion of Professor Hicks' Paper, pp.305-6, in: *The Theory of Capital - Proceedings of a Conference held by the International Economic Association*, edited by F.A. Lutz & D.C. Hague, Macmillan & Co Ltd., London.
- STIGLER, G. (1951): "The Division of Labour is Limited by the Extent of the Market". *The Journal of Political Economy*, Vol. 84, (3):185-193.
- VELUPILLAI, K. (1995): "Irving Fisher on a "Fundamental Theorem" in Neo-Austrian Capital Theory". *Journal of institutional and theoretical economics: JI-TE = Zeitschrift fr die gesamte Staatswissenschaft*, Vol. 151, (3): 556-564,
- (2000): "The Wisdom and Wit of Sen's Contribution to Capital Theory: Some Sketches", *Review of Development and Change*, Vol. 5, (1): 1-23.
- WINTER, S. (2005): *"Towards an Evolutionary Theory of Production"*. K Dopfer, (eds.) *The Evolutionary Foundations of Economics*. Cambridge University Press, Cambridge.

Appendix 2A: AG Neo-Austrian Model

The Neo-Austrian model developed for this study is a hybrid version of the Amendola and Gaffard's model¹⁰. Let I be the number of firms operating in an economy producing m different products. The total output (B) produced (from time t) is the total input (A) used for production processes times the productive capacity (x) from time (1 to $n^c + n^u$). Where, n^c = Construction period and n^u = Utilization period.

$$\sum_{T=t-(n^c+1)}^{n^c+n^u} B(T) = \sum_{T=t-(n^c+n^u)}^{n^c+n^u} A(T).x(T)$$

An elementary process of production is defined by the input vector:

$$[A] = [a_j^i(t)]; \forall t = 1, 2, \dots, n^c + n^u; \forall i = 1, 2, \dots, I; \forall j = 1, 2, \dots, m;$$

$$a_j^i(t) = [a_j^{ic}(t), a_j^{iu}(t)];$$

$$a_j^i(t) = a_j^{ic}(t); \forall t = 1, 2, \dots, n^c; = \text{Input used for construction (c)}$$

$$a_j^i(t) = a_j^{iu}(t); \forall t = n^c + 1, \dots, n^c + n^u; = \text{Input used for utilization (u)}$$

where, a = Inputs (*in real terms*); i = Index of the firms; j = Index of the products;
 t = Time; n^c = Construction period; n^u = Utilization period.

And the output vector (B) is given as (*in real terms*),

$$[B] = [b_j^i(t)]; \forall t = 1, 2, \dots, n^c + n^u; \forall i = 1, 2, \dots, I; \forall j = 1, 2, \dots, m;$$

$$b_j^i(t) = 0; \forall t = 1, 2, \dots, n^c; b_j^i(t) = b_j^{iu}(t); \forall t = n^c + 1, \dots, n^c + n^u;$$

where, $b_j^i(t)$ = Output of the commodity j of the firm (i) at each given time (t).

The total productive capacity (x) is given as:

$$[x] = [x_j^{ic}(t), x_j^{iu}(t)]; \forall t = 0, 1, 2, \dots, n^c + n^u;$$

where,

$[x_j^{ic}] = x_j^{ic}(t); \forall t = 1, 2, \dots, n^c$ is the productive capacity during the construction phase
 (c);

¹⁰ For further details see Amendola and Gaffard 1998, 2006.

$[x_j^{iu}] = x_j^{iu}(t); \forall t = n^c + 1, \dots, n^c + n^u$ is the productive capacity for the utilization (u).

Wage Fund

In each given period the level of the activity (both investment and current production) of each firm (or of the representative firm) depends on its wage fund $W^i(t)$, which is constrained by available financial resources $F^i(t)$ or, alternatively, by available human resources $L^s(t)$:

$$W^i(t) = \min(F^i(t), w^i(t)L^s(t))$$

The minimum desired Wage Fund W^i for each firm (i) can be determined by the product of the wage rate (w) and the total input ($a.x$).

$$W^i(t) = \sum_{j=1}^m w_j^i(t) \cdot (a_j^i(t) \cdot x_j^i(t))$$

Human Resources

The labour required (L^d) at any given time is given by:

$$L^d(t) = \sum_{j=1}^m \sum_{h=1}^l (a_{h,j}^c(t) \cdot x_j^c(t)) + (a_{h,j}^u(t) \cdot x_j^u(t)); \forall t = 1, 2, \dots, n^c + n^u; \forall h = 1, 2, \dots, l;$$

Where, h is the skill level of the labour L required to carry out production.

The labour supply (L^s):

$$L^s(t) = L^s(t-1) \cdot (1 + \iota) \cdot (1 + \lambda)$$

where,

ι = Rate of growth of population;

λ = Rate of change of wage rate for skill (h).

Learning process:

$$\tilde{L}_h^s(t) = L_h^d(t)(1 + \xi) \text{ if } L_h^s \geq L_h^d$$

$$\tilde{L}_h^s(t) = L_h^d(t)(1 - \xi) \text{ if } L_h^s < L_h^d$$

Where,

ξ is the learning coefficient.

Financial Resources

The available financial resources F of a firm i at any given time (t) is calculated by,

$$F^i(t) = m^i(t-1) + h^i(t-1) + f_s^i(t) - c^i(t) - h_d^i(t)$$

where,

$$m(t) = \sum_j \min[p_j(t)s_j(t); p_j(t)d_j(t)] = \text{Money proceeds from sales;}$$

Where, p = price; d = demand; s = supply

$h^i(t-1)$ = Idle money balance from the previous time period;

$h_d^i(t) = \rho(t)[m(t-1) + h^i(t-1) + f(t)]$ = Firms desired idle monetary balance at time (t)

ρ = Desired level of idle balanced by the producers ($\rho > 0$)

f = External financial resources;

c = Take out.

Aggregate Demand and Supply

Households (H) are presumed to spend all their revenues (both wages and social revenues - the take-out) unless they are rationed on the final goods market.

The current demand for final output $y(t)$ is given by,

$$y(t) = W(t) + c(t) + h^H(t-1) - h_d^H(t);$$

Where,

W = Wage fund;

c = Take out, that is the resources withheld from financing of production.

$h^H(t-1)$ = The monetary idle balances of households at time $t-1$.

$h_d^H(t) = \sigma(t)[W(t) + c(t) + h^H(t-1)]$ = Desired household idle monetary balance at time(t)

σ = Desired level of idle balanced by the households ($\sigma > 0$)

The demand for commodity (in real terms):

$$d_j(t) = y_j(t) / p_j(t);$$

where, $y_j(t) = \delta_j y_j(t)$; $\sum_j^m \delta_j = 1$;

δ_j is a utility index.

The supply of commodity (in real terms):

$$s_j(t) = y_j^*(t) / p_j(t);$$

where,

$y_j^*(t) = y_j(t-1)[1 + g_m(t-1)]$; is the money value of final demand expected in the current period.

g_m is the mean growth rate.

Price:

$$p(t) = p(t-1).(1 + g_{p_j}(t))$$

Market-determined price change:

$$g_{p_j}(t) = k_j(\phi_j(t-1))$$

Where,

$$\phi_j(t) = (d_j(t) - s_j(t)) / s_j(t);$$

k =reaction coefficient for different commodity;

$s_j^i(t)$ = Supply of firm (i);

$p_j^i(t)$ = Price of products fixed by firm (i).

Wage:

$$w(t) = w(t-1).(1 + g_{w_h}(t))$$

Market-determined wage rate changes:

$$g_{w_h}(t) = v_h(\psi_h(t-1))$$

Where,

$$\psi_h(t) = (L_h^d(t) - L_h^s(t)) / L_h^s(t)$$

v = Reaction coefficient for different skill level;

L_h^s = Labour supply for skill level (h);

L_h^d = Labour demand for skill level (h)

Appendix 2B: Simulation parameters

The innovation happens at time $t=0$;

$$n^c = 10;$$

$$n^u = 10;$$

$$m = 1;$$

$$a^{ic} = (10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10); \forall i$$

$$a^{iu} = (8 \ 8 \ 8 \ 8 \ 8 \ 8 \ 8 \ 8 \ 8 \ 8 \ 8); \forall i$$

$$b^i = 100; \forall i$$

Without viability conditions	With viability conditions
$\iota = 0.01;$ $\rho = 0.15;$ $\sigma = 0.1;$ $g_m = 0.02;$ $\lambda = 0.3;$ $g_{fs} = 0.01;$ $v = .25;$ $k = .25;$ $\eta = 1;$ $l = 1;$ $\xi = 0$	$\iota = 0.01;$ $\rho = 0.2;$ $\sigma = 0.1;$ $g_m = 0.02;$ $\lambda = 0.5;$ $g_{fs} = 0.05;$ $v = .25;$ $k = .25;$ $\eta = 1;$ $l = 1;$ $\xi = .5$

Chapter 3

The Time-to- Build Tradition* in Business Cycle Modelling*

Written jointly with K. Vela Velupillai

-
- Prepared in homage to celebrate the eightieth anniversary of the publication of Tinbergen's path-breaking *Ein Schiffbauzyklus* and the sixtieth anniversary of the publication of Goodwin's *Non-linear Accelerator and the Persistence of Business Cycles*.
 - The second author had the pleasure and privilege of direct and indirect instruction and years of inspiration on the matters dealt with in this chapter by some of the pioneers of the relevant theory. In particular, of course, Richard Goodwin and Björn Thalberg, but also Trygve Haavelmo, Nicholas Kaldor and, especially, Jan Tinbergen (alas only very late in his noble life; see footnote 1, in the main text). Stefano Zambelli's influence, via innumerable discussions with the second author for over a quarter of a century, and through his important written works on Frisch and Kalecki, is pervasive. He is, however, not responsible for any remaining infelicities in this chapter.

Abstract

An important frontier of business cycle theorising is the 'time-to-build' tradition that lies at the heart of Real Business Cycle theory. Kydland and Prescott (1982) did not acknowledge the rich tradition of 'time-to-build' business cycle theorising - except in a passing, non-scholarly, non-specific, reference to Böhm-Bawerk's classic on Capital Theory (Böhm-Bawerk [1899]), which did not, in any case, address cycle theoretic issues. The notion of 'time-to-build' is intrinsic to any process oriented production theory which is incorporated in a macrodynamic model. We provide an overview of this tradition, focusing on some of the central business cycle classics, and suggest that the Neo-Austrian revival should be placed in this class of dynamic macroeconomics, albeit 'traverse dynamics' is itself to be considered as a fluctuating path, from one equilibrium to another.

Keywords: Time-to-build, Business Cycle Theory, Nonlinear Difference-Differential Equation Models, Traverse.

3.1 The Time-to-Build Tradition in Business Cycle Theory

"That wine is not made in a day as long been recognized by economists (e.g., Böhm-Bawerk [1891]). But, neither are ships nor factories built in a day. A thesis of this essay is that the assumption of multiple-period construction is crucial for explaining aggregate fluctuations.

Our approach integrates growth and business cycle theory.

One very important modification to the standard growth model is that multiple periods are required to build new capital goods and only finished capital goods are part of the productive capital stock. Each stage of production requires a period and utilizes resources. Half-finished ships and factories are not part of the productive capital stock. "

Kydland & Prescott (1982), p. 1345.

Apart from the gratuitous reference to Böhm-Bawerk's classic on capital theory, the oldest referenced paper in the Kydland & Prescott (henceforth, K & P) 'classic' is to the descriptive - questionnaire-based - article by Thomas Mayer (1960), which, in turn, refers only to work by that author, and none of them of any vintage earlier than 1953.

Thus, the whole noble business cycle theoretic tradition incorporating variations on the theme of 'time-to-build' - meaning by this not just the time length required to complete the building of plant to produce capital goods that can, in turn, be used in the production process, but also the lead and lag times involved between decisions to build, orders to be placed, delivery to be undertaken, and so on - all the way from Tinbergen's classic *Ein Schiffbauzyklus* (Tinbergen [1931])¹ to the Keynesian tradition of nonlinear

¹ Tinbergen's initiation into economics was partly due to the encouragement of his Physics mentor, the great Paul Ehrenfest, who advised his young assistant to contact Wicksell, in 1925, which he duly did on 23 June, 1925. Velupillai wrote Tinbergen on 3rd February, 1984, asking whether Tinbergen remembered this letter and, if so, for a copy of Wicksell's response, if there was one. Tinbergen's response to Velupillai, dated 10/2/1984, together with his letter to Wicksell, are both available on request. Wicksell died on 3rd May, 1926. It is interesting to note, as pointed out by

business cycle theories (Goodwin [1951], Strotz, et.al [1953]) is ignored. In between, there were the classics by Frisch (1933), Frisch & Holme (1935) and the series of pioneering contributions by Kalecki (1935, 1936, 1939), which also are ignored. These two traditions share a mathematical formalism in that the dynamic equation that these works reduce their rich macroeconomics to can be encapsulated in a canonical nonlinear difference-differential equation².

If we take the reference to Böhm-Bawerk in K & P, in the context of business cycle theory, seriously, then a reasonable expectation³ would have been some mention of the rich, albeit controversial, tradition of Austrian Business Cycle theory linking 'Böhmian' capital theory, in the form of the *period of production*, with *industrial fluctuations*. The classic of this genre is, of course, Hayek's controversial little masterpiece, *Prices and Production* - which was, subject to searching criticisms, from many points of view, by Sraffa (1932), Hansen & Tout (1933), Hill (1933) and, above all, given the *provenance* of

Hollestelle (2006; p. 790):

"Ehrenfest's hand can also be seen in Tinbergen's famous analyses of the shipbuilding cycle, in which Tinbergen used the invention of his mentor, the adiabatic principle, to describe the periodic behaviour of the cycle."

The truth of this interesting observation can be verified in footnote 1, p.156, of the Tinbergen classic (italics added):

"D.h. der Wert nimmt im Laufe der betrachteten Periode langsam ab. Streng genommen ist also eigentlich auch eine Funktion von t. Wegen seiner langsam Veränderlichkeit ist es aber weitaus die einfachste Behandlungsweise, a voräufig als konstant zu betrachten und in der dann gefundenen Lösung als veränderlich einzusetzen. *Diese Methode ist der Physik bekannt als die Methode der adiabatischen Variabeln.* (Vgl. P.Ehernfest, Ann.d.Phys. Bd.51 [1926].S.327)."

² With characteristic perspicacity, referring to Tinbergen (op.cit), Schumpeter 'hit the nail on the head' (Schumpeter [1939], p.533; italics added):

"This cycle [The cycle in Shipbuilding], made famous by Professor Tinbergen, serves to illustrate a lag phenomenon *incident to all time-consuming construction of plant and equipment* and therefore differs (also in other respects) materially from the hog case."

Schumpeter's 'time-consuming construction' is what K & P have 'dubbed' 'time-to-build'. A little bit of scholarship could prevent a great deal of square-wheel reinventions.

³ Even a 'rational' one!

K & P, Frank Knight's series of critical essays on the *Austrian Theory of Capital* (Knight [1933], [1934] [1938])⁴. Indeed, this particular Hayekian theory comes closest to being an *Equilibrium Real Business Cycle Theory*, presaging and being a predecessor of modern RBC theory, but with at least two caveats: the first, is that the latter is not underpinned by a serious capital theory the way Hayek's attempted to be⁵; second, the former did not have - nor seek - the kind of theoretical technology that came to clothe modern RBC theory.

Then, there is the whole tradition of *replacement cycles*, initiated in Marx (1893)⁶ and elegantly summarised in the language of linear algebra by Bródy (1970). It is this tradition that links up most coherently with the *traverse dynamics* and the general viability⁷ of such paths of Amendola and Gaffard (1998). As a matter of fact, it is this *Marxian tradition* - we may refer to it this way, rather than as 'replacement cycles', which suggests only the purely technical aspects of durable goods replacements - that should be contrasted with the *Neo-Austrian tradition* and, indeed, should be considered the foundation for the Amendola & Gaffard (op.cit) exercises⁸.

⁴ Kaldor, whose intellectual adherence to Hayek's theory of capital and industrial fluctuations was most eloquently defended, especially against Frank Knight's penetrating criticisms, eventually turned against the Austrian visions, first in his brilliant criticism of Hayek's 'Concertina Effect' (Kaldor [1942]) and finally acknowledged his agreement with Knight at the famous 'Corfu Conference on Capital Theory' (Kaldor [1961], p.294):

"Professor Haberler would know that he [Kaldor] had himself at one time defended Wicksell from the attacks of Professor Knight. He was now convinced that all he had written in defence of neo-classical theory was wrong and that Professor Knight was right."

⁵ 'Attempted to be' is a serious qualification, given, in particular, Sraffa's devastating 'Wicksellian critique' (op.cit) of Hayek's claim that he was building on the foundations of Wicksell's reformulation of Böhm-Bawerk's capital theory.

⁶ A concise, but characteristically erudite, summary of this tradition, linking it also the line of research begun by Tinbergen, is given in Schumpeter (op.cit, chapter IV, § E).

⁷ By general viability is meant the *real, financial and human resource* (i.e., *labour*) feasibility of any traverse dynamics of a multisectoral dynamic economic system. The coherence between the two approaches can be gleaned from chapter 1.3 of Bródy (op.cit).

⁸ After all, Böhm-Bawerk's *magnum opus* was itself devised and presented as an alternative to the Marxian system. The viability of any traverse dynamics in Marx's extended reproduction schemes took into account the durable good, labour and financial feasibility of any such path, whether

Finally, there is the masterly work by Haavelmo, *A Study in the Theory of Investment* (Haavelmo [1960]), which may be referred to as a *non-stochastic* (ibid, chapter I, §3) *macrodynamic* foundation of investment theory, in complete contrast to the K & P approach. Haavelmo's '*Study*' is, in fact, a synthesis of all of the above mentioned approaches to encapsulate notions of 'time-to-build' in its manifestations of aggregate fluctuations. A particular application of Haavelmo's framework, in terms of the interaction of the 'time-to-build' and delivery time, placed in the context of a nonlinear Keynesian business cycle model of the Goodwin-type (op.cit), can be found in a remarkable - and much neglected - series of contributions by Björn Thalberg (1961, 1966).

The chapter, which should be considered a small contribution to *the doctrine-history of an aspect of business cycle theory*, is organised as follows. The next section contains a synopsis of the four pioneering business cycle theories, where some notion of 'time-to-build' played a crucial role in determining the final mathematical form of the equation that underpinned fluctuations of one sort or another. Section 3 is an attempt to summarise the mathematical and economic lessons to be learned from 'time-to-build' modelling as an ingredient of business cycles. The concluding section is a brief methodological reflection on the lessons to be learned from 'time-to-build' modelling in what we call *phenomenological macroeconomics*. The addendum consists of simulation results of a canonical 'time-to-build', nonlinear difference-differential equation model with increasing 'precision' incorporated, in the form of retaining higher order terms of the Taylor series. The classical nonlinear, endogenous, business cycle result of the genesis of a stable limit cycle, independent of initial conditions, is lost when the precision of the approximating equation is improved.

'steady' or not, whether initialised on a steady state path or not and, in any case, the notion of equilibrium was far richer than supply/demand consistency.

3.2 The Canonical Difference-Differential Equations in Business Cycle Theory

"The roots of the algebraic equation $\sum \alpha_v x^v = 0$ play a well-known role in the solution of the differential equation $\sum \alpha_v y^{(v)}(x) = 0$

Over a number of years a variety of economic and engineering problems has led to a study of difference-differential equations of which

$$\sum_{\mu=0}^m \sum_{\nu=0}^n a_{\mu\nu} y^\nu(x + \mu) = 0 \quad (3.1)$$

is a basic example. Here the algebraic equation is replaced by the transcendental equation

$$\sum_{\mu=0}^m \sum_{\nu=0}^n a_{\mu\nu} z^\nu e^{\mu z} = 0 \quad (3.2)$$

which has an infinity of roots. Sums of terms of the type $A_z e^{2z}$ over some or all of the roots of (3.2) (with grouping of terms if necessary to secure convergence) provide solutions of (3.1)."

Wright (1961), p.136

The four canonical difference-differential equation models in macrodynamics are those that first appeared in Tinbergen (op.cit), Frisch (1933), Kalecki (1935) and Goodwin (1951). They are, discussed below.

i. *Tinbergen (1931)*⁹

$$f'(t) = -af(t - \vartheta) \quad (a > 0) \quad (3.3)$$

Where:

⁹ Tinbergen, too, intellectually honest though he was, succumbed to the pointless temptation to add the well known caveat of all mathematical economics exercises (ibid, p.155):

"Schlie[ss]lich mu[ss] die Lösung noch der Bedingung genügen da[ss] sie überhaupt einen ökonomischen Sinn hat: sie soli also z.B. reell und endlich sein."

$f(t)$: total freight tonnage at time t (t : continuous, $t \in \mathfrak{R}$)

$f'(t)$: rate of change of freight tonnage (= ship building);

ϑ : a parameter, indicating the time period between decision to order extra tonnage and delivery of new ships ($\vartheta \in \mathfrak{R}$)

a : reaction coefficient, $a \in \mathfrak{R}$

Tinbergen's remarkable originality here was the behavioural assumption underpinning the accelerator dynamics encapsulated in (3.3): it was what later (in Goodwin [1951]) came to be called the 'flexible accelerator' (or the 'non-linear accelerator') with the equivalent of the difference between a 'normal' and 'actual' level of freight tonnage driving a positive feedback in the rate of ship building.

ii. *Frisch (1933)*

$$\ddot{x}(t) = \left(\frac{s\mu}{\varepsilon} - \lambda r \right) \dot{x}(t) + \left(\frac{s\mu}{\varepsilon} \right) \dot{x}(t - \varepsilon) + \frac{sm}{\varepsilon} [x(t) - x(t - \varepsilon)] \quad (3.4)$$

Where:

x : 'yearly production of consumer's goods';

m : 'the *total* depreciation on the capital stock associated with the production of a unit of consumer's goods', $m \in \mathfrak{R}^+$;

μ : 'the size of the capital stock that is needed directly and indirectly in order to produce one unit of consumption per year', $\mu \in \mathfrak{R}^+$;

ε : 'technically given constant' - essentially the 'time-to-build' parameter, $\varepsilon \in \mathfrak{R}^+$;

S : the *encaisse désirée* parameter for the production of capital goods, $s \in \mathfrak{R}^+$.

This is, of course a linear difference-differential equation - but the economic and mathematical reasons given for its genesis, in Frisch (1933), are untenable. Frisch begins by assuming the equivalent of a non-linear 'flexible accelerator' relationship between the production of consumption goods and the *encaisse désirée*,¹⁰ but assumes, 'as a first approximation the relationship to be linear', and works with:

$$\dot{x} = c - \lambda\omega \quad (3.5)$$

Where:

$$c, \lambda \in \mathfrak{R}^+$$

Had Frisch removed the 'first approximation' of a 'linear relationship', the resulting dynamics in the production of consumption goods can be shown to have the form (Velupillai [1992], p.64, equation (10)):

$$[1 - s\mu g(x, \dot{x})]\ddot{x} - (r + sm)g(x, \dot{x})\dot{x} = 0 \quad (3.6)$$

Where:

$$f'(\cdot) = g(x, \dot{x})$$

It can be seen, by a simple inspection of (3.6), that even a 'linear' approximation of

¹⁰ It may well be worth quoting Frisch in detail on this point simply because it has, to the best of our knowledge, never been made explicit (ibid, pp. 179-180; italics in the original):

"In the boom period when consumption has reached a high level, ... , consumption is one of the elastic factors in the situation. It is likely that this factor is one that will yield first to the cash pressure. To begin with this will only be expressed by the fact that the rate of increase of consumption is slackened. Later, consumption may perhaps actually decline. Whatever this final development it seems plausible to assume that the *encaisse désirée* ω will enter into the picture as an important factor which, when increasing, will, after a certain point, tend to diminish the rate of increase of consumption."

$g(x, \dot{x})$ results in a second-order, non-linear, differential equation. This equation is capable of generating the kind of oscillation Frisch thought should be the object of study, for the interaction between theory and observation. But ignoring the natural strategy of removing the 'first approximation', Frisch claims that the dynamic equation for the production of consumption goods that was generated by sticking to the 'first approximation' of linearity - the linear, nonflexible, accelerator - 'is too simple to give rise to oscillations' (Frisch, op.cit, p. 180), and he goes on:

"The system considered above [i.e., with the 'first approximation' of linearity] is thus too simple to be able to explain developments which we know from observation of the economic world. There are several directions in which one could try to generalize the set-up so as to introduce a possibility of producing oscillations."

After mentioning the possibility of taking the routes suggested by Keynes, Fisher and Marx (essentially, Kalecki), he opts for what he calls 'Aftalion's point of view with regard to production' (ibid, p. 181):

"The essence of this consists in making a distinction between the quantity of capital goods whose production is started and the activity needed in order to *carry to completion* the production of those capital goods whose production was started at an earlier moment. The essential characteristics of the situation that thus arises are that the activity at a given moment does not depend on the decisions taken at that moment, but on decisions taken at earlier moments. By this we introduce a new element of discrepancy in the economic life that may provoke cyclical oscillations."

Thus enters the untenable reason for the introduction of the 'time-to-build' as-

sumption - realistic though it may be - in a macro dynamic theory, albeit in a non-representative agent, non-optimum, yet entirely deterministic macro dynamic context.

In this sense of sticking to an untenable - both from economic and mathematical points of view - 'approximation', Frisch's later criticism of Kalecki seems highly questionable (Frisch & Holme [1935], p.225):

"The imposition of the condition [by Kalecki (1935)] that the solution shall be undamped is in my¹¹ opinion not well founded. It is more correct, I think, to be prepared to accept any damping which the empirically determined constants will entail, and then explain the maintenance of the swings by erratic shocks. This would be an explanation along the lines indicated in my paper in the Cassel volume."

Moreover, even this methodological point by Frisch (& Holme) - that 'it is more correct'¹² to 'explain the maintenance of the swings by erratic shocks' reiterated as a dogmatic credo for mathematical modelling of business cycles at the frontiers of macrodynamics, based on the so-called substantiation in Frisch (1933), has been shown to be vacuous by Zambelli's fundamental result that the famous 'Rocking Horse' does not rock (Zambelli [2007])¹³.

iii. *Kalecki (1935)*

$$\dot{I}(t) = \frac{m}{\theta}[I(t) - I(t - \theta)] - n[I(t - \theta) - U] \quad (3.7)$$

¹¹ It may well be a simple 'slip of the tongue' that 'my' is used - but not just once and not only 'my', but also 'I' - in a *joint* paper! The 'my' obviously refers to Frisch (and not Holme).

¹² What is the epistemological status of an assertion like 'more correct' in this context? It is a pity that Frisch's distinguished colleague, Trygve Haavelmo, debunked the methodological adherence of substantiating a theory on the basis of consistency with observations (Haavelmo [1940]), only half a decade *later*.

¹³ Indeed, the infelicities in Frisch's highly celebrated **Cassel Festschrift** paper extends even to an important mis-attribution even of Wicksell's original reference to the 'Rocking Horse' (cf. Velupilai, op.cit., footnote 4, p. 70).

Which, by writing:

$$J(t) \equiv I(t) - U \quad (3.8)$$

Can be represented more simply as:

$$\dot{J}(t) = \frac{m}{\theta} [J(t) - J(t - \theta)] - nJ(t - \theta) \quad (3.9)$$

Where:

$I(t)$: Investment orders at time t , ($\in \Re$);

U : (constant) depreciation factor ($\in \Re^+$);

θ : the average gestation lag, for the economy as a whole, between decisions to invest (order) and delivers of final (capital) goods;

$m, n (\in \Re)$: linearization parameters of the non-linear Investment function (see, equation (100, p. 331, *ibid*):

$$\frac{I(t)}{K(t)} = \phi \left(\frac{C_1 + A}{K} \right) \quad (3.10)$$

Where:

$K(t)$: capital stock at time t ;

A : gross accumulation equal to the production of capital goods;

C_1 : constant part of the consumption of capitalists;

Several comments should be added to the general tendency to refer to (3.7) (or, more frequently to (3.9)) as 'Kalecki's model of the cycle' which is, 'from a mathematical point of view ... a differential equation with a delay parameter' (Szydlowski [2002], p.698). The main economic point is that there is no mathematical reason, underpinned by any compelling economic reason, for 'Kalecki's model of the economic cycle' to be anything other than a straightforward high-order difference equation. Secondly, there is no justification for the linearization mentioned above. Thirdly, the non linearized Kalecki model of the business cycle would be given by (see Velupillai [1997], equations (21) & (22), p. 261:

$$\frac{K(t) - K(t-1)}{K(t-\theta)} = \phi \left\{ \frac{C_1 + U + \frac{1}{\theta} [K(t) - K(t-\theta)]}{K(t-\theta)} \right\} - U \quad (3.11)$$

This is a non-linear difference equation and, paradoxically, even if ϕ is now linearized, the final equation will remain a *non-linear difference equation*! Sixty years ago, in his masterly review of one of the great modern classics of endogenous macroeconomic cycle theory, Hicks (1950), Richard Goodwin reflected on such equations with characteristic prescience (Goodwin [1950], p.319, footnote 6):

"Combining the difficulties of difference equations with those of non-linear theory, we get an animal of a ferocious character and it is wise not to place too much confidence in our conclusions as to behavior."

To substantiate Goodwin's prescience on being careful 'not to place too much confidence in our conclusions as to behavior', we could add the following conjecture¹⁴ :

¹⁴ This is a comprehensively non-rigorous 'conjecture', as stated. However, it is quite easy to state this more rigorously by showing, first, the equivalence - in some formal sense between this dy-

Conjecture 1: For any ‘economically interesting’¹⁵ nonlinear function ϕ , the attractors of (3.11) are algorithmically undecidable.

The only reason why 'Kalecki's model of the economic cycle' is 'from a mathematical point of view ... a differential equation with a delay parameter', i.e., a difference-differential equation, is that Kalecki chose to sum the total of orders allocated during a period $(t - \theta, t)$ continuously - for which he could not have had any kind of economic data - rather than in *discrete time*. Had he chosen the latter part, the result would have been (3.11), above.

iv. Goodwin (1951)

$$\varepsilon \dot{y}(t + \theta) + (1 - \alpha)y(t + \theta) = O_A(t + \theta) + \varphi[\dot{y}(t)] \quad (3.12)$$

Where:

y : aggregate income;

θ : one half the construction time of new equipment;

$\varphi(\dot{y})$: the flexible accelerator;

O_A : the sum of autonomous outlays ($\beta(t)$ and $l(t)$);

A more direct way to look at this would be to write it out as:

$$\varepsilon \dot{y}(t + \theta) + (1 - \alpha)y(t + \theta) = \varphi[\dot{y}(t)] \quad (3.13)$$

namical system and a suitably initialised Turing Machine and, then, invoking the theorem of the *Halting Problem for Turing Machines*. It must be emphasised that we are not under the sanguine impression that any discrete dynamical system implies, and is implied by, an algorithm in its formal, recursion theoretic or constructive sense. However, respecting the natural data types in economics would entail that the discrete dynamical system (11) can be considered a natural algorithm, but, of course, subject to the classic theorem of the *Halting Problem for Turing Machines*.

¹⁵ Defining ‘economically interesting’ may well be subject to a variant of Berry’s Paradox, but we will not delve into these deep waters and rely on a common sense interpretation.

Where, now, $O_A(t + \theta)$ is assumed to be a constant and $y(t)$ is redefined as a deviation from its unstable, repelling, equilibrium value, $\frac{[\beta(t) + l(t)]}{(1 - \alpha)}$ and time units are shifted by

θ . This equation is a *non-linear difference-differential equation*, derived with impeccable macroeconomic logic. Unfortunately, Goodwin decided to approximate this by 'expanding the two leading terms in a Taylor series and dropping all but the first two terms' (ibid, p.12), to derive the famous (unforced) Rayleigh-van der Pol non-linear differential equation:

$$\varepsilon\theta \ddot{y} + [\varepsilon + (1 - \alpha)\theta] \dot{y} - \varphi(\dot{y}) + (1 - \alpha)y = 0 \quad (3.14)$$

Fortunately, however, in an early electro-analogue (as distinct from an analytical or digital) study of (12), Strotz, et.al., (1953), found a multiplicity of limit cycles and a breakdown of the notion of 'independence of initial conditions' of such cycles, and reached the interesting and important conclusion¹⁶ that (pp. 406- 7; italics added):

"The multiplicity of cycles that has been observed [in the analogue simulations] *can be ascribed to the presence of the difference term*. Had Goodwin approximated his nonlinear difference-differential equation by using the first four terms of the Taylor series expansion of ['the two leading terms'], the resulting approximating equation would have been a nonlinear differential equation of the fourth order, which we believe would have had two limit cycles solutions rather than one, both dependent on initial conditions. Improving the approximation by retaining more terms of the Taylor's expansion would increase the order of the differential equation and this would

¹⁶ As a part of illustrating the multiplicity of solutions and the dependence on initial conditions of Goodwin's model, we have replicated Strotz, et.al., (1953) simulations and analysed the model for higher orders using a digital computer. (See appendix 3 A) The results show that as the order of the differential equation increases the system tends to have multiple solutions, all depending upon the initial conditions, thus emphasizing the need for further investigation.

increase the number of solutions provided by the approximation. To the extent that this is generally true of nonlinear mixed systems, economic theory encounters a methodological dilemma. If mixed systems seem to be required, this implies that we must in general expect a multiplicity of solutions. The resulting indeterminacy must then be overcome by specifying the initial conditions of the model."

This conclusion is the most important one in the whole tradition of 'time-to-build' modelling, in the context of business cycle theory. It identifies and demonstrates almost exactly the nature of the role played by 'time-to-build' assumptions, within the context of a macro dynamic theory, in generating endogenous cycles and - instead of independence of initial conditions - shows independence of *ad hoc shockeries*¹⁷, or, exogenous shocks. Unlike in the case of the other three pioneering formal contributions to this tradition, considered above, in this approach very few - if any - arbitrary, ad hoc, approximations, without economic rationale, were made in deriving the final form of the dynamic equation in the considered variable.

3.3 Mathematical and Economic Considerations in Solving Non-Linear Difference-Differential Equations

" Thus from the standpoint of stability of self-excited oscillations, a linear d. d. e [difference-differential equations] is unable to account for the observed facts, just as it was impossible to account for the existence of self-sustained oscillations on the basis of an ordinary linear d.e [differential equation]

Hence, if one tries to fit the oscillations appearing in retarded systems into the framework of the linear theory of d.d.e., one has exactly the same difficulty that was experienced in the theory of ordinary d.e. when one tried to fit self-sustained oscillations into a similar linear process.

Obviously, the only issue from this situation is to investigate the *non-*

¹⁷ In the felicitous phrase coined by Richard Day to describe Lucasian business cycle theory.

linear d.d.e. In fact, all observed oscillations ... start spontaneously from rest as soon as a certain threshold value of a parameter is reached; moreover, they generally exist not only for one isolated value of the parameter (as indicated by the linear theory), *but for a certain interval* of these parameter values; finally, oscillation persists with a definite stationary amplitude for a given value of parameter."

Minorsky (1962 [1974]), pp.52-3; italics in the original

Kalecki's equation, (9), can be rewritten in the form:

$$\dot{J}(t) - \frac{m}{\theta} J(t) + \left(\frac{m}{\theta} + n \right) [J(t - \theta) + J(t + \theta)] = 0 \quad (3.15)$$

In more general notation, this can be written as:

$$\dot{x} - ax + bx_{\theta} = 0 \quad (3.16)$$

Where:

$$x_{\theta} \equiv x(t - \theta) \text{ (or, depending on the context, } x_{\theta} \equiv x(t + \theta) \text{)}$$

Had Kalecki done what Goodwin did, then, first shifting the time coordinate by θ units, (16) can be rewritten as:

$$\dot{x}_{\theta} - ax_{\theta} + bx = 0 \quad (3.17)$$

Then, a Taylor series expansion of the leading term gives:

$$\dot{x}_{\theta} = \dot{x}(t + \theta) = \dot{x}(t) + \frac{\theta}{1!} \ddot{x}(t) + \frac{\theta^2}{2!} \dddot{x}(t) + \dots \quad (3.18)$$

Then, 'approximating' the linear difference-differential Kalecki equation by an 'equiva-

lent' purely differential equation will retain the linear form and the above strictures of Minorsky can be shown very easily to be substantiated. Thus, Frisch was correct in his criticism of Kalecki, only because the latter's aim was to generate an endogenous cycle theory. Had Kalecki not approximated his ϕ (in equation (3.10), above), the Frisch critique would have been as inapplicable as it was to the final form of the Goodwin equation (3.12) which was, partly, devised to counter the Frischian, 'exogenous', *ad hoc shockeries*, methodology in mathematical business cycle theorising. From equation (3.18) the meaning of what Strotz et.al. (op.cit) did can also be gleaned.

Conversely, had Goodwin worked only with the linear accelerator, the general form of his differential-difference equation, in the above notation, would have been

$$\ddot{x} + px + \lambda x_\theta = 0 \quad (3.19)$$

The characteristic equation of which would be:

$$f(z) = z^2 + pz + \lambda e^{-\theta z} \quad (3.20)$$

Substituting $z = \alpha + i\omega$ in (3.20), separating the real and imaginary parts respectively and considering only harmonic values (i.e., $\alpha = 0$), we get:

$$\cos \beta_1 = \frac{\omega_1^2}{\lambda_1} \quad \text{and} \quad \sin \beta_1 = \frac{\rho \omega_1}{\lambda_1} \quad (3.21)$$

$$\cot \beta_1 = \frac{\omega_1}{p} = \left(\frac{1}{p\theta} \right) \beta_1 \quad (3.22)$$

and,

$$\beta^4 + \theta^2 p^2 \beta^2 - \lambda^2 \theta^4 = 0 \quad (3.23)$$

Analysing these equations give the basis for the Minorsky strictures in the opening quote for the following reason. There are two sets of roots: one set, β', β'', \dots , independent of the variable parameter λ , can be called the set of *fixed roots*; the second set is given by the positive root of (3.20):

$$\beta_{11}(\lambda) = +\sqrt[2]{-\frac{p^2\theta^2}{2} + \sqrt[2]{\frac{p^4\theta^4}{4} + \lambda^2\theta^4}} \quad (3.24)$$

Thus, $\beta_{11}(\lambda)$ is a monotonically increasing function of λ , with $\beta_{11}(0) = 0$. Hence, as λ increases continuously, from $\lambda = 0$, $\beta_{11}(\lambda)$ also moves continuously and for some value, say λ_1 could coincide with one of the above mentioned *fixed roots*, say β' ; i.e.,

$$\beta' = \beta_{11}(\lambda) \quad (3.25)$$

and so on for, respectively¹⁸, β''' and λ''' , β'''' and λ'''' , and so on. At these equalities, (3.19) and (3.20) have a common harmonic root:

$$\beta_1 = \theta\omega_1 \quad (3.26)$$

and:

$$f(i\omega_1) = 0 \quad (3.27)$$

As λ continues to increase, to a discrete sequence of values of λ , say, $\lambda_1', \lambda_1'', \dots$ there will correspond also a discrete sequence of harmonic frequencies, say, $\omega_1, \omega_1', \dots$. The key result is that *the only point at which the dynamics can remain in a stationary state is precisely when λ is equal to a harmonic value.*

This is the thrust of Frisch's objection to Kalecki and Goodwin's indictment against linear theory and the meaning of the opening strictures against linear theory by

¹⁸ In view of (3.16) only the first, third, ... , fixed roots are relevant.

Minorsky. This is also the kind of analysis that can make sense of Frisch's epistemological phrase on a 'more correct' theory. Essentially, this parallels the idea that structurally unstable dynamical systems - such as the Lotka-Volterra equations - should not be harnessed for modelling naturally occurring dynamical systems since they are highly unlikely to be meaningfully observable. Finally, this is also the way meaning can be attached to the results of Strotz et.al (op.cit).

Three concluding observations may be made.

What of the general non-linear difference-differential equation theory and why have economists shunned modelling in this framework? Almost seventy years of deep research on the general theory of non-linear difference-differential equation studies, from Wright (1946) and Brownell (1950, by way of the classic textbook of Bellman & Cooke (1963) and the monograph of Mohammed (1978), to Hale (1993) and beyond, has gone unheeded in macrodynamics. Why? We have no coherent answer to this simple - even simplistic - question. The natural mathematical dynamic framework for modelling 'time-to-build' processes in the context of business cycle theory appears to be the general non-linear difference-differential equation¹⁹. This is what we tried to show in the discussion of the 'Kalecki model of the economic cycle', above.

Secondly, where does this leave the kind of linear dynamical systems that underpin the 'time-to-build' tradition emanating from the Kydland & Prescott research program? Are they not subject to the 'Minorsky strictures'? Indeed, they are - and to even more analytical strictures because the K & P tradition also claims computability. But de-

¹⁹ Even more compellingly, given the nature of economic data types - that economic variables and parameters can, at best, only be rationally valued - it must be the obvious way to model any dynamic process in economics. Such a formalization could easily be encapsulated within the general scheme of Diophantine Dynamics, a branch of Computable Economics.

veloping these strictures has to be left for a different exercise.

Thirdly, what of the Neo-Austrian 'traverse dynamics', as an example of 'time-to-build' dynamics as a 'disequilibrium process'? Before responding to this rhetorical query, it may be useful to recall yet another of Tinbergen's important reflections on an issue that is of relevance here. In Tinbergen (1943, p. 45)²⁰:

“[T]he theory of the business cycle contains the certain controversies derive from the part attributed to positions of equilibrium in the explanation of the business cycle; there are here two different and contrasting views: (a) the business cycle represents a movement around an equilibrium; (b) it is a movement between between two equilibriae²¹. The first view is expressed in many econometric models (Note: I may refer to Kalecky's²² and my own work; but there are many other examples.)”

Thus, it is clear that all four pioneering theories considered in the previous section belong to the first of the two classifications suggested by Tinbergen²³. It is our view that the Neo-Austrian approaches should be considered in the second class.

Now to the third, rhetorical, query. Amendola and Gaffard (op.cit, p.25) note that:

“[I]n the analysis of [Hicksian] Traverse, .. the adoption of a superior technique [is considered] as a process taking place sequentially over time. The explicit consideration of the time structure of the production process and if its intertemporal complementarity (with a focus on *the phase of construc-*

²⁰ The original is in Danish, but Velupillai was given, by Richard Goodwin, the original typescript of an English translation prepared by Tinbergen for Goodwin. The quote here is from this typescript. Incidentally, Goodwin himself considered this characteristically elegant paper by Tinbergen one of the pioneering contributions to the nonlinear theory of the business cycle (op.cit, p.2, footnote 3).

²¹ This is the spelling in Tinbergen's translation.

²² This is the spelling in Tinbergen's translation.

²³ Obviously, New Classical dynamics - whether of business cycles or anything else – belong also to the first of Tinbergen's two categories.

tion of a 'new' productive capacity and on its coming necessarily before the phase of utilization) allows to illuminate the fact that a change of the technique in use necessarily implies a change in the age structure of productive capacity and hence a dissociation of inputs from output and of costs from proceeds. We are in fact here clearly in an 'out-of-equilibrium' context”

This kind of 'traverse dynamics' is a path from one growth equilibrium - or one steady state growth path - to another. In the 'time-to-build' tradition that is tied to endogenous business cycle theory, on the other hand, 'traverse dynamics' is not a 'disequilibrium' thread linking two equilibrium configurations. As a matter of fact, we subscribe to the view that this particular 'traverse dynamics' vision, clearly and candidly expressed and described by Amendola and Gaffard, is an incoherent vision; our stand is substantiated by rigorous demonstration by Gunnar Myrdal against the Lindahlian concept of periods of temporary equilibria linked by points or time at which varieties of instantaneous changes occur (Myrdal [1931]). It is not surprising that this is a vision 'resurrected' by Hicks, who did more than anyone else to work within the Lindahlian framework of temporary equilibria, separated by periods during which disequilibria can emerge.

Unless the Neo-Austrian notion of 'traverse dynamics' is placed within the context of business cycle theory, where 'traverse' is 'time-to-build' and is intrinsic to the dynamics of the system, it will remain, at best, a pseudo-dynamic process with provable indeterminacies. In particular, it is easy to prove that 'traverse dynamics', when formalised effectively, is undecidable in the precise sense of computable economics.

But this is a wholly additional consideration, beyond the scope of the narrow focus on the 'time-to-build' tradition in business cycle theory, that was the theme of this chapter.

3.4 Brief Concluding Methodological Reflections

"I must not be too imperialistic in making claims for the applicability of maximum principles in theoretical economics. There are plenty of areas in which they simply do not apply. Take for example my early paper dealing with the interaction of the accelerator and the multiplier. This is an important topic in macroeconomic analysis.

....

My point in bringing up the *accelerator-multiplier* here is that it *provides a typical example of a dynamic system that can in no useful sense be related to maximum problems*. By examining the sick we learn something about those who are well; and by examining those who are well we may also learn something about the sick. The fact that the accelerator-multiplier cannot be related to maximizing takes its toll in terms of the intractability of the analysis.

Samuelson, 1970 [1972], pp. 12-13; italics added.

In two methodological senses RBC modelling, incorporating the 'time-to-build' assumption, is perfectly coherent: in basing its foundations in optimization and in interpreting observed behaviour as optimum – rational - reactions to exogenous disturbances to an equilibrium configuration. Thus, also, belonging to the first of Tinbergen's above two classificatory characterizations, but with the added proviso that even when 'out-of-equilibrium', behaviour is rational. Obviously, the same cannot be claimed by any of the four theories of the business cycle discussed in section 2. They may be described as being fluctuations in aggregate variables in *phenomenological* macroeconomics²⁴, where not maximization but 'conservation' principles are invoked. In terms of concepts used in RBC modelling, this refers to what is called 'calibration' in that research tradition. Calibrating, for example, the parameters of an aggregate production function of the Cobb-Douglas

²⁴ A phrase we have coined on the basis of the hint from Tinbergen's use of Ehrenfest's adiabatic principle in the formalization and analysis of the *Schiffbauzyklus*. For further discussions and definitions, see Velupillai (2008).

type would be equivalent to generating conservative cycles in phenomenological macroeconomics. This is the kind of assumption that leads to 'relaxation oscillations' in non-linear models of the business cycle in phenomenological macroeconomics. Failure of this kind of conservation principle - for example in linear dynamic models - leads to unstable, non-cyclical, dynamics (as pointed out by Frisch's critique of Kalecki's model).

Tinbergen's two-fold characterization of macroeconomic dynamics may not be exhaustive. Tinbergen, in common with all analytical economists who came before and after him, characterized the interpretation of aggregate fluctuations on the basis of one or the other of equilibrium norms: either the observed fluctuations are a deviation from an equilibrium (or equilibria); or, they are movements between equilibria. But is it really true that these are the only ways to characterise any observable aggregate dynamics? Surely, it is also possible that observed fluctuations are independent of any equilibrium norm? In other words, is it possible to construct (observable) dynamical systems that can be studied without any equilibrium norm? We conjecture that non-maximum dynamical systems - i.e., dynamical systems 'that can in no useful sense be related to maximum problems' - are those that display intrinsic dynamics, without any anchoring in any kind of equilibria. A constructible example of such dynamical systems are those that are capable of '*computation universality*' (cf., Velupillai [2011]).

We have referred to Myrdal's critique of Lindahl's temporary equilibrium dynamics²⁵ as an example of an incoherence in Neo-Austrian 'traverse dynamics'. This is part of a more formal criticism of any kind of inter-period (dis)equilibrium dynamics linked by alternative equilibria, as in Lindahl-Hicks or in Hicksian Neo-Austrian 'traverse dynamics'. In terms of formal dynamical systems theory the critique is about the dynam-

²⁵ Unfortunately, this critique appears only in the Swedish version (Myrdal, op.cit, pp. 227-230) of *Monetary Equilibrium*, and was removed from both the German and English translations.

ics at a boundary separating basins of attraction. Now, dynamical systems capable of computation universality reside only at the boundaries of basins of attraction. All formal macroeconomic dynamic models are constrained to lie within one or another of the basin of attraction of a given dynamical system and, therefore, eventually analysable in terms of equilibrium norms. This is not the case for dynamical systems that reside on the boundaries of basins of attraction - i.e., dynamical systems capable of computation universality²⁶.

The rich tapestry of dynamics implied by incorporating interesting 'time-to-build' assumptions in macroeconomics, particularly in its phenomenological versions, could give rise to wholly new, non-equilibrium, non-maximum, research paradigms. The lessons to be learned from the classics are inexhaustible and, at least in this sense, the role of the history of economic thought should not be underestimated.

²⁶ A part of what we have in mind is discussed cogently in Pincock (2009) in terms of boundary layer dynamics of the *Navier-Stokes equation*.

References

- AMENDOLA, M. AND J. LUC GAFFARD (1998): *Out of Equilibrium*, Clarendon Press, Oxford.
- BELLMAN, R. AND K.L. COOKE (1963): *Differential-Difference Equations*, Academic Press, New York.
- BÖHM-BAWERK, E. (1891): *The Positive Theory of Capital*, translated by William A. Smart, Macmillan & Co Ltd., London.
- BRODY, A. (1970): *Proportions, Prices and Planning*, North-Holland Publishing Company, Amsterdam.
- BROWNELL, F. H. (1950): Non-Linear Delay Differential Equations, chapter III, pp. 89-148, in: *Contributions to the Theory of Nonlinear Oscillations*, Volume I, edited by Solomon Lefschetz, Annals of Mathematics Studies, Number 20, Princeton University Press, Princeton, NJ.
- FRISCH, R. (1933): Propagation and Impulse Problems in Dynamic Economics, pp. 171-205, in: *Economic Essays in honour of Gustav Cassel*, George Allen & Unwin Ltd., Museum Street, London.
- FRISCH, R. AND H. HOLME (1935): The Characteristic Solution of a Mixed Difference and Differential Equation Occurring in Economic Dynamics, *Econometrica*, Vol. 3, #2, April, pp.225-239.
- GOODWIN, R.M (1950): A Non-Linear Theory of the Cycle, *The Review of Economics and Statistics*, Vol. 32, # 4, November, pp.316-320.
- (1951): The Nonlinear Accelerator and the Persistence of Business Cycles, *Econometrica*, Vol. 19, #1, January, pp. 1-17.
- HAAVELMO, T. (1940): The Inadequacy of Testing Dynamic Theory by Comparing Theoretical Solutions and Observed Cycles, *Econometrica*, Vol. 8, #4, October, pp. 312-321.
- (1960): *A Study in the Theory of Investment*, The University of Chicago Press, Chicago, Illinois.
- HAYEK, F. VON (1931): *Prices and Production*, George Routledge & Sons, Ltd., Broadway House, 68-74 Carter Lane, London.
- HANSEN, A.H AND H. TOUT (1933): Annual Survey of Business Cycle Theory: In-

- vestment and Saving in Business Cycle Theory, *Econometrica*, Vol. 1, #2, April, pp.119-147.
- HILL, M. (1933): The Period of Production and Industrial Fluctuations, *The Economic Journal*, Vol.43, #142, December, pp.599-610.
- HOLLESTELLE, M.J (2006): Paul Ehrenfest as a Mediator, pp.787-792, Chapter 26, in: *The Global and the Local: The History of Science and the Cultural Integration of Europe*, Edited by M. Kokowski, Proceedings of the 2nd ICESHS, Cracow, Poland, September, 6-9.
- KALDOR, N. (1942): Professor Hayek and the Concertina Effect, *Economica*, N.S., Vol. 9, #36, November, pp.359-382.
- (1961): Comments in the Discussion of Professor Lutz's Paper, p.294, in: *The Theory of Capital - Proceedings of a Conference held by the International Economic Association*, edited by F.A. Lutz & D.C. Hague, Macmillan & Co Ltd., London.
- KALECKI, M. (1935): A Macrodynamic Theory of Business Cycles, *Econometrica*, Vol. 3, #3, July, pp.327-344.
- (1936): Comments on the Macrodynamic Theory of Business Cycles, *Econometrica*, Vol. 4, #4, October, pp.356-360.
- (1939): *Essays in the Theory of Economic Fluctuations*, Allen & Unwin Ltd., London.
- KNIGHT, F. (1933): Capitalistic Production, Time and the Rate of Return, pp.327-342, in: *Economic Essays in honour of Gustav Cassel*, George Allen & Unwin Ltd., Museum Street, London.
- (1934): Capital, Time and the Interest Rate, *Economica*, N.S., Vol.1, #3, August, pp.257-286.
- (1938): On the Theory of Capital: In Reply to Mr Kaldor, *Econometrica*, Vol. 6, #1, January, pp.63-82.
- KYDLAND, F. AND E.C. PRESCOTT (1982): Time to Build and Aggregate Fluctuations, *Econometrica*, Vol. 50, #6, November, pp.1345-1370.
- MARX, K. (1893 [1967]): *Capital: A Critique of Political Economy*, Volume II, Bk. II: The process of Circulation of Capital, edited by F. Engels, Progress Publishers, Moscow.
- MAYER, T. (1960): Plant and Equipment Lead Times, *The Journal of Business*, Vol. 33, #2, April, pp.127-132.

- MINORSKY, N. (1962 [1974]): *Nonlinear Oscillations*, Robert E. Krieger Publishing Company, Huntington, New York.
- MOHAMMED, S. E. A (1978): *Retarded Functional Differential Equations: A Global Point of View*, Research Notes in Mathematics #21, Pitman Publishing Limited, London.
- MYRDAL, G. (1931): Om penningteoretisk jämvikt. En studie över den "normala räntan" i Wicksells penninglära, *Ekonomisk Tidskrift*, Arg. 33, haft 5/6, pp.191-302.
- PINCOCK, C. (2009): Towards a Philosophy of Applied Mathematics, Chapter 8, pp. 173-194, in: *New Waves in Philosophy of Mathematics*, edited by Otávio Bueno & Øystein Linnebo, Palgrave Macmillan, Houndmills, Basingstoke, Hampshire, UK.
- SAMUELSON, P.A (1970, [1972]): Maximum Principles in Analytical Economics, *Nobel Memorial Lecture*, December 11, 1970. Reprinted in: *The Collected Scientific Papers of Paul A. Samuelson*, Volume III, edited by Robert C. Merton, Chapter 130, pp. 2-17; The MIT Press, Cambridge, Mass.
- SCHUMPETER, J.A (1939): *Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process*, Vol. I & Vol. II, McGraw-Hill Books Co., New York.
- SRAFFA, P. (1932): Dr Hayek on Money and Captial, *Economic Journal*, Vol. 42, #1, March, pp.42-53.
- STROTZ, R. H, J. C. MCANULTY & J. B. NAINES, JR., (1953): Goodwin's Nonlinear Theory of the Business Cycle: An Electro-Analog Solution, *Econometrica*, Vol. 21, #3, July, pp. 390-411.
- SZYDLOWSKI, M. (2002): Time-to-build in dynamics of economic models I: Kalecki's model, *Chaos, Solitons and Fractals*, Vol. 14, #5, September, pp.697-703.
- THALBERG, B. (1961): An Analysis of a Market for Investment Goods, pp.161-174, Chapter 9, in: *The Theory of Capital - Proceedings of a Conference held by the International Economic Association*, edited by F.A. Lutz & D.C. Hague, Macmillan & Co Ltd., London.
- (1966): A Trade Cycle Analysis: Extensions of the Goodwin Model, *Studentlitteratur*, Lund
- TINBERGEN, J. (1931): Ein Schijfbauzyklus, *Weltwirtschaftliches Archiv*, Vol. 34, Issue 1, pp.152-164.

- (1943): Ligevaegtstyper og Konjunkturbevaegelse, *Nordisk Tidsskrift for Teknisk Økonomi*, Vol. 32, # 2-4, pp.45-63.
- VELUPILLAI, K. (1992): Implicit Nonlinearities in the Economic Dynamics of 'Impulse and Propagation', chapter 5, pp. 57-71, in: *Nonlinearities, Disequilibria and Simulation: Essays in Honour of Björn Thalberg*, edited by Kumaraswamy Velupillai, The Macmillan Press Ltd., Houndmills, Hampshire, UK.
- VELUPILLAI, K. VELA (1997): Exploring The Original Kalecki Model T', chapter 20, pp.254-264, in: *Markets, Unemployment and Economic Policy: Essays in honour of Geoff Harcourt*, Volume two, edited by Philip Arestis, Gabriel Palma and Malcolm Sawyer, Routledge, London.
- (2008): The Mathematization of Macroeconomics, *Economia Politica [Journal of Analytical and Institutional Economics]*, Vol. XXV, # 3, pp. 283–316.
- (2011): Dynamic Complexity, Nonlinearity and Emergence: An Algorithmic Interpretation, *Journal of Economic Surveys*, Vol. 25, #1 (Forthcoming).
- WRIGHT, E. M (1946): The Non-Linear Difference-Differential Equation, *Quarterly Journal of Mathematics*, Vol. 17, #1, pp. 245-252.
- (1961): Stability Criteria and the Real Roots of a Transcendental Equation, *Journal of the Society for Industrial and Applied Mathematics*, Vol. 9 , #1, March, pp.136-148.
- ZAMBELLI, S. (2007): The Rocking Horse That Never Rocked: Frisch's "Propagation and Impulse Problem", *History Of Political Economy*, Vol. 39, #1, pp.145-166.

Appendix 3A: Simulating Goodwin's Nonlinear Business Cycle model

The aim of this appendix is to study Goodwin's 'time-to-build' underpinned nonlinear accelerator model, with more 'precise' approximations, and to investigate its dynamics through simulations. One of the very insightful, simulational, studies on Goodwin's model can be found in the work of Strotz, et.al., (1953). Using an electro-analog computer, Strotz, et.al., analysed the *formal properties* of Goodwin's model (see equation A.1).

$$\epsilon y'(t + \theta) + (1 - \alpha)y(t + \theta) = O_A(t + \theta) + \phi[y'(t)] \quad (3A.1)$$

Where:

y : aggregate income;

θ : one half the construction time of new equipment;

$\phi(y')$: the flexible accelerator;

O_A : the sum of autonomous outlays ($\beta(t)$ and $l(t)$);

They did so by simulating the nonlinear *difference-differential* system for various initials conditions and *higher order approximations of the Taylor series approximation of the above canonical equation*. The results showed that Goodwin's model is *sensitive* to 'initial conditions'¹ and there are "at least twenty-five other limit cycles that are also solutions to the same equation, indicating that there are an infinite number of additional solutions." (ibid, p: 398) Moreover, as Strotz, et.al., pointed out, if Goodwin had *not* approximated his non-linear *difference-differential* model by a second order *differential*

¹ We are not referring to the well known property of nonlinear dynamical systems known as 'sensitive dependence to initial conditions' (SSIC). Here, we simply mean that the reduction of (A.1) to the Rayleigh-van der Pol type nonlinear differential equation shown the existence of limit cycles independent of initial conditions. Taking better approximations to A.1 shows that this *independence* breaks down.

equation (3A.2), by taking only the first two terms but with more terms of the Taylor series expansion of the two leading terms, the model would have exhibited a *richer* dynamics.

$$\varepsilon\theta y''(t) + [\varepsilon + (1 - \alpha)\theta]y'(t) - \phi[y'(t)] + (1 - \alpha)y(t) = 0 \quad (3A.2)$$

In fact, “[i]mproving the approximation by retaining more terms of the Taylor's expansion would increase the order of the differential equation and this would increase the number of solutions provided by the approximation” (ibid, p: 407), all depending upon the initial conditions.

This result emphasized, and continues to emphasise the need for further studies of a simulational kind, to learn how to approach the analytical solutions and properties of its attractors and their dependence on the structure of sets of initial conditions. This appendix is structured such that, first, we replicate Strotz, et.al, results, by using a digital computer², and then go beyond *order 4* to investigate how the system would behave and evolve over time.

Simulations:

In 1953, Strotz, et.al., simulated Goodwin's model, by using an electro-analog computer, and found the model to have multiple solutions depending on the initial conditions. They systemically altered Goodwin's initial parameter values and orders of the system to see if the set of solutions changed or not. As the computers in 1950s were in a developing stage they could not analyse for a wide range of values and there was “an *error* in the quantitative analysis of the circuit” (ibid, 398). Therefore in this exercise, we have replicated the simulation, for the parameters (see Table 1), in Strotz, et.al., paper by using a digital computer (see figure 1 *a, b, c – 5 a, b, c*; the top figure shows the phase plot and the below one shows the cycle plot).

² All the simulations have been carried out using Matlab. The code used for simulating the differential equation is *ode45*, which uses 4th and 5th order Runge-Kutta formulas.

Figure 1 shows the evolution of Goodwin's model when α is changed to .4 (see fig 1.a) and 0.733 (see fig 1.a), instead of 0.6 (1.b) as used by Goodwin, while all the other parameters remain the same. The results show that when α takes a value 0.6 we get a limit cycle of length 11 years but when α is changed to 0.4 the length of the cycle increases to 9 years. Moreover, when α is increased to 0.733 the cycle lengthens to 18 years (fig. 1c). In the next step, we simulated the model for various values of κ , for example, 1.58 and 8.42, as done by Strotz et al., and we have found that the limit cycle was 7.75 years (for $\kappa = 1.58$, fig. 2a) and 49 years (for $\kappa = 8.42$, fig. 2a). Thirdly, the value of ε is changed to 0.349 the cycle length becomes a little longer than Goodwin's cycle and when ε value is kept as 0.82 the cycle length was shortened to 9 years (fig. 3.a,b,c). For next set of simulation, θ is changed from 1 year to half a year and one and a half year. The simulations show that when θ is 0.5 year and 1.5 years the economy's cycle length is 12- years and 14+ years respectively. Lastly, the upper limit of the induced investment function ($\bar{\phi}$), instead of 9 billion, is changed to 6 billion and 15 billion and the results show that the cycle length decreased marginally, when the upper limit was 6 billion, and the cycle length increased marginally when the upper limit was set to be 15 billion. In fact the simulations reinforce the sensitivity of initial conditions and illustrates the economic system's traverse to different limit cycles. Like Strotz et al., we also found α , ε , and $\bar{\phi}$ to be sensitive and θ to be less sensitive. But interestingly the simulations show that κ seems to be as sensitive as α , ε and $\bar{\phi}$, unlike what Strotz et.al. observed.

The results obtained in our analysis showed a great deal of similarities with the results obtained by Strotz, et.al., but with some minor differences. This might be due to the difference in structure of the input equation that is being fed in the computer and also because of the processing limitation of the computer itself. For example, Strotz, et.al, use equation (A.1) to build an electrical circuit to simulate the nonlinear *difference-differential* equation, whereas we take the Taylor series expansion of the two leading terms (i.e., $y'(t + \theta)$ and $y(t + \theta)$) and approximate the nonlinear *difference-differential*

equation to a nonlinear *differential* equation and then simulate the system. Moreover, the higher modes in Strotz, et.al., model are given according to the orders of oscillations (in terms of frequency) while in our system the orders increase according to the number of terms retained in the Taylor series expansion of the two leading terms. Moreover, as Strotz, et.al., write (ibid, p: 398), “[t]here are several characteristics of the higher-mode solutions which are peculiar to the apparatus [i.e., Electro-Analog computer] used and which introduce an error in the quantitative analysis of the circuit.” Because of the limitation of their apparatus, they did not have any provisions to control the initial values of $y(t)$, $\dot{y}(t)$, and $\phi[\dot{y}(t-\theta)]$ at $t=0$; therefore, whenever the system is operated, the solution took a form of some higher modes and so they only analysed the modes for which the results are replicable. For example, the solutions obtained for the systems of mode higher than 4 were *unstable*, and due to the problem of replication of results, they confined their analysis to modes up to 4 (ibid, p:401). The discrepancy in the results might be due the above limitations but unlike *their* electro-analog computer, we were able to go beyond order 4.

As an example, we have illustrated few cases in which the nonlinear *difference-differential* equation is approximated by retaining 3, 4 and 5 terms in the Taylor’s expansion of the two leading terms and the nonlinear *differential* equations thus obtained are of order 3, 4 and 5 respectively (see 3A.3, 3A.4, and 3A.5).

$$\varepsilon \frac{\theta^2}{2} y''''(t) + C_2 y''(t) + C_1 y'(t) - \phi[y'(t)] + (1 - \alpha)y(t) = 0 \quad (3A.3)$$

$$\varepsilon \frac{\theta^3}{6} y'''''(t) + C_3 y''''(t) + C_2 y''(t) + C_1 y'(t) - \phi[y'(t)] + (1 - \alpha)y(t) = 0 \quad (3A.4)$$

$$\varepsilon \frac{\theta^4}{24} y''''''(t) + C_4 y''''''(t) + C_3 y''''(t) + C_2 y''(t) + C_1 y'(t) - \phi[y'(t)] + (1 - \alpha)y(t) = 0 \quad (3A.5)$$

Where,

$$C_1 = \varepsilon + (1 - \alpha)\theta;$$

$$C_2 = \varepsilon\theta + (1 - \alpha)\frac{\theta^2}{2};$$

$$C_3 = \varepsilon\frac{\theta^2}{2} + (1 - \alpha)\frac{\theta^3}{6};$$

$$C_4 = \varepsilon\frac{\theta^3}{6} + (1 - \alpha)\frac{\theta^4}{24}.$$

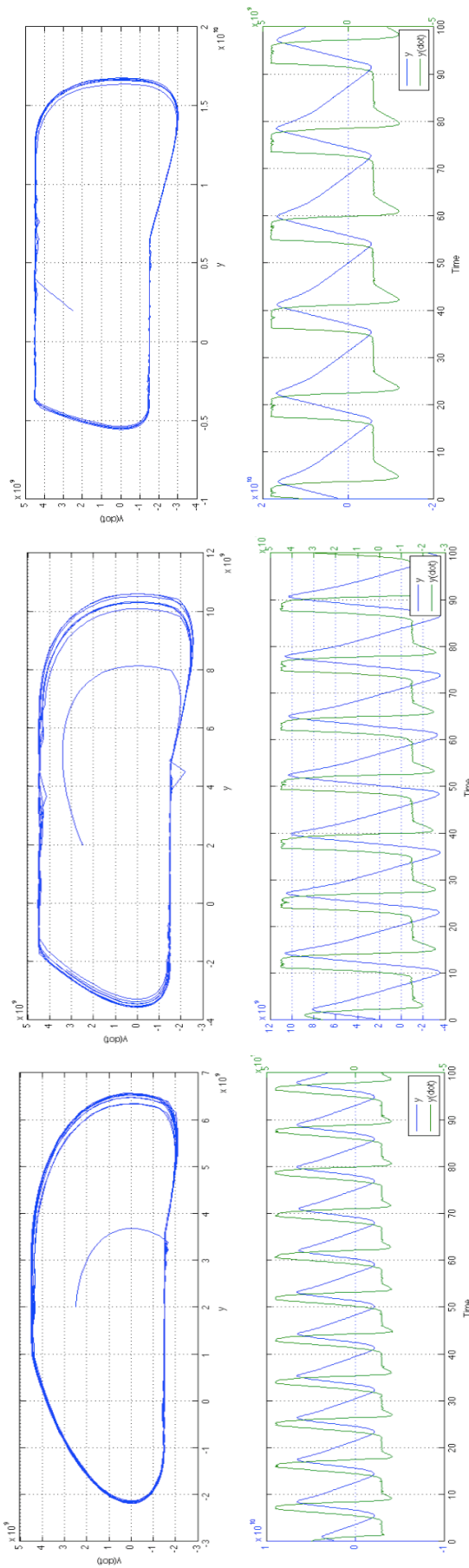
The above nonlinear *differential* equations have been simulated, using Matlab, for Goodwin parameters and the results are shown in figure 6. It is very interesting to see how the system behaves when more terms in the Taylor's expansion are retained. Moreover, as the orders increase, the system tended to have more number of solutions and became more *sensitive* to the initial conditions. Commenting on the problem of multiplicity of solutions, Strotz, et.al., insightfully noted that there is not one but at least 25 different limit cycles as solutions for Goodwin's model, and in this exercise, by simulating the model for orders up to 5, we have found that there are at least 45 different limit cycles, all depending upon the initial conditions. The simulational results reinforce the results of Strotz, et.al., and emphasize the need for further understanding of Goodwin's nonlinear model and its fundamental dependence on a 'time-to-build' structure.

Summary:

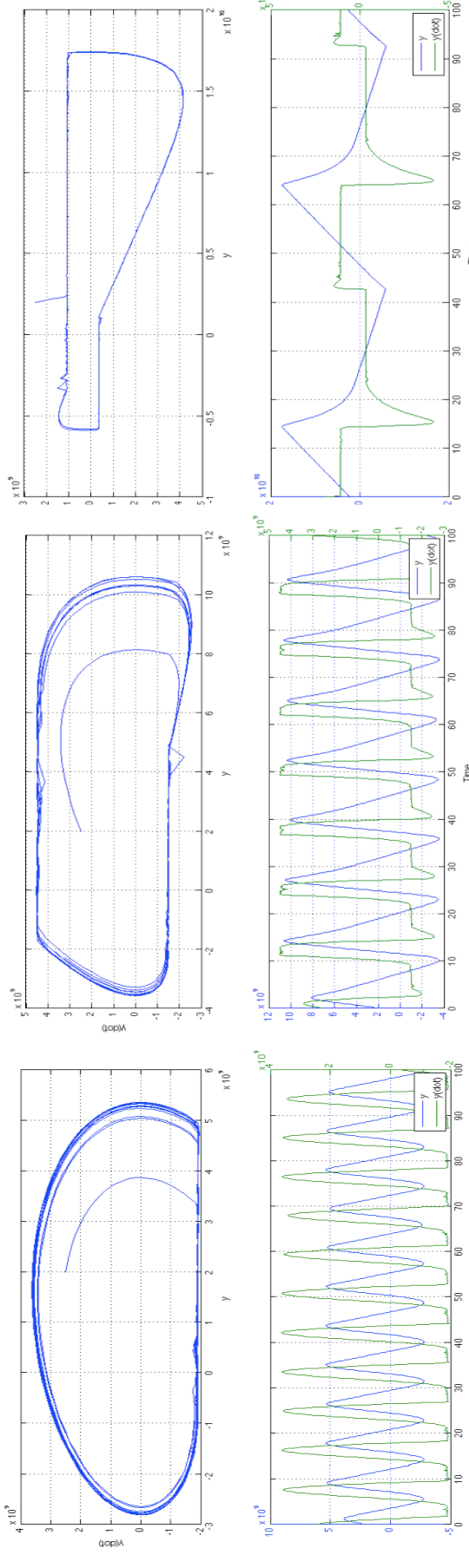
“The problem is to determine what kinds of "initial conditions" lead to the various possible cycles, and then to determine whether these conditions can occur. This presents an analytical problem of great *complexity*, but one that must be solved if non-linear mixed models are to provide unambiguous answers to problems in economic theory.”

Strotz, et.al., (1953, p: 408; italics added)

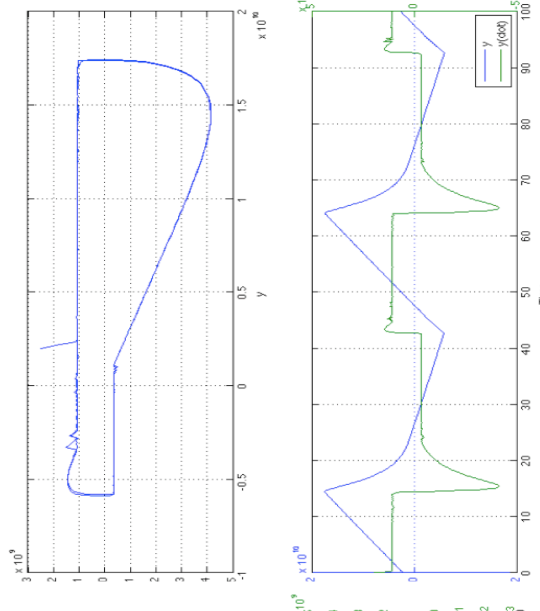
The multiplicity of solutions, depending upon the initial conditions, an economic system can have will decide what kind of possible paths the system can traverse over time. The dependence on the initial conditions and the role of ‘time-to-build’ function in business cycle theory emphasize the importance of further investigating these nonlinear systems. This exercise is one such attempt to illustrate the richness of the nonlinear difference-differential system that is based on a ‘time-to-build’ structure used for modelling and analysing macroeconomic dynamics.



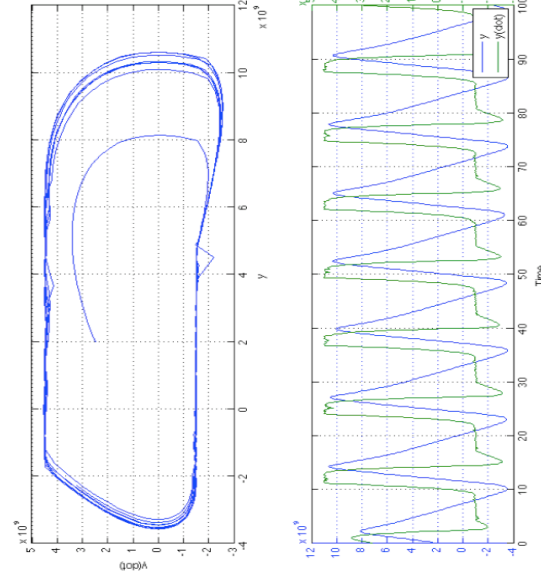
1a



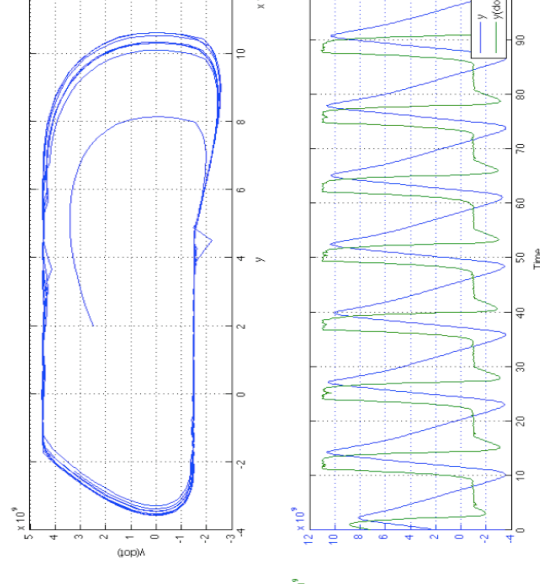
1b



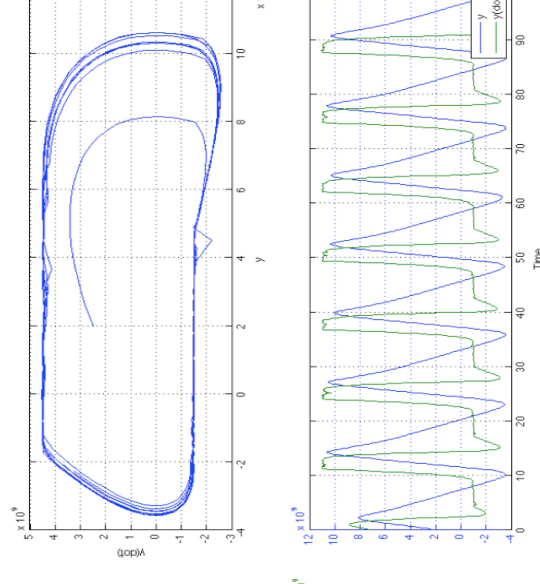
1c



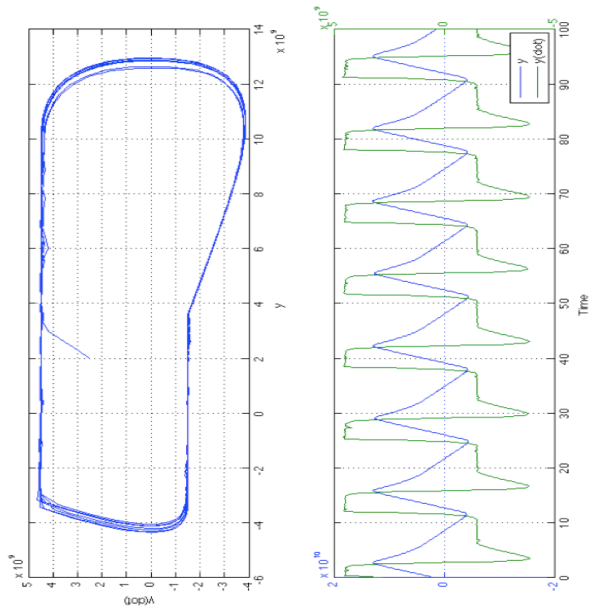
2a



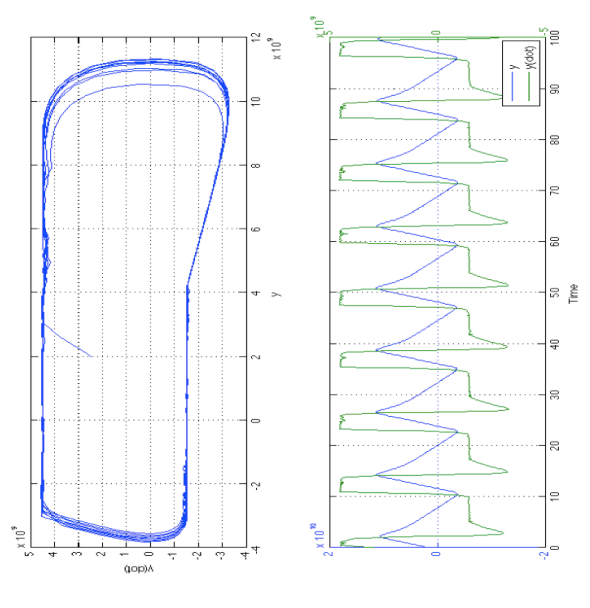
2b



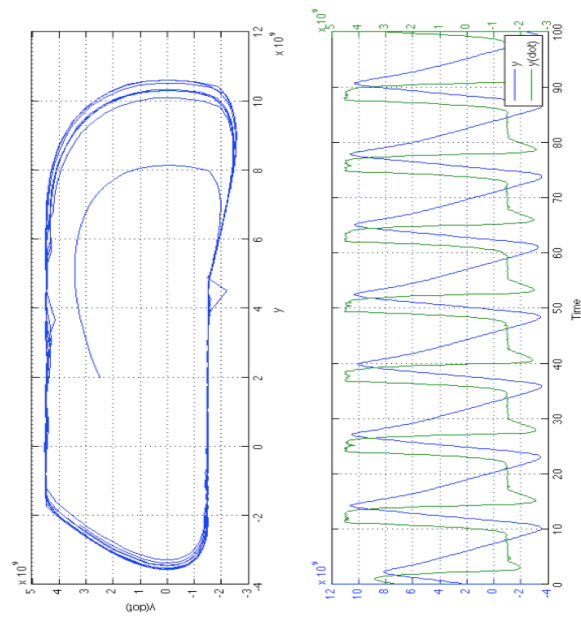
2c



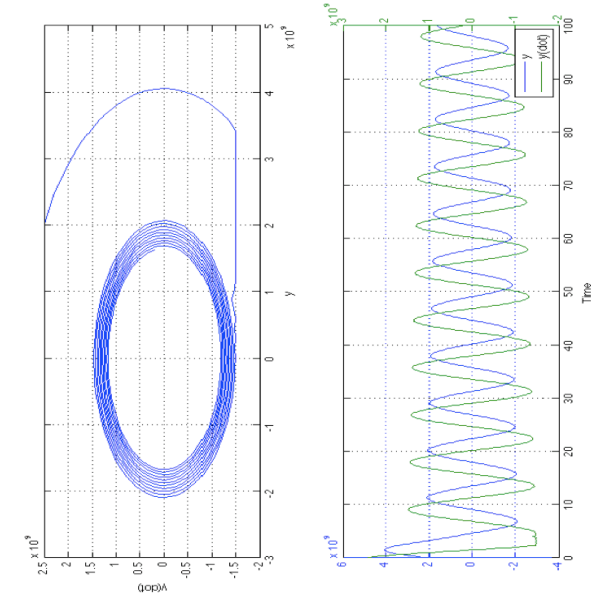
3a



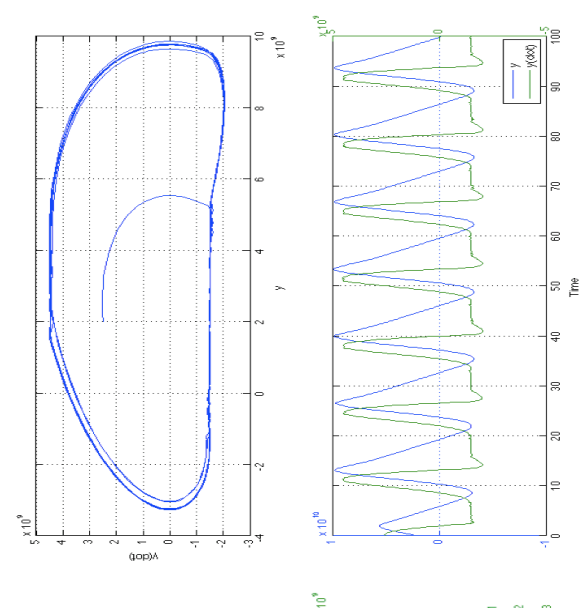
3b



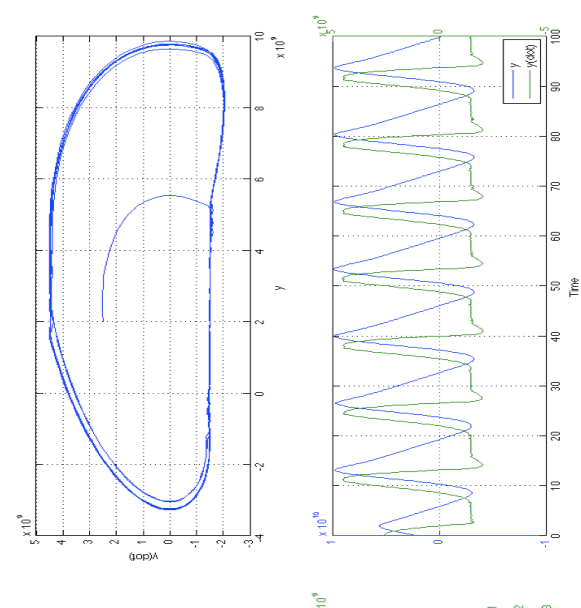
3c



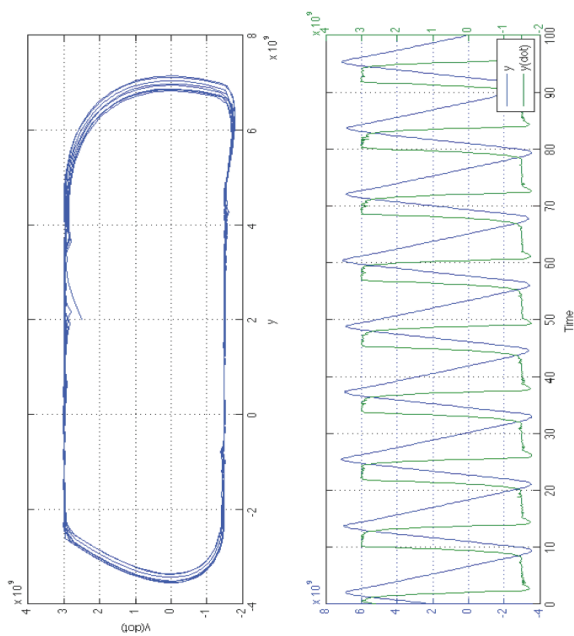
4a



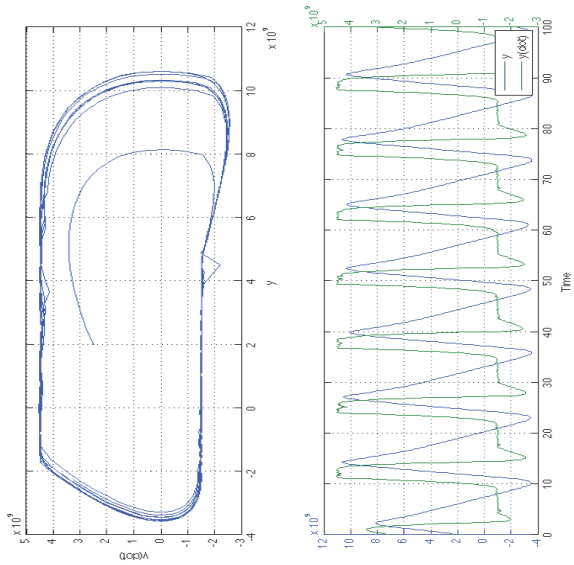
4b



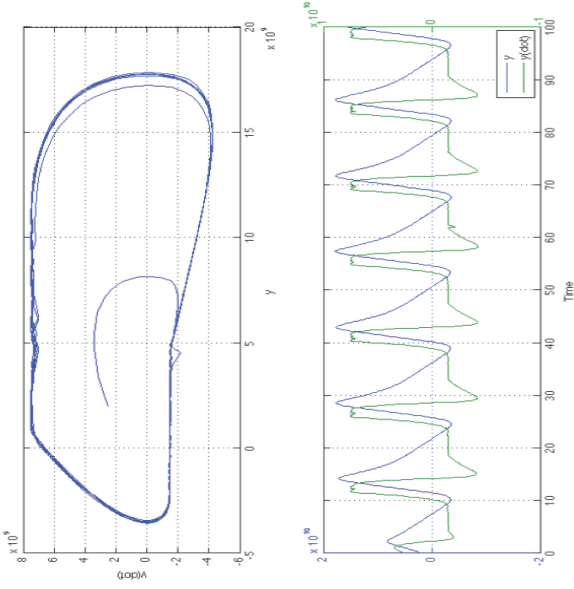
4c



5a



5b



5c

Table 1:

Case	α	κ	ε	θ	$\bar{\phi} - \underline{\phi}$
$I(b)-5(b)$	0.6	2.0	0.5	1.0	9: -3
1 (a)	0.400	2.00	0.500	1.00	9: -3
1 (c)	0.733	2.00	0.500	1.00	9: -3
2 (a)	0.600	1.58	0.500	1.00	9: -3
2 (c)	0.600	8.42	0.500	1.00	9: -3
3 (a)	0.600	2.00	0.349	1.00	9: -3
3 (c)	0.600	2.00	0.802	1.00	9: -3
4 (a)	0.600	2.00	0.500	0.50	9: -3
4 (c)	0.600	2.00	0.500	1.50	9: -3
5 (a)	0.600	2.00	0.500	1.00	6: -3
5 (c)	0.600	2.00	0.500	1.00	15: -3

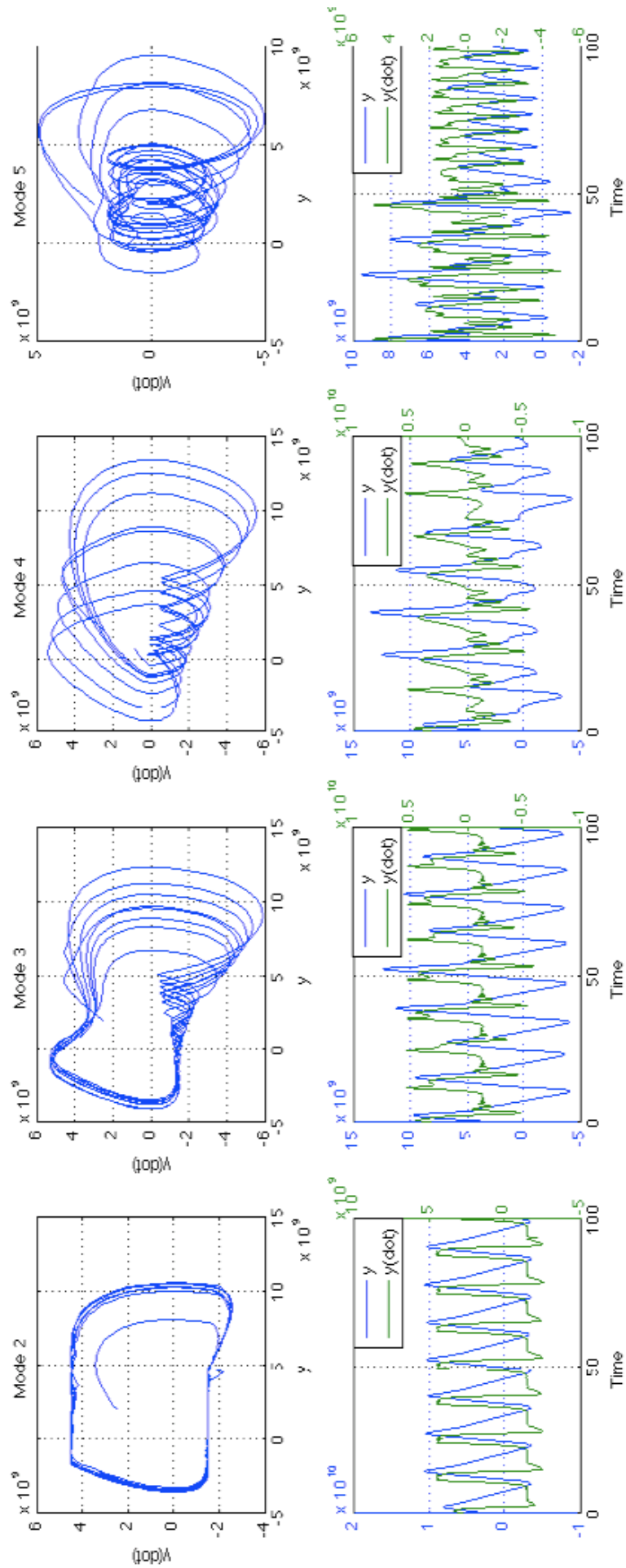


Figure 6

Chapter 4

***“Carry-on-activity”* and Process Innovation**

Written jointly with Stefano Zambelli

4.1 Introduction

In this chapter we study a theoretical model where R&D is the search for a better production process. In order to make the concept of process innovation precise we adopt a particular interpretation of the Church-Turing thesis to claim that anything which is computed, that is: anything which is ‘produced’, can be in principle computed-produced by a Universal Turing Machine or, equivalently, by a Universal Constructor (see Chaitin, 2010: 73). Following the work of Velupillai (2000, 2010 and Zambelli (2004, 2005) we assume that the output can be encoded as a string, which is the input, the specific Turing Machine and the output to be fed to the Universal Turing Machine. The working of the Turing Machine is interpreted as being equivalent to the carry-on to completion of the product which does require the employment of labour. That is, a process innovation is here going to be defined precisely as the generation of the same output-string with a different Turing Machine. Between the different production processes the most efficient one is here going to be the one able to produce the output with a lower algorithmic complexity (see Zambelli, 2004, pp.165-7 and 2005, pp.238-46). Lower algorithmic complexity means lower necessity of labour to complete production. Whether a more efficient process is going to be implemented by the firms depends on its efficiency, i.e., its algorithmic complexity, but also on the market conditions and R&D policies. In this chapter an endogenous model of process innovation is presented and studied.

The production of a commodity requires the use of means of production and time. As discussed in the previous chapters the idea that it takes time to build has been at the core of the Austrian and Neo-Austrian traditions (see Hicks, 1973; Amendola and Gaffard, 1998 and 2006), but also in other traditions (see Frisch, 1933, Kalecki 1935). The point that is central in these approaches is that between the moment in which a decision

on production is taken and the moment in which the product is brought to completion market conditions can change so as to modify the economic convenience of production. It is not only time that elapses, but resources (mostly labour) have to be used and financed at the same time as production takes place. This asynchrony between the carry-on of production and the delivery of the output for market sale may be fundamental in generating (explaining) business cycles and/or in shaping the traverse.

Both Frisch (1933) and Kalecki (1935) have captured this asynchrony and the work efforts and investments in what they have termed the carry-on-activity. That is: a function of time describing at each point of time the activity necessary for the product to be brought closer to completion. Practically this carry-on-activity does represent the production process.

Carry-on-activity (known also as manufacturing process management) is a little more with respect to what was discussed in chapter 3. There we have discussed time to build, following the tradition of Tinbergen (1931), Frisch (1933), Kalecki (1935) and Goodwin (1951), by focusing our attention on the lag that exists between the time in which a production decision is taken and the moment in which the product is completed. We left out, following the literature, the problem of the different shapes of the carry-on-activity by assuming that the efforts to complete a unit of output are constant all through the period of production. Amendola and Gaffard (1998, 2006), which we can take as an example of the Neo-Austrian tradition (Hicks, 1973), do the same and the work efforts necessary to complete a unit of output are constant through time. The fact that market condition may cause the interruption of the completion of the product is obviously a different matter with respect to varying working efforts.

Kydland and Prescott (1982) in their famous time to build model have a production function which is point input - point output. *Practically* to produce it does not take time

and hence they have no carry-on-activity function: production occurs inside the period and/or *at most* there is one unit of time lag between in the moment in which production takes place and the output is completed.

In previous chapters we have discussed the importance of viability and of the traverse, but have left out the issue of the carry-on-activity.

Also Romer (1986, 1990, 1993), in his works on the endogenous creation of knowledge, has emphasized the importance of ideas and knowledge and has put focus on its creation, but has not considered the fact that to produce it takes time and effort. Innovations do not involve the production process. In his Chemistry set metaphor¹, Romer discusses the *curse of dimensionality*², but leaves out discoveries and new ideas that innovate with respect to the production process.

Even Velupillai (2000: Ch. 9, 2010), and Zambelli (2004, 2005) who have expanded the implications of Romer’s (1993: 68) suggestion to characterize an idea as being equivalent to a bit string by considering the generating process of a Turing Machine do not focus on the *time and effort* necessary to bring to completion a unit of output (Production) or on the time and efforts to generate a new productive idea (R&D).

¹ As Romer (1993: 68) explains,

[W]ithin the metaphor of the chemistry set, it is obvious what one means by an idea. Any mixture can be recorded as a bit string, an ordered sequence of 0s and 1s of length 100. The bit at position j is set to one if element j is included in the mixture... an idea is the increment of information that comes from sorting some of the bit strings into two broad categories: useful ones and useless ones. To represent this information we can add two more bits on the end of each bit string describing a mixture. These are set 00 if we know nothing about its properties, 10 if it is a useful mixture, and 01 if it is useless.

² For example, the number of different combinations of the chemistry set is 2^{100-1} (or approximately $6.3 \cdot 10^{29}$) and another such example is that a shirt is made in 52 steps, so there are 52! ways of making a shirt, and since it is a large set, and not every method has been tried out so there is always some scope for improvement (Romer, 1993).

In the following, expanding from the above mentioned works of Velupillai and Zambelli, we will present a model in which process innovations and the processes themselves are endogenously generated as Turing Machine equivalents which it “take time to compute”, hence that it takes time to produce.

The importance of the Turing Machine as a mechanism encoding ideas and innovations is due to an adherence to a particular view of the Church-Turing thesis that allows us to consider productive discoveries as equivalent to the strings generated as the result of *halting* Turing Machines. One of the fathers of Algorithmic Information Theory, Greg Chaitin (1995: x) has claimed that a “universal Turing Machine is, from a physicist’s point of view, just a physical system with such a rich repertoire of possible behaviours that it can simulate any other physical system.” If we consider the production of goods and services equivalent to the transformation of ideas into physical processes aimed at the production of commodities the importance of the Turing Machine metaphor should be evident.

Hence, by modelling the Research and Development activities as a concurrent search of new Turing Machines and as Turing Machines we are bound to deal with the *halting* problem of the TMs and are able to capture the intrinsic uncertainty of the R&D innovation processes (Zambelli, 2004; 2005). In so doing we avoid stochastic *ad-hockeries*. The key, then, is to conceptualise the dynamics of production and innovation process as a dynamically interacting phenomena and study its evolution. As we will see *the production process depends on the degree and magnitude of the innovation process, but also the innovation process depends on the production process.*

The conceptualization of production as a computational process forces us to view the system as being intrinsically dynamical and to model the evolutionary process in an insightful way. In this context to the concepts of viability and that of the traverse, as in-

troduced in the previous chapters, being essential in distinguishing between the innovation and its economic feasibility, is given a rigorous content. The Neo-Austrian approach is very useful framework that enables us to understand the dynamic behaviour of the economy but limits itself in explaining the generation of new ideas, knowledge, and innovation. In this chapter, we focus our attention on the aspects of knowledge creation, which is *process innovation* and *labour saving mechanization*. In the characterization of a process, we will follow the Romer-Velupillai-Zambelli approach mentioned above by considering a product to be produced with a process which is a bit string. Process innovations will be equivalent to *robotisation (automation)* of production.

The conceptualization of innovational processes as a process in time, using the Turing Machine metaphor (Zambelli 2004, 2005), enables us to model the evolution of process innovation in an insightful and non-stochastic way. Moreover, this endogenous model of process innovation will provide us with a valuable tool to study the concurrent effects of reducing the labour necessary to produce and the time to build. Therefore, the model of process innovation is then infused into a synthesis of traditional macroeconomic models where ‘*time-to-build*’ plays a central role.

4.2 The model. Time-to-Build, carry-on-activity and process innovation

In algorithmic information theory, you measure the complexity of something by counting the number of bits in the smallest program for calculating it:

program → *Universal Computer* → output

If the output of a program could be a physical or a biological system, then this complexity measure would give us a way to measure the difficulty of explaining how to construct or grow something, in other words, measure either traditional smokestack or newer green technological complexity:

software → *Universal Constructor* → physical system

DNA → *Development* → biological system

Chaitin (2010: 73)

Chaitin (2010) emphasizes that ideas, knowledge, innovations, technologies, and, even, economies be considered as bits of information which would enable the human/machine to produce/compute.

In this chapter we assume that the product is encoded as a long string of 0s and 1s. For example the following string of *n-digits*

01110010100010010100100011110010...10100100011000010101010101111101

can be seen as the encoding of a product. Such a string may be produced by several TMs. Note that there exists at least one TM which is able to produce the above string. This TM is the trivial TM that, operating on a blank tape, does print the string in a sequential manner. How to construct this TM is discussed in Zambelli (2005). If we define algorithmic complexity in terms of the quadruples (i.e., states) of the TM we can consider as the worst case for the production of the above *n-digits* string that in which the TM to produce it, has algorithmic complexity precisely equal to *n*. When a TM which has lower algorithmic complexity with respect to the above worst case is found, by construction, we reduce the known complexity of the string. We can say that we have a better way to produce the string. Here we make a direct analogy with the production process. We link the algorithmic complexity of the TM generating the *n-digits* string with the labour efforts necessary to conduct the computation. A process innovation is going to be defined as *robotization* of production and hence as a reduction (saving) of labour efforts.

The analogy with computation by TMs and production is straight forward. Labour is necessary to compute-produce the output and is necessary to operate the machines that allow a reduction of the working efforts through the use of a TM which has lower complexity. Once the above is granted a crucial problem is that of modelling the discovery process. How can the new *labour effort reducing machines* can be discovered? The link

with R&D is straight forward. R&D activity is defined as labour devoted to the scope of finding new TMs with lower complexity with respect to the known ones.

The R&D unit carries out research in order to find if there is any shorter algorithm than that of the object itself. As in the so called ‘real world’, the R&D unit has a task and characteristics which are very different from the production unit. The production unit does not face uncertainty in the sense that the task of production is simply that of implementing well known processes whose outcome is well defined. Using the TM metaphor we can say that the *n-digits* string can be produced with *certainty*, using an already halted TM.

On the other hand, the work of the R&D unit is subjected to high degrees of uncertainty. The outcome of trying out of new TMs and discovering whether that TM allows *robotization* of the process is highly uncertain and it is due to the intrinsic uncertainty related with the structure of the generated string, the innovation, that it is likely to be different from the encoding of the output or it is due to the existence of the halting problem, i.e., to the fact that the outcome of the tried out specific TM is unknown because it is not known whether the TM will halt or not.

4.2.1 The search of process innovations through investment R&D: the search process.

The labour employed by a firm at any given time depends upon the firm’s strategy to carry out just production with the given production process and/or whether the firm is willing to search for better production processes, i.e. a better carry-on-activity. If the firm decides to invest in R&D it will have to allocate resources, i.e. labour, for the search of this new process.

Without entering here into the details of the actual search for new TMs, let just say that the search process is equivalent to *data compression* where by data compression

means reduction of the number of states necessary for the production of the string or subsets of it.

We assume that the production of the output q

- is encoded by a digital string;
- that it has time to build equal to ε periods
- that for each period labour efforts are required to proceed in the computation;
- the efforts per unit of time necessary to bring production to completion depends on the available carry-on-activity;
- a process innovation is an improvement on the carry-on-activity and depends on whether a new TM with a lower algorithmic complexity has been found;
- R&D is the search for these new TMs – an increase in the number of researchers determines an increase in the number of TMs tried out.

In a way the above procedure is somewhat equivalent with the scheduling of work tasks necessary in order to complete an output. Below a scheduling of actual production with the associated work effort is reproduced. This is an example of an actual work efforts used for the production of a unit of output ([http://en.wikipedia.org/wiki/Scheduling_\(production_processes\)](http://en.wikipedia.org/wiki/Scheduling_(production_processes))); see figure 4.1).

In this chapter the above chart can be seen as the labour efforts necessary to further the computation of a TM. Innovations are reductions of the labour efforts that are made possible; thanks to the discovery of a lower states TMs. Process innovations are the generation of new charts.

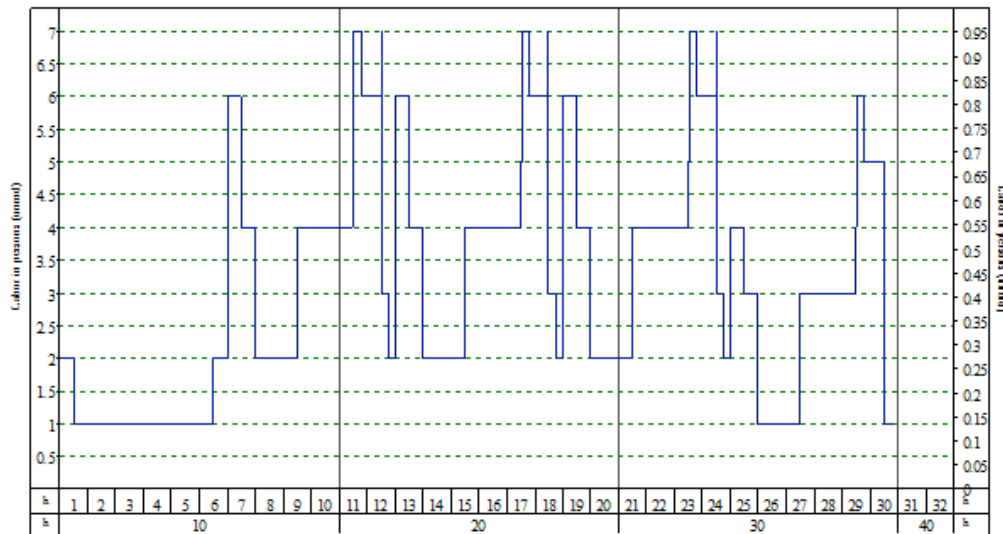


Figure 4.1 Labor vs Time Chart

4.2.2 Production Process and Process Innovation

Production of goods, or ideas or knowledge or innovation, is essentially a transformation of a set of inputs, according to a set of rules (i.e., *algorithm*), to produce the desired output. The crucial difference between the production and innovation process is that the production processes are *deterministic* in nature while the innovational processes are *not*. That is, for the production processes that are being carried out with the inputs and the set of rules, we know *a priori* that it will produce the desired output. In the case of the innovational process, it is impossible to know, *a priori*, if the process that is being carried out will halt by producing an innovation or not. When we conceptualize production as a process, in time, then the time to build characteristics of the production takes a central stage all the factors that affect it.

Preliminary assumptions:

- Events occur in discrete time;
- There are a finite number of firms (n) that produce an identical output (q);

- The output is perishable, i.e. once produced it cannot be stored;
- The output is consumed by the workers;
- During each time period the total output, $Q(t)$, is sold at a uniform price, $p(t)$.
- Workers do not save and use all their income to buy the output;
- Producers buy labour at a given wage, $w(t)$. Wages are paid before the labour is delivered – i.e. the firms either have previously accumulated funds or borrow funds (either from the workers themselves or from a bank);
- The completion of a production, according to the production *blueprint*, requires time - i.e. it requires several periods (ε_i) to be completed - and it requires work efforts which vary across periods (carry-on activity);
- The whole production knowledge and output description is encoded in terms of Turing Machines. Innovations and/or discoveries of new processes are in terms of the discoveries of halting TMs (as in Velupillai, 2000: Ch. 9, 2010: Ch. 10, and Zambelli, 2004, 2005).
- Each firm can decide how much to devote to the production of the final output or to R&D. The discovery by one firm is patented and cannot be used by other firms.
- There is an authority that decides:
 - on the interest rate and financial capital taxation rate. Hence (see below), the authority determines a net interest rate $r(t)$ which can be also negative because it is the net between interest payments and taxation payments;
 - on bankruptcy rules.

Firms decide production at time t , which will be completed at time $t + \varepsilon_i$. ε_i is the individual firm's *period of production*, which could be different with respect to the different firms and will depend on the specific research efforts and market condition. In this respect we follow the early mathematical formulation of Frisch (1933) and Kalecki (1935). Decisions of production (orders³) made at time $t - \varepsilon_i$ are planned to be delivered at time t .

$$Q^P(t) = \sum_{i=1}^n q_i^P(t) \text{ Aggregated actual real planned output (planned deliveries)} \quad (4.1)$$

³ In the context of this chapter production decisions and 'orders' are equivalent. We are keeping the term 'orders' so as to make a reference to what is described in Ch. 3 of the present dissertation.

$$O^P(t) = \sum_{i=1}^n o_i^P(t - \varepsilon_i) \text{ Aggregated real planned production (orders)} \quad (4.2)$$

Clearly we have that the firms planned output, $o_i^P(t - \varepsilon_i)$, will become actual output $q_i(t)$ only in the case in which the plans are fulfilled. Planned production may not become actual output simply because there may be a lack of financing or because the prospects have changed and what was expected to generate *gains* at time t is suddenly expected to generate *losses* (see Chapter 2 of the present dissertation) or because between the moment $t - \varepsilon_i$ and t there has been a process innovation that makes the adoption of a new process more convenient. Hence, the actual production realized or the deliveries (4.1a) will be

$$Q(t) = \sum_{i=1}^n q_i(t) \text{ Aggregated actual real output (actual deliveries)} \quad (4.1a)$$

During each period the financial wealth of a firm will be given by:

$$FW_i(t) = \text{Rev}_i(t) - \text{Exp}_i(t) + FW_i(t - 1) \quad (4.3)$$

Clearly there will be firms that will have a positive financial net worth (credit) and others that will have a negative financial net worth (debt). $\text{Rev}_i(t)$ are the revenues of firm i at time t and $\text{Exp}_i(t)$ are the expenditures at time t .

The revenues at time t are the revenues from sales of produced goods and the revenues due to interest payments from others

$$\text{Rev}_i(t) = \text{Rev}_i^q(t) + \text{Rev}_i^F(t) \quad (4.4)$$

While the expenditure at time t is given by payment of wages and the payment of interests to others

$$\text{Exp}_i(t) = \text{Exp}_i^q(t) + \text{Exp}_i^F(t) \quad (4.5)$$

Assuming the same interest rate for assets and liabilities we have the following ‘financial cost’:

$$\text{FC}_i(t) = \text{Rev}_i^F(t) - \text{Exp}_i^F(t) = r(t)FW_i(t - 1) \quad (4.6)$$

Clearly whether $FC_i(t)$ would be positive or negative depends on whether $r(t)$ is positive or negative and/or on whether $FW_i(t-1)$ is positive or negative.

The revenues from sales of the produced output is given by

$$\text{Rev}_i^q(t) = p(t)q_i(t) \quad (4.7)$$

In this model the expenditure for production purposes is expenditure in the only factor of production, which in this model is labour:

$$\text{Exp}_i^q(t) = w(t)L_i^{\text{tot}}(t) = w(t)(L_i^q(t) + L_i^{\text{R\&D}}(t)) \quad (4.8)$$

The output q_i can be produced by several alternative techniques which all include different working efforts. In essence there are different processes that imply different work efforts distributed in different sequences in between periods. A technique can be described by an array of time indexed labour inputs, the *carry-on-activity*, that are necessary in order to bring to completion a project, $\ell^{Z_i} = [\ell(0), \ell(1), \ell(2), \dots, \ell(\varepsilon^{Z_i})]$. Each firm has a production possibility sequence which is described by: $(\ell^{Z_i}, \varepsilon^{Z_i})$. This would be the firm production process as long as a process innovation will not take place. Therefore a decision of production made at time $t - \varepsilon_i^{Z_i}$, to produce a unit of the quantity q_i^t would require a proportional $\omega_i^{t-\varepsilon_i^{Z_i}}$ use of labour

$$\ell(t - \varepsilon_i^{Z_i}), \quad \ell(t - \varepsilon_i^{Z_i} + 1), \quad \ell(t - \varepsilon_i^{Z_i} + 2), \quad \dots, \quad \ell(t) \rightarrow 1 \quad (4.9)$$

$$\omega_i^{t-\varepsilon_i^{Z_i}} \ell(t - \varepsilon_i^{Z_i}), \quad \omega_i^{t-\varepsilon_i^{Z_i}} \ell(t - \varepsilon_i^{Z_i} + 1), \quad \omega_i^{t-\varepsilon_i^{Z_i}} \ell(t - \varepsilon_i^{Z_i} + 2), \quad \dots, \quad \omega_i^{t-\varepsilon_i^{Z_i}} \ell(t) \rightarrow q_i^t \quad (4.10)$$

The focus in this chapter is on *process innovation*. Process innovation can take the form of different work efforts per unit of time or of a reduction of the period of production, or both. In the sequel we will assume that the period of production is fixed. Hence a discovery is the discovery of new Turing Machines that implies a reduction in the work efforts per unit of time.

The discovery can take place if a firm invests in R&D. A discovery is the discovery of a new *halting* TM. Each period the firm i makes a decision of production $o_i(t)$ which implies a completion at $t + \varepsilon_i^z$. At every given point of time, the firm decides what percentage of its labour force is to be employed for the new and on-going projects and for conducting new R&D activities. The total labour demand of firm i at time (t) is given by

$$L_i^{tot}(t) = L_i^q(t) + L_i^{R\&D}(t) = \sum_{j=0}^{\varepsilon_i^z} \omega_i^z(t - \varepsilon_i^z + j) \ell_i^z(j) + L_i^{R\&D}(t) \quad (4.11)$$

The *total* labour demand will be given by:

$$L^D(t) = \sum_{i=1}^n L_i^{tot}(t) \quad (4.12)$$

Assuming that the labour supply is inelastic to wages we have: $L^S(t) = L^D(t)$.

The labour employed by the firms at every given point of time depends on the revenue obtained from the sale of output commodities. If the revenue generated is less than the cost incurred (i.e. amount of wages paid for the labour), then the financial imbalance will reflect on the new projects, and in some cases even the on-going projects may have to be truncated due to shortage of credit or external financial resources.

The total output will be given by:

$$Q(t) = \sum_{i=1}^n q_i(t) \quad (4.13)$$

The output $Q(t)$ is not necessarily equal to what is planned, $Q^P(t)$, as in equation (4.1).

The reason is that during production the firm may interrupt the production, or a process innovation may change the time to delivery of the original decision. Given our assumptions we have the following market clearing price determination:

$$p(t) = \frac{w(t)L^D(t)}{Q(t)} \quad (4.14)$$

Here we assume also that the process innovation can be patented and hence the innovation is associated to the individual firm. Obviously for a given level of employment the firm is confronted with a trade-off between deciding to use the labour force in the production of the final output or in R&D activity. Both variables $L^D(t)$ and $Q(t)$ are the result of past decisions made by the different n firms and hence they all depend on the level of technological progress, on the individual firm's access to technology. The natural and unavoidable asynchronicity between the moment in which a decision of production is taken and the moment in which the product is completed (i.e. it is delivered to the market) makes it impossible for the firms to know whether their decisions will determine a positive or a negative cash flow – the whole will depend on the overall discoveries occurring during the period, the financial and taxation conditions, the employment levels and the taxation rates, and so on. Furthermore, the model closure will require a specification of at the firm level of the decisions of production, the allocation of resources to R&D. Once the closure has been provided the dynamics of the model can be studied through repeated simulations. Simulation will be parametric. This means that by changing some values of the parameters different dynamic evolution will be generated. Once the simulation is started our approach requires that no stochastic element is introduced, ever.

First we will study the dynamics of the model through several simulation runs and with policy variables unchanged.

Second we will run the same simulations as above, but with changed policy variables. That is with different policy rules for:

- the interest rate and financial capital taxation rate, $r(t)$;
- bankruptcy rules.

Furthermore, please note that in the context of this chapter credit is created exogenously.

We have that $\sum_{i=1}^n FW_i(t) = 0$ and that $\sum_{i=1}^n FC_i(t) = 0$.

The total stock of credit money is then given by:

$$M(t) = \sum_{i=1}^n \begin{cases} FW_i(t) & \text{if } FW_i(t) > 0 \\ 0 & \text{if } FW_i(t) \leq 0 \end{cases} \quad (4.15)$$

Clearly the policy variable $r(t)$ (which is both an interest rate and a capital tax rate) can influence the distribution of financial wealth and will have an effect on the dynamic evolution and on the innovation process (carry-on-activity and so on) and on the variables such as aggregate output and $M(t)$.

4.3 Simulations

The above model can be studied through simulations. Here we are assuming that the common feature of the n firms is the production of the same output, but that the technological innovations and the organizational structure of the firms are different and their development and economic survival depends on the activity of the R&D department. Clearly the firm has, on one hand, to complete production, and on the other the firm has to invest in R&D. Investment in R&D is investment in the future and the revenues from this activity are highly uncertain because they will depend not only on whether the R&D activity is successful, but also on what the market conditions and the economic state of the firm will be in the future, i.e. at the moment in which the discovery will be made. The market conditions would depend on the different strategies of the n firms and on the discoveries. Firms with low investment in R&D will have, in the short run, better market performance, but it is not said at all that the same firm will not have high R&D perfor-

mance in the future. Clearly, the higher the revenues from sales of the output the higher would be in absolute terms the *capacity* to invest in R&D. On the contrary an aggressive firm investing most of its resources in R&D might end up doing very little R&D simply because of lack of revenues of the sales of the physical output.

Having the above example in mind one can speculate on whether it would be advisable to subsidise the activity of the more aggressive firm, which would almost certainly suffer in the short run. In the context of our model a subsidy requires financing through taxation. In order to focus on some important features, we will here assume that the nominal wage rate is fixed and that employment is also fixed. Labour income is also *not* taxed. What we have is that a subsidy does in fact imply a redistribution of purchasing power among the firms. The transfer from profit making firms, which accumulate financial wealth, can be made with a negative interest rate (that is the profit-capital tax rate is above the monetary interest rate) and assuming that at first the losses will be made by the firms that invest in R&D, this mechanism should allow for the detection of new techniques or better organizational structures. On the contrary a reinforcement of the positive economic performance can be obtained with a positive interest rate (that is when the monetary interest rate is above the profit-capital tax rate). Figure 4.2 reports three different aggregate evolutions of output whose difference is to be found in the three different exogenous tax-interest rates and *ceteris paribus*.

In our model an interest rate value different from zero implies a redistribution of financial wealth between the different firms. Given that the financial wealth is not accumulated, but is used to employ labour, different levels of financial wealth would imply different capacities to employ labour and hence different amounts of labour force to be used in the R&D department. The n firms are ordered in terms of their propensities to invest in R&D and it is not said that the firms having high relative investments in the R&D

departments will be the ones most successful. The difference in evolution reported in the

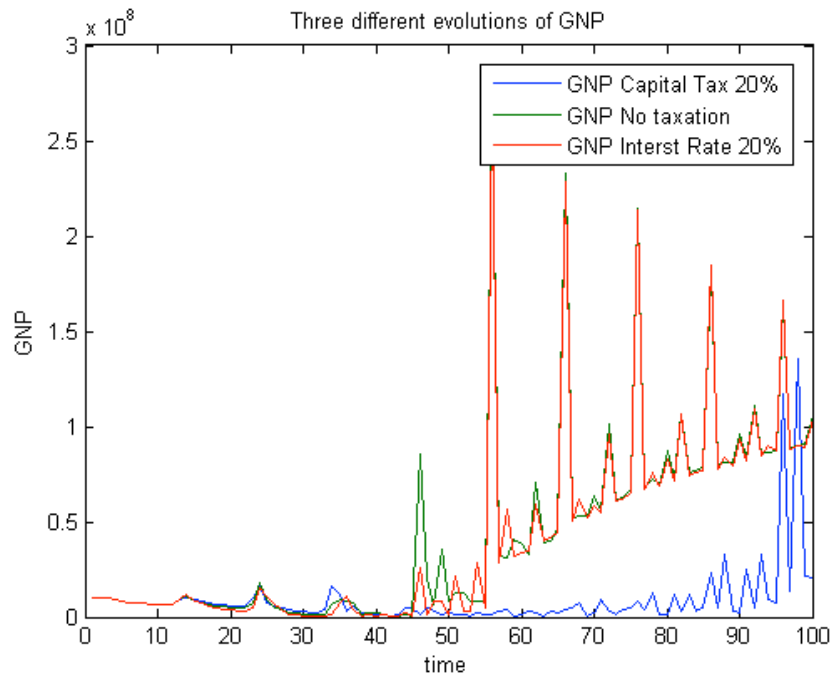


Figure 4.2

Figures is to be explained by the fact that the firm that may have implemented a very important innovation goes bankrupt before being allowed to innovate.

Another interesting feature of our experiments is to be found in the unavoidable short-run fall of output which is a consequence of allocating resources from physical production to production of knowledge (R&D). But when the firms’ R&D discovers new information bit strings that reduce the algorithmic complexity of the production blueprint by several orders of magnitude, we observe that the output of the firms increases drastically. As the innovative firm cannot save a portion of its profit for the next time period, the excess money is then reinvested in new projects and R&D activities. Therefore, when that new project, with innovative *carry-on* activity function, comes to completion there will be a significant increase in the output, but as the economic system as a whole adapts over time and the output is smoothed out until a new discovery or innovation is made.

This behaviour captures the disruptive nature of innovation and its effects on production, labour and money.

It is interesting to note that when the economy decides to employ a portion of the production labour for R&D activities, the GNP decreases thus increasing the prices of commodities. The firms that do not carry out R&D make profit and, with the profits, the firm employs more labour for production. The scenario changes only when the R&D firms start to innovate, thus reducing the cost of production, while the cost of production of the firm with no R&D remains the same.

Figure 4.3 reports a typical dynamic evolution that follows the decision to invest in R&D. Without R&D the output would have continued as a straight line, but the divergence of resources towards R&D does decrease production and hence total sales. It is only after a transient period that, thanks to the increase in productivity, we observe an increase in total output (See figure 4.4).

In our model the knowledge lost when bankruptcy occurs cannot be transferred

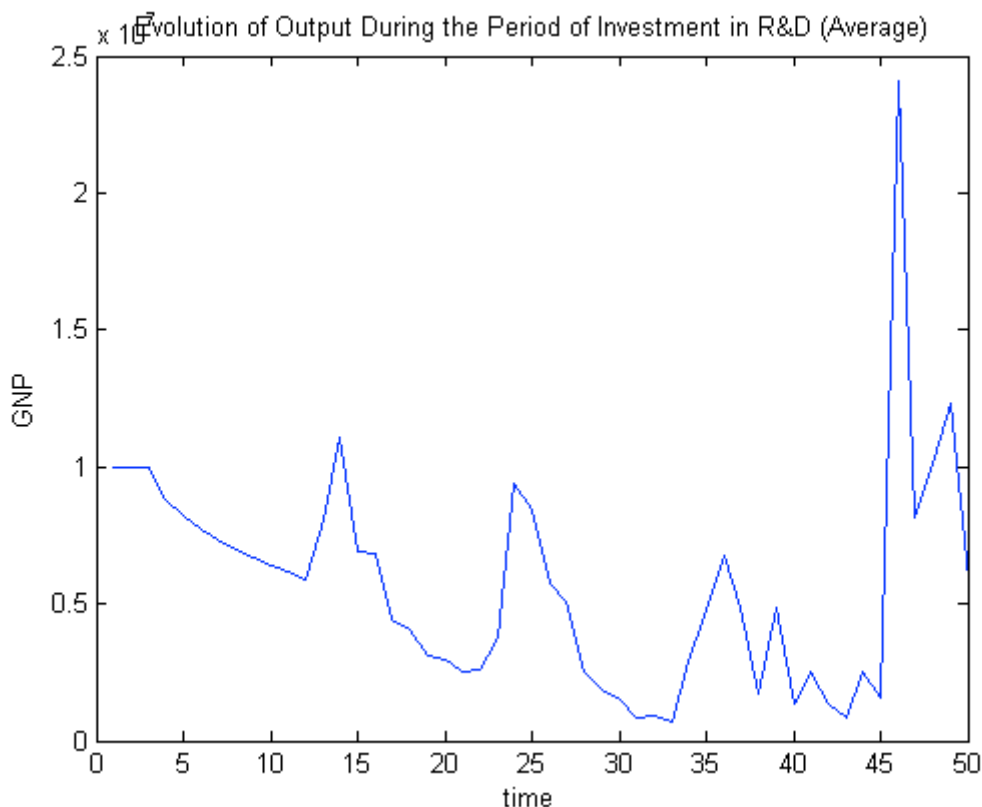


Figure 4.3

asa bonus to the other firms, it is simply lost. Another leakage is determined when the firm is forced to cut down or truncate a previously started production plan. The work effort put into past projects, when not brought to completion, is lost. The competition among the different firms, the drive to find better processes and the specific competition of our model (captured by market clearing conditions) leads in almost all cases to the sur-

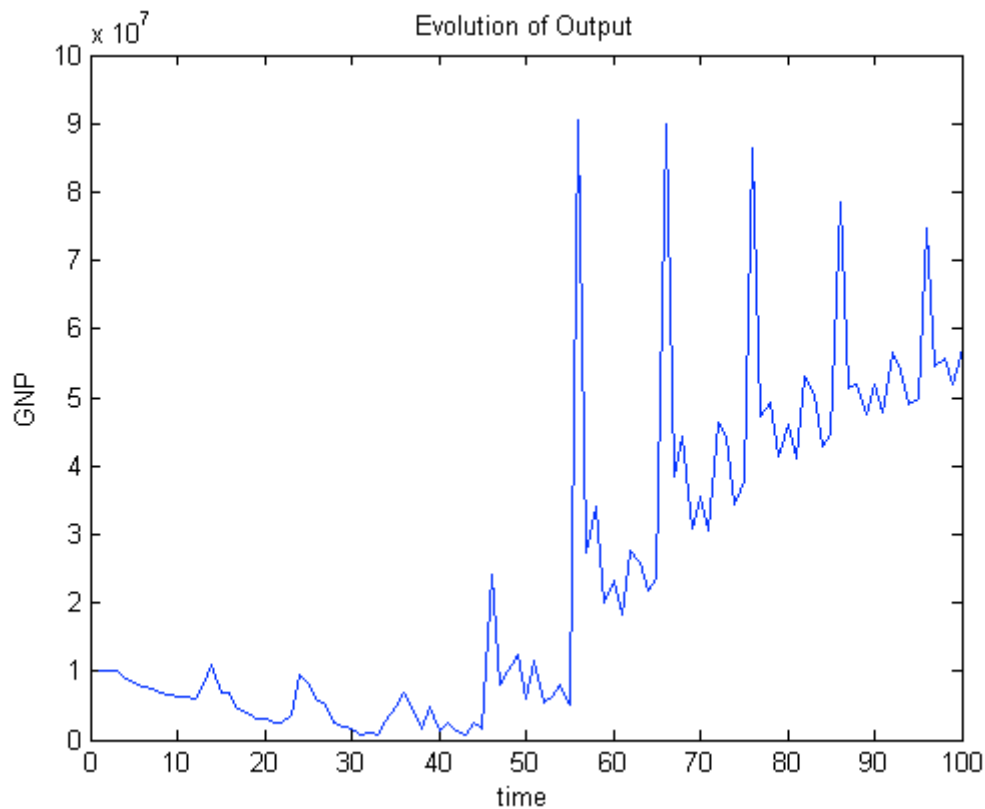


Figure 4.4

vival of just one firm that would end up employing the whole labour. Given that we have only one output which is produced and that the firms and workers have access to the same market this result is not surprising at all.

Figures 4.2 to 4.4 show different evolutions in terms of different interest rates.

In Figure 4.5 we report a graph where statistics concerning different performances of aggregate output are collected as a function of the interest rate $r(t)$ (from -20% to $+20\%$). The graph reports the average value of the output for 100 different runs where the

only different is the TMs space. As can be seen from the graph we can conclude, in the context of our model that the policy of transferring resources from profit making firms to loss making firms with redistribution through taxation (negative interest rate) is not rewarding. This is not an obvious result. We would have expected the contrary. Clearly during the period in which a firm invests in R&D the firm is likely to make lower revenues from sales and hence lower resources would be available for investment in R&D. Transferring resources towards these loss making firms is a policy that should allow faster discoveries. But our result indicates that this is not the case. Positive interest rates imply that firms that are incurring into losses have to borrow from the profit making firms. In the context of our model, it turns out that a trade-off between resources to be devoted to production of the final good and to R&D is advisable. What our results indicate is that the support to profit making firms allows for a sort of Schumpeterian mechanism in which the firms that do innovate first, but that do at the same time invest in production, are those that are most successful. In our case this leads to an increase also of the total output. As Figure 4.4 indicates this type of mechanism implies first a drop in production (what Aghion and Howitt would, maybe erroneously, call destruction) and subsequently an increase of the output. The result of Figure 4.5 indicate that after 100 periods the highest average production is to be associated with interest rates higher than 6%.

Another very interesting result can be derived from studying the number of times in which a firm has survived. We are working with 10 firms characterized by their R&D intensity going from 0 R&D to 45 % of the employed labour. Given the truly and deep uncertainty of the innovation process captured by our characterization of knowledge in terms of the TM and to the importance of the market conditions present at the moment in which a discovery is made, it would be interesting to have statistics concerning the per-

formance of the different firms, i.e a measure of the times in which the different firms

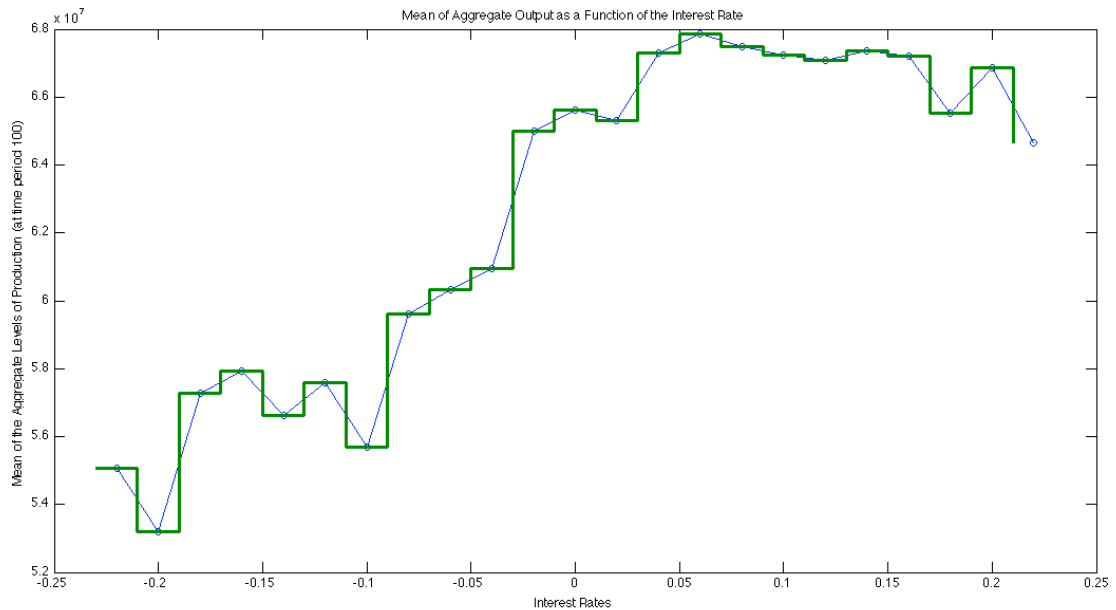


Figure 4.5

would turn out to be successful. Or, equivalently, a statistic concerning the number of times in which a firm would go bankrupt.

We have found that in our model the “strategy” which turns out to be the most successful is to invest around 15% of the available labour efforts into R&D.

It should be noted that with a positive rate of interest the firms that decide to carry out more R&D but fail to innovate goes bankrupt quickly while the firm with no R&D activities survives for a longer period. In these situations, credit policy may play a vital role in determining the survivability of the R&D firms. But having a negative rate of interest may reduce the aggregate output of the economy so it would be interesting to further investigate the dynamics of the economy when more credit is introduced within the economic system, along with various other viability and bankruptcy mechanisms. The simulations show how an economy evolves in disequilibrium and it would be a very interesting to further enhance and investigate the model for different policy parameters.

4.4 Conclusions

Our scope has been that of investigating how to make endogenous the carry-on-activity function. What we have elaborated on is the observation that process innovation is nothing else than a change (improvement) of the carry-on-activity function. The model of section 4.2.2 has very interesting characteristics among which is the fact that the distinction between the moment in which a discovery is made, the moment in which the investment decisions are made and the way in which these decisions of production are brought to completion have all relevance for the determination of the individual firms and of the market conditions which in turn do influence the formation of the carry-on-activity function. Frisch (1933) and Kalecki (1935) did operate with constant and exogenously given carry-on-activity functions – and we have shown how to remove that feature.

Our approach may have some relevance also for modern (real) business cycles, Kydland-Prescott-style, for endogenous growth models, Romer-style or for the creative-destruction models of the Aghion-Howitt type. In all these works the issue that it does take time to build and that production has to be seen as a process is not discussed in a meaningful matter. In all these models technological progress is just a simple and immediate change of the Cobb-Douglas production function (or minor variants of it).

Clearly, what we have presented here is an embryonic model and improvements can be made. In particular it would be appropriate to relax the assumption of full employment of labour resources and it would be particularly interesting to consider a model with multiple products where innovations are to be seen not only as process innovations, but also as product innovations.

The whole exercise requires keeping in mind the richness of the TM's equivalent computations and discoveries.

References

- AMENDOLA, M. AND GAFFARD, J. (1998): *Out of Equilibrium*, Oxford University Press, Oxford.
- (2006): *The Market Way to Riches: Behind the Myth*, Edward Elgar, Cheltenham, UK ; Northampton, MA.
- CHAITIN, G. (1995): ‘Foreword’, in Calude C.: *Information and Randomness: An Algorithmic Perspective*, Springer, Berlin, pp. IX–X.
- (2010): ‘The Information Economy’, in Zambelli S. (ed.) *Computable, Constructive and Behavioural Economic Dynamics: Essays in honour of Kumaraswamy (Vela) Velupillai*, Routledge, pp. 73-78.
- FRISCH, R. (1933): Propagation and Impulse Problems in Dynamic Economics, pp. 171-205, in: *Economic Essays in honour of Gustav Cassel*, George Allen & Unwin Ltd., Museum Street, London.
- GOODWIN, R.M. (1951): The Nonlinear Accelerator and the Persistence of Business Cycles, *Econometrica*, Vol. 19, #1, January, pp. 1-17.
- HICKS, J. (1973): *Capital and Time: A Neo-Austrian Theory*, Clarendon Press, Oxford.
- KALECKI, M. (1935): A Macrodynamic Theory of Business Cycles, *Econometrica*, Vol. 3, #3, July, pp.327-344.
- KYDLAND, F. AND E.C. PRESCOTT (1982): Time to Build and Aggregate Fluctuations, *Econometrica*, Vol. 50, #6, November, pp.1345-1370.
- ROMER, P. (1986): ‘Increasing Returns and Long Run Growth’, *Journal of Political Economy*, 94, pp. 1002–37.
- (1990): ‘Endogenous Technological Change’, *Journal of Political Economy*, 98, pp. S71–S102.
- (1993): ‘Two strategies for economic development: using ideas and producing ideas’, in Proceedings of the World Bank Annual Conference on Development Economics, IBRD, World Bank, Washington, DC.
- TINBERGEN, J. (1931): Ein Schifffbauzyklus, *Weltwirtschaftliches Archiv*, Vol. 34, Issue 1, pp.152-164.
- VELUPILLAI, K. VELA. (2000): *Computable Economics*, Oxford University Press.

- (2010): *Computable Foundations for Economics*, Routledge, Oxon.
- ZAMBELLI, S. (2004): 'Production of Ideas by Means of Ideas: a Turing Machine Metaphor', *Metroeconomica*, 55 (2–3), pp. 155–79.
- (2005): 'Computable Knowledge and Undecidability: A Turing Machine Metaphor', in Velupillai K. Vela (ed.) *Computability, Complexity and Constructivity in Economic Analysis*, Blackwell, pp. 233–63.

Chapter 5

Conclusion

Modelling and understanding the evolutionary dynamics of economic systems, during out-of-equilibrium situations, is the broad theme of this dissertation. In particular, emphasis has been placed on the limitations of orthodox theory and the importance of ‘time-to-build’ framework for modelling the traverse. Traverse is a path taken by the economic system taken to move from one state to another. This process of traverse is essentially a disequilibrium process. The orthodox theory, which is based on the concept of equilibrium, is ineffective in modelling this dynamic process and so we investigate the possible tools by which we can analyse the disequilibrium process in a more insightful manner. In the first chapter of the dissertation, we have discussed in detail the origins and pioneering works in traverse analysis in detail. Moreover, we have investigated the different paths taken by the researchers, who further developed these theories, and discussed the limitations of their approaches. ‘time-to-build’ framework not only plays a vital role in the models of traverse, but also plays pivotal role in the macro-dynamics models. In this dissertation, we have undertaken studies to further develop the ‘time-to-build’ framework, by removing some of its strict assumptions and to make it more tractable for modelling macro-dynamics.

In chapter 2, we have taken up the Hicksian Neo-Austrian model and analysed the importance of crucial factors - such as - production, labour and monetary resources, within a dynamic environment. During disequilibrium, due to asynchronicity between demand and supply, the viability of the economic system will be at stake and so viability creating mechanisms are to be employed in order for the system to sustain. Having explained the importance of viability conditions, we trace back and discuss the various notions of viability that are used in the economic literature. The origin of a formal definition of viability was traced back to the work of Hawkins and Simon. Even though Hawkins and Simon first formalized the notion of viability their definition captures only a limited

aspect of this multidimensional phenomenon. It is only in Sraffa's magnum opus that we find a deep and enlightening capsulation of the notion of viability. Sraffa's notion of *viability of a system* is analogous of *survival of a system*; therefore his definition captures all the aspects of viability. Though the viability aspects of the Sraffian system have been widely researched, similar studies have not been carried out to analyse the viability aspects of Hicks' Neo-Austrian 'time-to-build' system. So, we have analysed the viability aspects of a Neo-Austrian model by developing viability conditions for its three main structures - production, labour, and monetary structures.

To demonstrate the importance of viability creating mechanisms, we have simulated the Neo-Austrian model, with and without viability conditions, to show the different possible paths that a transitional economy can choose in order to reach the desired state. In the absence of viability creating mechanisms, the economic system fails to maintain a positive capital value of the production processes and eventually fails to sustain. On the other hand, when the viability creating production, labour and monetary structures are introduced, the system is able to sustain and evolve for a longer time. The results emphasize the need for organizations to viably create and manage its economic structures during such disequilibrium. Moreover, the results also illustrate the importance of viability creating mechanisms that are to be employed in order to reduce the bottlenecks and coordination problems that might spring up during such indeterminate times. We have also shown a case to illustrate how a non-viable system can be brought to a nearly viable state by using the concept of near-decomposability. As a policy maker, one can decompose (or nearly decompose) the sectors in such a way that the non-self-replacing sectors impact on the other sectors can be reduced and help the crucial sectors sustain in a viable manner. The nearly viable system can sustain in a short run. But, unless the viability creating mechanisms and/or new innovations are incorporated, the system will fail to sustain in a long

run. The main contributions of the second chapter are to trace the notions of viability and to develop the viability conditions for a Neo-Austrian model. Moreover, the concept of near decomposability and approximate viability has been explored to shed light on different ways by which the system can be brought to a viable and self-replacing state. Further analyses can be carried out to study various viability creating mechanisms that can be employed within an evolving economic system, which would enable the system to viably traverse the dynamic disequilibrium trajectory to reach a desired new state or equilibrium.

In the third chapter, we have discussed the importance of the time-to-build tradition in modelling macro-dynamics and in particular business cycle theory. The use of ‘time-to-build’ function in modelling business cycles can be traced back to the classic works of Tinbergen, Frisch, Kalecki, Goodwin, and Kydland and Prescott. In this chapter, we have analysed how the ‘time-to-build’ framework enters in their linear or non-linear difference differential systems and how different mathematical tools were used in modelling economic fluctuations. We also analyse that the different kinds of tools that the mathematical economists used for modelling business cycles. For example, we have shown that Frisch’s difference-differential system was full of nonlinearities but he approximated to make his nonlinear model a linear one. Similarly, Kalecki’s original model is a difference equation system but he uses a differential form, for encapsulating the ‘time-to-build’ framework of capital accumulation, to make his difference equation system to a differential-difference type system. We have also shown that if Goodwin had not approximated his nonlinear difference-differential system to a nonlinear differential system, by taking first two terms of the Taylor’s Series of the two leading terms, he would have got a system that would have shown rich dynamics and dependence on initial conditions. We have further explored the importance of ‘time-to-build’ tradition in modelling business cycle and emphasized the need of relaxing some of the strict assumptions that underlay these

models. As soon as we start to relax some of the assumptions, we notice that the nonlinearities, which are intrinsic in these models, take the centre stage of the analysis and the model becomes highly sensitive to initial conditions. To demonstrate the rich dynamics of nonlinear models, we took Goodwin's nonlinear model and simulated it to show how the dynamics of the model changes, depending upon the initial conditions, when more terms are taken into account in the Taylor's series approximation.

As a future work, it would be interesting to investigate why the modeller chose to use a particular kind of structure or mathematical tools for modelling business cycles. Further investigating the above-mentioned aspects of these models, while relaxing their strict assumptions, would enable the researcher to envisage and encapsulate their rich dynamics and also to model the economic phenomena in a more insightful manner.

In chapter 4, we have focused on the 'time-to-build' framework and further developed the framework by endogenizing technology and innovation using Turing Machine metaphor. Hicks, in *Capital and Time*, while expanding his simple model, presents a detailed discussion of the many possible ways in which the Neo-Austrian 'time-to-build' model can be enhanced. Taking up this task, Amendola and Gaffard improved the Neo-Austrian model by making it more comprehensive and suitable for studying the dynamics of the traverse in a more tractable way. But their 'time-to-build' model, like many other 'time-to-build' models, fall short of explaining how the technological change actually emerges within the system because they assume the technological change as exogenously given or treat it as being stochastic or probabilistic.

In order to remove this *ad hoc* assumption we have used the Turing Machine metaphor to model the innovational process in a non-stochastic way that would encapsulate the uncertainties of technological change and innovations in a more insightful manner. As every production process, let it be production of commodities or innovations, is quintes-

essentially a computational process. Therefore, by Church-Turing thesis, for every computing process there exists at least one Turing Machine that would perform the same task. Moreover, for a given algorithm, when fed into a Turing Machine, it is impossible to predict *a priori* if the Turing Machine will halt or not. It is the famous *Unsolvability of the Halting Problem for Turing Machines*. If we model the R&D processes as Turing Machine processes, due to the Non-Halting problem for Turing Machines, it would be impossible to predict *a priori* if the R&D activity that is being carried out will stop/halt finding an innovation or not. By modelling the R&D processes as Turing Machine processes we would be able to model the intrinsic uncertainty of the R&D activities, in a more insightful and non-stochastic manner. In this chapter, we have taken a ‘time-to-build’ model and modelled the R&D processes in terms of Turing Machines to study the dynamics of the model. This enhanced ‘time-to-build’ model, with endogenous evolution of technology, is then simulated for various different bankruptcy policies and interest rates, in order, to study the traverses of the evolving economic system. The main contribution of this chapter is to endogenize innovations within a ‘*time-to-build*’ framework so that the model captures the dynamics of the economic phenomena in the Schumpeterian way, without *ad-hoceries*. The model simulation demonstrates rich varieties of possible paths a system can traverse during disequilibrium, depending upon the production and R&D strategies and other control variables – such as different interest rates policies and bankruptcy policies. This enhanced and more tractable ‘time-to-build’ model can then be applied to existing models where the ‘time-to-build’ framework plays a pivotal role.

Economists *taming* the complex dynamic economic system should develop and employ new tools and techniques to better capture the economic phenomena, with its intrinsic nonlinearities and indeterminacies. In this dissertation, we have presented three main chapters discussing the importance of ‘time-to-build’ framework for disequilibrium

analysis and emphasizing the need to relax the strict assumptions that underlay this framework. We illustrate some of the many different ways by which we can harness the richness of computability and algorithmic complexity theory in modelling evolutionary economic systems in a more tractable manner. In this way, we are able to provide new tools to model the out-of-equilibrium economies in a more insightful and phenomenological spirit. The need for shifting the focus from equilibrium macroeconomics to *phenomenological* macroeconomics is the credo of this dissertation.