PhD oral defense
26th March 2012

Spacetime metrology
with LISA Pathfinder

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LISA Pathfinder

- Partnership of many universities worldwide
- Univ. of Trento plays the role of Principal Investigator
- Currently in the final integration
- Planned to flight in the next years

SpaceCraft

Housing courtesy of

Test Mass

Optical Bench

Vibration tests
Outline of the talk

In the framework of this ESA mission, we talk about:

- **Theoretical characterization** of the LISA arm *without/with* noise
- **Dynamical modeling** for LISA Pathfinder, closed-loop equations of motion
- **Data analysis**: simulation and analysis of some experiments, system calibration, estimation of the acceleration noise, ...
LISA, a space-borne GW detector

LISA is the proposed ESA-NASA mission for GW astrophysics in 0.1 mHz – 0.1 Hz

- astrophysical sources at cosmological distances
- merger of SMBHs in galactic nuclei
- galactic binaries (spatially resolved and unresolved)
- EMRIs
- GW cosmic background radiation

LISA measures the relative velocities between the Test Masses (TM) as Doppler frequency shifts

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LISA Pathfinder, in-flight test of geodesic motion

- Down-scaled version of LISA arm ~40 cm
- In-flight test of the LISA hardware (sensors, actuators and controllers)
- Prove geodesic motion of TMs to within $3 \times 10^{-14} \text{ ms}^{-2}/\text{VHz} @ 1 \text{mHz}$ differential acceleration noise requirement
- Optically track the relative motion to within $9 \times 10^{-12} \text{ m}/\text{VHz} @ 1 \text{mHz}$ differential displacement noise
LISA Pathfinder, in-flight test of geodesic motion

Residual acceleration noise

~ 1 order of magnitude relaxation

LPF requirement

LISA requirement

Frequency [Hz]

\[ \sqrt{\text{PSD}} \text{[m s}^{-2} \text{ Hz}^{-1/2}] \]

\[ 10^{-11} \]

\[ 10^{-12} \]

\[ 10^{-13} \]

\[ 10^{-14} \]

\[ 10^{-15} \]

\[ 10^{-4} \]

\[ 10^{-3} \]

\[ 10^{-2} \]

\[ 10^{-1} \]

\[ 9 \times 10^{-12} \text{ m/Hz} \]

\[ 3 \times 10^{-14} \text{ ms}^{-2}/\text{Hz} \]
Spacetime metrology without noise

How does the LISA arm work?

It is well-known in General Relativity that the Doppler frequency shift measured between an emitter sending photons to a receiver – we name it Doppler link – is

\[ \delta \omega_{e \rightarrow r} = k_\mu \Delta v^\mu_{e \rightarrow r} \]

The differential operator implementing the parallel transport of the emitter 4-velocity along the photon geodesic can be defined as

\[ \Delta v^\mu_{e \rightarrow r} = v^\mu_r (x^\alpha_r) - v^\mu_e (x^\alpha_e \xrightarrow{\text{parallel}} x^\alpha_r) \]

and one can show that it is given by

\[ \Delta v^\mu_{e \rightarrow r} = \delta v^\mu_{e \rightarrow r} + \int_\gamma \Gamma^\mu_{\alpha \beta} v^\alpha_e \, dx^\beta \]

The Doppler link measures

- relative velocity between the particles (without parallel transport)
- the integral of the affine connection (i.e., curvature + gauge effects)
Doppler link as differential accelerometer

For low velocities and *along the optical axis*, differentiating the Doppler link it turns out

\[ \dot{\omega}_{e \rightarrow r} = k_\mu \Delta v^\mu_{e \rightarrow r} \]

\[ \dot{\omega}_{e \rightarrow r} = k_\mu \delta a^\mu_{e \rightarrow r} + k_\mu \int_\gamma \frac{d\Gamma^\mu_{\alpha\beta}}{d\tau} v^\alpha_e \, dx^\beta \]

The Doppler link (and the LISA arm) can be reformulated as a

differential time-delayed accelerometer

measuring:

- The spacetime curvature along the beam
  \[ \delta a_R = c^2 \int_\gamma R_{0110} \, dx \]

- Parasitic differential accelerations
  \[ k_\mu \delta a^\mu_{e \rightarrow r} \]

- Gauge “non-inertial” effects
  \[ \delta a_{\text{gauge}} = \frac{1}{2} c^2 \int_\gamma (h_{00,00} + 2h_{01,00} + 2h_{01,01} - h_{00,11}) \, dx \]
Doppler response to GWs in the weak field limit

To prove that this reasoning is correct, it is possible to evaluate the Doppler response to GWs in the TT gauge

\[
h_{\mu\nu} = \begin{pmatrix}
0 & \ldots & h_+ & \ldots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
h_+ & \ldots & 0 & \ldots & 0
\end{pmatrix}
\]

obtaining the standard result already found in literature

\[
\frac{\delta \omega_h}{\omega_c} = \frac{1}{2} \left[ h(t) - h(t - \delta x/c) \right]
\]

where \( h \) is the “convolution” with the directional sensitivities and the GW polarizations

\[
h(t, \theta, \phi) = \xi_+ (\theta, \phi) h_+(t) + \xi_\times (\theta, \phi) h_\times(t)
\]

The parallel transport induces a **time delay** to the time of the emitter
Noise sources within the LISA arm

- LPF is a down-scaled version of the LISA arm, which is a sequence of 3 measurements:

\[
\frac{\delta \omega_{2\rightarrow 1}}{\omega_e} = \frac{1}{c} \cdot \left( \delta v_{SC} + v_{TM2}^{(SC)} - v_{TM1}^{(SC)} \right)
\]

- In LISA there are 3 main noise sources:
  - laser frequency noise from armlength imbalances, compensated by TDI, not present in LPF
  - parasitic differential accelerations, characterized with LPF
  - sensing noise (photodiode readout), characterized with LPF
Laser frequency noise

In space, laser frequency noise cannot be suppressed as on-ground: arm length imbalances of a few percent produce an unsuppressed frequency noise of $10^{-13}/\text{VHz} \ (2 \times 10^{-7} \ \text{ms}^{-2}/\text{VHz}) @1\text{mHz}$

Doppler links

\[
\sigma_k(t) = s_k(t) + S_k(t) - s_p'[t - T_p^2[k]] \\
\sigma'_k(t) = s'_k(t) + S'_k(t) - s_p^2[t - T_p[k]]
\]

Time-Delay Interferometry (TDI) suppresses the frequency noise to within $10^{-20}/\text{VHz} \ (2 \times 10^{-15} \ \text{ms}^{-2}/\text{VHz}) @1\text{mHz}$

It is based upon linear combinations of properly time delayed Doppler links

\[
X_k(t) = \sigma'_k(t) + \sigma_{p^2[k]}(t - T_p^2[k]) + \sigma_k(t - 2T_p^2[k]) + \sigma'_p[k](t - T_p^2[k] - 2T_p[k]) \\
- [\sigma'_k(t) + \sigma_p[k](t - T_p^2[k]) + \sigma'_k(t - 2T_p^2[k]) + \sigma_{p^2[k]}(t - T_p[k] - 2T_p^2[k])]
\]
Total equivalent acceleration noise

- Differential forces (per unit mass) between the TMs, due to electromagnetics & self-gravity within the SC, $S_{n,\delta f/m}(\omega)$
- Sensing noise (readout, optical bench misalignments), $S_{n,\delta x}(\omega)$

We can express all terms as total equivalent acceleration noise

$$S_{n,\delta a}(\omega) = S_{n,\delta f/m}(\omega) + \omega^4 S_{n,\delta x}(\omega)$$

will be demonstrated by LPF to be within $3 \times 10^{-14}$ ms$^{-2}$/VHz @1mHz
Dynamics of fiducial points

- **Fiducial points** are the locations on TM surface where light reflects on.
- They do not coincide with the centers of mass.
- As the TMs are not pointlike, extended body dynamics couples with the differential measurement (cross-talk).

![Diagram of TM and SC connections](image)
LISA Technology Package

We switch to the real instrument...

15 control laws are set up to minimize:

**SC jitter and differential force disturbances**
## Degrees of freedom

### Science mode, along the optical axis $x$:

- TM$_1$ is in **free fall**
- the SC is forced to follow TM$_1$ through **thruster actuation**
- TM$_2$ is forced to follow TM$_1$ through **capacitive actuation**

<table>
<thead>
<tr>
<th>Coordinate</th>
<th>Control</th>
<th>Sensor</th>
<th>Actuator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>Drag-free</td>
<td>$o_{x_1} = IFO[x_1]$</td>
<td>FEEP</td>
</tr>
<tr>
<td>$y_1$</td>
<td>Drag-free</td>
<td>$o_{y_1} = GRS[y_1]$</td>
<td>FEEP</td>
</tr>
<tr>
<td>$z_1$</td>
<td>Drag-free</td>
<td>$o_{z_1} = GRS[z_1]$</td>
<td>FEEP</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>Drag-free</td>
<td>$o_{\theta_1} = GRS[\theta_1]$</td>
<td>FEEP</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>Elect. suspension</td>
<td>$o_{\eta_1} = IFO[\eta_1]$</td>
<td>GRS</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>Elect. suspension</td>
<td>$o_{\phi_1} = IFO[\phi_1]$</td>
<td>GRS</td>
</tr>
<tr>
<td>$x_2$</td>
<td>Elect. suspension</td>
<td>$o_{x_{12}} = IFO[x_{12}]$</td>
<td>GRS</td>
</tr>
<tr>
<td>$y_2$</td>
<td>Drag-free</td>
<td>$o_{y_2} = GRS[y_2]$</td>
<td>FEEP</td>
</tr>
<tr>
<td>$z_2$</td>
<td>Drag-free</td>
<td>$o_{z_2} = GRS[z_2]$</td>
<td>FEEP</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>Elect. suspension</td>
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<td>$\phi_2$</td>
<td>Elect. suspension</td>
<td>$o_{\phi_{12}} = IFO[\phi_{12}]$</td>
<td>GRS</td>
</tr>
<tr>
<td>$\theta_{SC}$</td>
<td>Attitude</td>
<td>$o_{\theta_{SC}} = ST[\theta_{SC}]$</td>
<td>GRS</td>
</tr>
<tr>
<td>$\eta_{SC}$</td>
<td>Attitude</td>
<td>$o_{\eta_{SC}} = ST[\eta_{SC}]$</td>
<td>GRS</td>
</tr>
<tr>
<td>$\phi_{SC}$</td>
<td>Attitude</td>
<td>$o_{\phi_{SC}} = ST[\phi_{SC}]$</td>
<td>GRS</td>
</tr>
</tbody>
</table>
Closed-loop dynamics

In LPF dynamics:

- the relative motion is expected to be within ~nm
- we neglect non-linear terms from optics and extended body Euler dynamics
- the forces can be expanded to linear terms

Since the linearity of the system, it follows that the equations of motion are linear, hence in frequency domain they can be expressed in matrix form (for many dofs)

\[
\ddot{x} = g(x, t) \Rightarrow \\
\ddot{x} = g_0 - kx + \ldots \Rightarrow \\
(d^2 / dt^2 + k)x = g_0
\]

Dynamics operator
Closed-loop dynamics

As said, the equations of motion are linear and can be expressed in matrix form

**Dynamics**
\[ Dq = g \]

**Sensing**
\[ o = Sq + o_n \]

**Control**
\[ g = f_n + A[f_i - C(o - o_i)] \]

"forces produce the motion"
"positions are sensed"
"noise, actuation and control forces"

The *generalized* equation of motion in the sensed coordinates
\[ \Delta o = f_n + DS^{-1}o_n + A(f_i + C o_i) \]

where the 2nd-order differential operator is defined by
\[ \Delta = DS^{-1} + AC \]
Dynamical model along the optical axis

- direct forces on TMs and SC
- force gradients
- sensing cross-talk
- actuation gains

guidance signals:
- reference signals that the controllers must follow ("the loops react")

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Dynamical model along the optical axis

The model along $x$ can be mapped to the matrix formalism...

\[
q = \begin{pmatrix} x_1 \\ x_{12} \end{pmatrix} \quad \quad S = \begin{pmatrix} 1 & 0 \\ S_{21} & 1 \end{pmatrix} \quad \quad o = \begin{pmatrix} o_1 \\ o_{12} \end{pmatrix}
\]

\[
s^2 x_1 + s^2 x_{SC} + \omega_1^2 x_1 + \Gamma_x (x_2 - x_1) = f_1 \\
+ s^2 x_2 + s^2 x_{SC} + \omega_2^2 x_2 - \Gamma_x (x_2 - x_1) = f_2 - C_{sus}(s) o_{12}
+ s^2 x_{SC} - \tilde{m}_1 \omega_1^2 x_1 - \tilde{m}_2 \omega_2^2 x_2 = f_{SC} + C_{df}(s) o_1
\]

\[
- \tilde{m}_1 f_1 - \tilde{m}_2 f_2
+ \tilde{m}_2 C_{sus}(s) o_{12}
\]
Two operators

The differential operator

\[ T_{o \rightarrow f} = \Delta \]

- estimates the \textbf{out-of-loop equivalent acceleration} from the sensed motion
- subtracts known force \textbf{couplings, control forces} and \textbf{cross-talk}
- subtracts \textbf{system transients}

The transfer operator

\[ T_{o_1 \rightarrow o} = \Delta^{-1} A C \]

- solves the \textbf{equation of motion} for applied control bias signals
- is employed for system calibration, i.e. \textbf{system identification}
Suppressing system transients

The dynamics of LPF (in the sensed coordinates) is described by the following linear differential equation

\[ \Delta o = f \]

- \[ \Delta o_s = f \]
- \[ \Delta o_t = 0 \]

**Particular solution (steady state):**
- depends on the applied forces
- can be solved in frequency domain

**Homogeneous solution (transient state):**
- depends on non-zero initial conditions
- is a combination of basis functions
  \[ o_t = \sum c_k \phi_k \]
Suppressing system transients

Applying the operator on the sensed coordinates, transients are suppressed in the estimated equivalent acceleration

\[ \Delta o = \Delta (o_s + o_t) = \Delta o_s + \sum_k c_k \Delta \phi_k = f \]

But, imperfections in the knowledge of the operator (imperfections in the system parameters) produce systematic errors in the recovered equivalent acceleration

\[ \delta f = \delta \Delta (o_s + o_t) \]

System identification helps in mitigating the effect of transients
Cross-talk from other degrees of freedom

Beyond the dynamics along the optical axis...

\[
o \approx o_0 + \delta o
\]

cross-talk from other DOFs

\[
o_0
\]

nominal dynamics along \(x\)

All operators can be expanded to first order as "imperfections" to the nominal dynamics along \(x\)

To first order, we consider only the cross-talk from a DOF to \(x\), and not between other DOFs.

Cross-talk equation of motion

\[
\Delta_0 \delta o \approx \delta T_{f_n \rightarrow f} f_n + \delta T_{o_n \rightarrow f} o_n + \delta T_{f_i \rightarrow f} f_i + \delta T_{o_i \rightarrow f} o_i
\]
Data analysis during the mission

The **Science and Technology Operations Centre (STOC)**:

- interface between the LTP team, science community and Mission Operations
- telecommands for strong scientific interface with the SpaceCraft
- quick-look data analysis
Data analysis during the mission

Data for this thesis were simulated with both analytical simplified models and the Off-line Simulation Environment – a simulator provided by ASTRIUM for ESA. It is realistic as it implements the same controllers, actuation algorithms and models a 3D dynamics with the couplings between all degrees of freedom.

During operational exercises, the simulator is employed to:
- check the mission timeline and the experiments
- validate the noise budget and models
- check the procedures: system identification, estimation of equivalent acceleration, etc.

Data were analyzed with the LTP Data Analysis Toolbox – an object-oriented Matlab environment for accountable and reproducible data analysis.
Multi-input/Multi-Output analysis

We are able to recover the information on the system by stimulating it along different inputs

In this work two experiments were considered allowing for the determination of the system:

Exp. 1: injection into drag-free loop
Exp. 2: injection into elect. suspension loop
LPF model along the optical axis

- For simulation and analysis, we employ a model along the optical axis.

- Transfer matrix (amplitude/phase) modeling the system response to applied control guidance signals.

"the spacecraft moves": drag-free loop  
"the second test mass moves": suspension loop cross-talk
LPF parameters along the optical axis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Typical value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1^2$</td>
<td>-1x10^{-6} s^{-2}</td>
<td>Parasitic stiffness constant: force gradient (per unit mass) between the TM and the SC</td>
</tr>
<tr>
<td>$\omega_{12}^2 = \omega_2^2 - \omega_1^2$</td>
<td>&lt;1x10^{-6} s^{-2}</td>
<td>Differential parasitic stiffness constant: force gradient (per unit mass) between the TMs</td>
</tr>
<tr>
<td>$S_{21}$</td>
<td>1x10^{-4}</td>
<td>sensing cross-talk from $o_1$ to $o_{12}$</td>
</tr>
<tr>
<td>$A_{\text{df}}$</td>
<td>1</td>
<td>thruster actuation gain: how forces on SC translate into real applied forces</td>
</tr>
<tr>
<td>$A_{\text{sus}}$</td>
<td>1</td>
<td>capacitive actuation gain: how forces on TM$_2$ translate into real applied forces</td>
</tr>
<tr>
<td>$\Delta t_1$</td>
<td>&lt;1 s</td>
<td>delay in the application of the $o_1$ control bias</td>
</tr>
<tr>
<td>$\Delta t_2$</td>
<td>&lt;1 s</td>
<td>delay in the application of the $o_{12}$ control bias</td>
</tr>
</tbody>
</table>
Equivalent acceleration noise

- at high frequency, dominated by sensing
- at low frequency, dominated by direct forces

Goal: prove the relevance of system identification for the correct assessment of the equivalent acceleration noise

<table>
<thead>
<tr>
<th>Diff. acc. source</th>
<th>$10^{-14}$ ms$^{-2}$/Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuation</td>
<td>0.75</td>
</tr>
<tr>
<td>Brownian</td>
<td>0.72</td>
</tr>
<tr>
<td>Magnetics</td>
<td>0.28</td>
</tr>
<tr>
<td>Stray voltages</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Estimation of equivalent acceleration noise

- System identification
- Parameters: $\omega_1^2, \omega_{12}^2, S_{21}, A_{df}$
- Optimal design
- Equivalent acceleration

Sensed relative motion: $O_1, O_{12}$

With system identification:

Without system identification:

- Diff. operator: $\Delta$

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Identification experiment #1

Exp. 1: injection of sine waves into $o_{i,1}$
- injection into $o_{i,1}$ produces thruster actuation
- $o_{12}$ shows dynamical cross-talk of a few $10^{-8}$ m
- allows for the identification of: $A_{df}$, $\omega_1^2$, $\Delta t_1$
Identification experiment #2

Exp. 2: injection of sine waves into $o_{i,12}$
- injection into $o_{i,1}$ produces capacitive actuation on $TM_2$
- $o_1$ shows negligible signal
- allows for the identification of: $A_{sus}$, $\omega_{12}^2$, $\Delta t_2$, $S_{21}$
Parameter estimation

Data production
- Noise run
  - $o_1$, $o_{12}$
  - Experiment 1
- $o_{i,1}$, $o_1$, $o_{12}$
  - Experiment 2

Data analysis
- Whitening filters
  - $w_1$, $w_{12}$

Modeling
- $\chi^2$
  - $T_{o_i \rightarrow o}$

Non-linear optimization:
- **preconditioned conjugate gradient search** explores the parameter space to large scales
- **derivative-free simplex** improves the numerical accuracy locally

Method extensively validated through Monte Carlo simulations

**Joint** (multi-experiment/multi-outputs) **log-likelihood** for the problem

$$
\chi^2(p) = \int \rho_r(\omega, p)^* S_n(\omega)^{-1} \rho_r(\omega, p) \, d\omega
$$

- **cross-PSD matrix**
- **residuals**

$$
\rho_r(\omega, p) = \rho_{exp}(\omega) - T_{o_i \rightarrow o}(\omega, p) \rho_{i}(\omega)
$$
Non-Gaussianities

Investigated the case of non-gaussianities (glitches) in the readout producing fat tails in the statistics (strong excess kurtosis)

Regularize the log-likelihood with a weighting function

$$\chi^2 = \sum_i \rho(r_{w,i})$$

$$\rho(r_{w,i}) = \begin{cases} 
  r_{w,i}^2, & \text{abs. deviation, log-normal distr.} \\
  |r_{w,i}|, & \text{mean squared dev. (aka, “ordinary least squares”), Gaussian distr.} \\
  \log(1 + r_{w,i}^2), & \text{log. deviation, Lorentzian distr.} 
\end{cases}$$

Better weighting for large deviations than squared and abs dev.
Non-Gaussianities

- $\chi^2$ regularizes toward $\sim 1$
- the parameter bias, in average, tends to $<3$
Under-performing actuators under-estimated couplings

Investigated the case of a miscalibrated LPF mission, in which:

- the TM couplings are stronger than one may expect
- the (thruster and capacitive) actuators have an appreciable loss of efficiency

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True</th>
<th>Best-fit</th>
<th>Guess</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1^2 [10^{-6} \text{ s}^{-2}]$</td>
<td>-3</td>
<td>-2.9998(2)</td>
<td>1.1</td>
</tr>
<tr>
<td>$\omega_{12}^2 [10^{-6} \text{ s}^{-2}]$</td>
<td>-2</td>
<td>-2.0000(1)</td>
<td>0.32</td>
</tr>
<tr>
<td>$S_{21} [10^{-3}]$</td>
<td>-1.5</td>
<td>-1.4998(1)</td>
<td>0.55</td>
</tr>
<tr>
<td>$A_{df}$</td>
<td>0.62</td>
<td>0.61994(8)</td>
<td>0.77</td>
</tr>
<tr>
<td>$A_{sus}$</td>
<td>0.6</td>
<td>0.599990(8)</td>
<td>1.3</td>
</tr>
<tr>
<td>$\Delta t_1 [\text{s}]$</td>
<td>0.6</td>
<td>0.6013(7)</td>
<td>1.8</td>
</tr>
<tr>
<td>$\Delta t_{12} [\text{s}]$</td>
<td>0.4</td>
<td>0.398(2)</td>
<td>0.95</td>
</tr>
</tbody>
</table>

The parameter bias reduces from $10^3$-$10^4 \sigma$ to <2 and the $\chi^2$ from $\sim 10^5$ to $\sim 1$
Under-performing actuators under-estimated couplings

Exp. 1 residuals

~2 orders of magnitude

Exp. 2 residuals

~3 orders of magnitude
Equivalent acceleration noise

Inaccuracies in the estimated system parameters produce **systematic errors** in recovered equivalent acceleration noise.

\[ \delta S_{n,f} \approx \delta \Delta S_{n,o} \Delta^* + \Delta S_{n,o} \delta \Delta^* \]

Perform an estimation of the equivalent acceleration noise in a miscalibrated LPF mission (as in the previous example):

- **without** a system identification (@ initial guess)
- **with** a system identification (@ best-fit)
- with the perfect knowledge of the system (@ true)
Equivalent acceleration noise

Thanks to system identification:
- the estimated acceleration recovers the “true” shape
- improvement of a factor ~2 around 50 mHz
- improvement of a factor ~4 around 0.4 mHz

\[ \Delta a \approx 1.5 \times 10^{-13} \text{ m s}^{-2} \text{ Hz}^{-1/2} \]
Equivalent acceleration noise

The agreement between estimated noise and models shows:

- the models explain the estimated acceleration spectra
- the accuracy of the noise generation

\[ \Delta a \approx 1.5 \times 10^{-13} \text{ m s}^{-2} \text{ Hz}^{-1/2} \]
Equivalent acceleration noise

The systematic errors are due to:
- 0.4 mHz, uncalibrated coupling forces ($\omega_1^2$ and $\omega_2^2$) and capacitive actuation ($A_{\text{sus}}$)
- 50 mHz, uncalibrated thruster actuation ($A_{\text{df}}$) and sensing noise ($S_{21}$)
Suppressing transients in the acceleration noise

As a consequence of changes of state and **non-zero initial conditions**, the presence of transients in the data is an expected behavior.

Transients last for about 2 hours in $\omega_{12}$
Suppressing transients in the acceleration noise

Comparison:
(i) between the acceleration estimated at the transitory (first $3\times10^4$ s) and steady state;
(ii) between the acceleration estimated without and with system identification

System identification helps in mitigating the transitory in the estimated acceleration noise

Dashed: steady state
Solid: transitory
Blue: without sys. ident.
Red: with sys. ident.

Data simulated with ESA simulator: suppression is not perfect since the limitation of the model
Design of optimal experiments

Goal: find optimal experiment designs allowing for an optimal determination of the system parameters

The LPF experiments can be optimized within the system constraints:
- shape of the input signals
- sensing range of the interferometer 100 µm
- thruster authority 100 µN
- capacitive authority 2.5 nN

\[ o_i(t) = \sum_{n=1}^{N_{inj}} a_n \sin(2\pi f_n t) \theta(t - t'_n) \theta(t''_n - t) \]

- \( T < 3\)h (experiment duration): fixed
- \( N_{inj} = 7 \): fixed to meet the exp. duration
- \( \delta t = 1200 \) s (duration of each sine wave): fixed
- \( \delta t_{\text{gap}} = 150 \) s (gap between two sine waves): fixed for transitory decay
- \( f_n \) (injection freq.): \textit{optimized}
- \( a_n \) (injection ampl.): varied according to \( f_n \) and the (sensing/actuation) contraints

input signals: series of sine waves with \textit{discrete frequencies} (we require \textit{integer number of cycles})

26/03/2012
Giuseppe Congedo - PhD oral defense
Design of optimal experiments

Fisher information matrix

\[ J(\theta) = \int o_i(\omega, \theta)^* \nabla_p T_{o_i \rightarrow o}(\omega, p_{est})^* S_n(\omega)^{-1} \nabla_p T_{o_i \rightarrow o}(\omega, p_{est}) o_i(\omega, \theta) d\omega \]

input parameters (injection frequencies)

noise cross PSD matrix

Estimated system parameters

input signals being optimized

modeled transfer matrix with system identification

Perform (non-linear discrete) optimization of the scalar estimator

\[ \phi(\theta) = \begin{cases} 
\det(J(\theta)) \\
\min(\text{eig}(J(\theta))) \\
\text{tr}(J(\theta)) 
\end{cases} \]

optimization criteria

minimizes “covariance volume”
Design of optimal experiments

With the optimized design:

- Improvement in precision:
  - factor 2 for $\omega_1^2$ and $\omega_{12}^2$
  - factor 4 for $S_{21}$
  - factor 5-7 for $A_{sus}$, $A_{df}$: important for compensating the SC jitter

- Some parameters show lower correlation than the standard experiments

- The optimization converged to only two injection frequencies (0.83 mHz, 50 mHz)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard</th>
<th>Optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\omega_1^2}$ [s$^{-2}$]</td>
<td>$4 \times 10^{-10}$ {1.4}</td>
<td>$2 \times 10^{-10}$ {0.68}</td>
</tr>
<tr>
<td>$\sigma_{\omega_{12}^2}$ [s$^{-2}$]</td>
<td>$2 \times 10^{-10}$ {0.41}</td>
<td>$1 \times 10^{-10}$ {2.0}</td>
</tr>
<tr>
<td>$\sigma_{S_{21}}$</td>
<td>$4 \times 10^{-7}$ {0.086}</td>
<td>$1 \times 10^{-7}$ {1.1}</td>
</tr>
<tr>
<td>$\sigma_{A_{df}}$</td>
<td>$7 \times 10^{-4}$ {1.6}</td>
<td>$1 \times 10^{-4}$ {0.50}</td>
</tr>
<tr>
<td>$\sigma_{A_{sus}}$</td>
<td>$1 \times 10^{-5}$ {1.7}</td>
<td>$2 \times 10^{-6}$ {0.28}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Standard</th>
<th>Optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corr[$S_{21}$, $\omega_1^2$]</td>
<td>-0.2</td>
<td>-0.03</td>
</tr>
<tr>
<td>Corr[$S_{21}$, $\omega_{12}^2$]</td>
<td>0.09</td>
<td>0.02</td>
</tr>
<tr>
<td>Corr[$A_{sus}$, $\omega_1^2$]</td>
<td>-0.7</td>
<td>-0.7</td>
</tr>
<tr>
<td>Corr[$\omega_1^2$, $\omega_{12}^2$]</td>
<td>-0.5</td>
<td>-0.5</td>
</tr>
</tbody>
</table>
Concluding remarks

- Theoretical contribution to the foundations of spacetime metrology with the LPF differential accelerometer
- Modeling of the dynamics of the LISA arm implemented in LPF
- Development of the parameter estimation method employed for system calibration and subtraction of different effects
- Relevance of system identification for LPF
Future perspectives

- More investigation in the reformulation of the Doppler link as a differential accelerometer
- Application of system identification to the analysis of some cross-talk experiments
- Investigation of non-stationary (transient) components in the noise
- Application of the proposed optimal design to the ESA simulator
- Currently under investigation, the application of system identification in the domain of equivalent acceleration
Thanks for your attention!
Additional slides...
Monte Carlo validation

1000 noise realizations, true (7) parameters kept fixed
- all parameters are in good agreement with a Gaussian PDF, as well as correlations and variances
- parameters are unbiased to within 1-2 standard deviations
- expected (Fisher) errors are approximately in agreement with the noise fluctuation
- the method statistically suppresses the signals (see above)
Monte Carlo validation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True</th>
<th>Mean best-fit</th>
<th>Best-fit std. dev.</th>
<th>Mean exp. std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1^2 \times 10^{-6} s^{-2}$</td>
<td>-1.303</td>
<td>-1.303006(7)</td>
<td>2x10^{-4}</td>
<td>1x10^{-3}</td>
</tr>
<tr>
<td>$\omega_{12}^2 \times 10^{-6} s^{-2}$</td>
<td>-0.698</td>
<td>-0.697998(6)</td>
<td>2x10^{-4}</td>
<td>5x10^{-4}</td>
</tr>
<tr>
<td>$S_{21} [10^{-4}]$</td>
<td>0.9</td>
<td>0.90004(9)</td>
<td>3x10^{-3}</td>
<td>4x10^{-4}</td>
</tr>
<tr>
<td>$A_{df}$</td>
<td>1.003</td>
<td>1.00297(1)</td>
<td>4x10^{-4}</td>
<td>4x10^{-3}</td>
</tr>
<tr>
<td>$A_{sus}$</td>
<td>0.9999</td>
<td>0.9999001(1)</td>
<td>4x10^{-6}</td>
<td>2x10^{-5}</td>
</tr>
<tr>
<td>$\Delta t_1 [s]$</td>
<td>0.06</td>
<td>0.059995(3)</td>
<td>9x10^{-5}</td>
<td>3x10^{-4}</td>
</tr>
<tr>
<td>$\Delta t_{12} [s]$</td>
<td>0.05</td>
<td>0.05000(3)</td>
<td>8x10^{-4}</td>
<td>1x10^{-3}</td>
</tr>
</tbody>
</table>
Whitening

System identification relies upon a proper estimation of whitening filters
Design of optimal experiments

For almost the entire frequency band, the maximum amplitudes are limited by the interferometer sensing.
Identification in the acceleration domain

\[ \chi^2 = \int (f_{\text{exp}} - f_{\text{mdl}})^* S_{n,f}^{-1} (f_{\text{exp}} - f_{\text{mdl}}) \, d\omega \]

\[ = \int (o_{\text{exp}} - o_{\text{mdl}})^* \Delta^* (\Delta^* S_{n,o}^{-1} \Delta^{-1}) \Delta (o_{\text{exp}} - o_{\text{mdl}}) \, d\omega \]

\[ = \int (o_{\text{exp}} - o_{\text{mdl}})^* S_{n,o}^{-1} (o_{\text{exp}} - o_{\text{mdl}}) \, d\omega \]

**Comment:** formally equivalent to standard identification in displacement, but the transitory is mitigated in equivalent acceleration.

**Implementation:** closed-loop optimization where the parameters enter into the estimated acceleration
Transient analysis

- injected a (Gaussian) force signal of amplitude $1.6 \times 10^{-13}$ ms$^{-2}$ and duration $\sim 1$ h
- PSD estimation does not detect any (statistically significant) excess
- time-frequency wavelet spectrogram: a factor $>2$ noise excess
- can be applied to the search of transients in LPF noise, like “modified gravity” signals